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Emergence of a non-scaling degree distribution in bipartite networks: A numerical and analytical study

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Abstract – We study the growth of bipartite networks in which the number of nodes in one of the partitions is kept fixed while the other partition is allowed to grow. We study random and preferential attachment as well as combination of both. We derive the exact analytical expression for the degree-distribution of all these different types of attachments while assuming that edges are incorporated sequentially, *i.e.*, a single edge is added to the growing network in a time step. We also provide an approximate expression for the case when more than one edges are added in a time step. We show that depending on the relative weight between random and preferential attachments, the degree-distribution of this type of network falls into one of the four possible regimes, which range from a binomial distribution for pure random attachment to an u-shaped distribution for dominant preferential attachment.

A bipartite network is a graph that connects two distinct sets (or partitions) of nodes, which we will refer to as the *top* and the *bottom* sets. An edge in the network runs between a *top* and a *bottom* node but never between a pair of *top* or a pair of *bottom* nodes (see fig. 1). Typical examples of this type of networks include collaboration networks such as the movie-actor [1–6], article-author [7–11] and board-director [12,13] networks. In the movie-actor network, for instance, the movies and actors are the elements of the *top* and the *bottom* sets, respectively, and an edge between an actor a and a movie m indicates that a has acted in m. The actors a and a' are *collaborators* if both have participated in the same movie, *i.e.*, if both are connected to the same node m'. The concept of *collaboration* can be extended to include diverse phenomena represented by bipartite networks such as the city-people network [14], in which an edge between a person and a city indicates that the person has visited that particular city, the word-sentence [15,16], bank-company [17] or donor-acceptor networks that accounts for injection and merging of magnetic field lines [18].

Several models have been proposed to synthesize the structure of bipartite networks when both the partitions grow unboundedly over time [1–4,16], It has been found that for such growth models, when each incoming *top* node preferentially attaches itself to the *bottom* nodes, the emergent degree distribution of the *bottom* nodes follows a power-law [1]. Another important property of bipartite networks is that the clustering coefficient cannot be measured in the standard way [2], and has to be measured as a cycle of four connections [19].

On the other hand, bipartite networks, where one of the partitions remains fixed over time (*i.e.*, the number of *bottom* nodes are constant), have received comparatively much less attention. Recently it has been shown through numerical simulations that restrictions in the growth rate of the partitions can lead to non-scaling degree distributions highly sensitive to the parameters of the growth model [20]. However, there is still no systematic and analytical study of this kind of networks. Realizations of this type of bipartite networks include numerous relevant systems such as the relation between the codons and genes, as well as amino acids and proteins, in biology, and elements and compounds, in chemistry. We can also include in this group those networks in which one partition can be considered to be in a pseudo-steady state while the other one keeps on growing at a much faster rate. For instance, it is reasonable to assume that for the city-people network [14], the city growth rate is close to zero compared

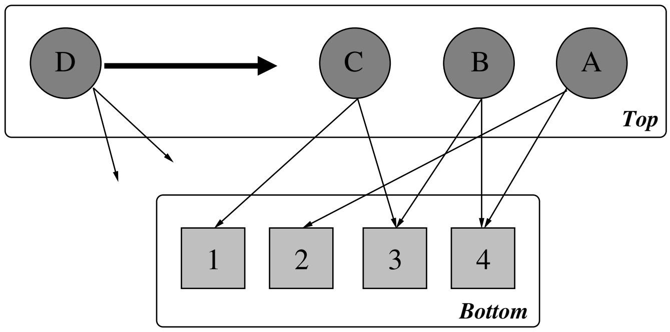

Fig. 1: Scheme of a growing bipartite network. In the example, the number of fixed *bottom* nodes is $N = 4$. Each *top* node arrives with $\mu = 2$ edges. The *top* node D represents a new incoming node.

with the population growth rate. Other examples of this type could be the phoneme-language network [21, 22], in linguistics, or train-station network [23,24] in logistics.

In this work we study the growth of bipartite networks in which the number of nodes in one of the partitions is kept fixed. We explore random and preferential attachment as well as a combination of both. We obtain an exact analytical expression for the degree distribution of the *bottom* nodes assuming that the attachment is sequential, *i.e.*, a single edge is added to the network in one time step. We also present an approximate solution for the case of parallel attachment, *i.e.*, when more than one edges are incorporated into the network in a given time step. We show that, depending on the relative weight of random to preferential attachment, the degree-distribution of this type of network falls into one of the four possible regimes, which range from a binomial distribution for pure random attachment to an u-shaped distribution for dominant preferential attachment. For combinations of random and preferential attachment the degree-distribution tends asymptotically with time to a beta-distribution.

The growth model. – We consider the case in which the *top* partition grows with time while the number of nodes in the *bottom* partition N is kept constant. We grow the network in the following way. At each time step a new node is added to the *top* set. Then, μ edges are connected from the new *top* node to the nodes in the *bottom* set (see fig. 1). The probability of attaching a new edge to the *bottom* node i is $\tilde{A}(k_i^t)$, where k_i^t refers to the degree of the *bottom* node i at time t. We refer to $\tilde{A}(k_i^t)$ as the attachment kernel and define it as

$$\tilde{A}(k_i^t) = \frac{\gamma k_i^t + 1}{\sum_{j=1}^{N} (\gamma k_j^t + 1)}, \qquad (1)$$

where the sum in the denominator runs over all *bottom* nodes, and γ is a model parameter which controls the relative weight of random to preferential attachment. γ can be thought of as $\gamma = 1/\alpha$, where α is a positive constant known in previous models as *initial attractiveness* [25].

There is a subtlety related to μ and the attachment kernel that is worth mentioning. The stochastic process can be performed in such a way that the attachment of the μ incoming nodes is done sequentially, *i.e.*, one edge is attached per time step. This implies that the denominator of $\tilde{A}(k_i^t)$ has to be updated for each incoming node (and hence an edge), and that t refers to the event of incorporating a new edge to the *bottom* set. Alternatively, the attachment of the μ new edges can be done in parallel. This implies that all the new μ edges have the same probability of attaching to *bottom* node i. In this case, t refers to the event of incorporating a new node to the *top* set.

There are two significant limits to consider: $\gamma = 0$ and $\gamma \to \infty$. For $\gamma = 0$, eq. (1) reduces to $\tilde{A}(k_i^t) = 1/N$, which implies that all *bottom* nodes have the same probability of being selected by an incoming edge. This limit corresponds to pure random attachment. For $\gamma \to \infty$ eq. (1) reduces to $\tilde{A}(k_i^t) = k_i^t / \Sigma_{j=1}^{N} (k_j^t)$, which means that higher degree *bottom* nodes have higher probability of being selected. This case corresponds to pure preferential attachment.

Stochastic simulations have been performed with the initial condition where all *bottom* nodes at time $t = 0$ have zero degree, *i.e.*, initially no edges are connected to the *bottom* nodes.

Evolution equation for sequential attachment. – Now we aim to derive an evolution equation for the degree distribution of the *bottom* nodes. We focus on sequential attachment. Let $p_{k,t}$ be the probability of finding a randomly chosen *bottom* node with degree k at time t. Recall that t refers to the t-edge attachment event. $p_{k,t}$ is defined as $p_{k,t} = \langle n_{k,t} \rangle / N$, where $n_{k,t}$ refers to the number of nodes in the *bottom* set with degree k at time t, and $\langle \ldots \rangle$ denotes ensemble average, *i.e.* average over realizations of the stochastic process described above. We express the evolution of $p_{k,t}$ in the following way:

$$p_{k,t+1} = (1 - A(k,t)) p_{k,t} + A(k-1,t) p_{k-1,t}\,, \qquad (2)$$

where $A(k,t)$ refers to the probability that the incoming edge lands on a *bottom* node of degree k. $A(k,t)$ can be easily derived from eq. (1) and takes the form

$$A(k,t) = \begin{cases} \frac{\gamma k + 1}{\gamma t + N} & \text{for } k \leqslant t, \\ 0 & \text{for } k > t, \end{cases} \qquad (3)$$

for $t > 0$ while for $t = 0$, $A(k,t) = (1/N)\delta_{k,0}$.

The reasoning behind eq. (2) is the following. The probability of finding a *bottom* node with degree k at time $t+1$ decreases due to the number of nodes, which have a degree k at time t and receive an edge at time $t+1$ therefore acquiring degree $k+1$, *i.e.*, $A(k,t)p_{k,t}$. Similarly, this probability increases due to the number of nodes that at time t have degree $k-1$ and receives an edge at time $t+1$ to have a degree k, *i.e.*, $A(k-1,t)p_{k-1,t}$. Hence, the net increase in the probability can be expressed as in eq. (2).

According to the stochastic simulations, we assume that at time $t=0$ all *bottom* nodes have zero degree, which implies that the initial condition $p_{k,t=0}=\delta_{k,0}$, where $\delta_{k,0}$ is the Kronecker delta function.

Exact solution for sequential attachment. – We express eq. (2) as

$$\mathbf{p}_{t+1}=\mathbf{M}_t\mathbf{p}_t=\left[\prod_{\tau=0}^{t}\mathbf{M}_\tau\right]\mathbf{p}_0\,, \tag{4}$$

where $\mathbf{p}_t$ denotes the degree distribution at time t and is defined as $\mathbf{p}_t=[p_{0,t}\ p_{1,t}\ p_{2,t}\ldots]^T$, $\mathbf{p}_0$ is the initial condition expressed as $\mathbf{p}_0=[1\,0\,0\,\ldots]^T$, and $\mathbf{M}_\tau$ is the evolution matrix at time τ which is defined as

$$\mathbf{M}_\tau=\begin{pmatrix}1-A(0,\tau) & 0 & 0 & 0\ldots\\ A(0,\tau) & 1-A(1,\tau) & 0 & 0\ldots\\ 0 & A(1,\tau) & 1-A(2,\tau) & 0\ldots\\ \vdots & \vdots & \vdots & \vdots\ \ddots\end{pmatrix}. \tag{5}$$

Since our initial condition is a vector with zeros in all positions except in the first one, all the relevant information, *i.e.*, the degree distribution of the *bottom* nodes, is in the first column of $\left[\prod_{\tau=0}^{t}\mathbf{M}_\tau\right]$. A close inspection of the evolution of this column reveals that the k-th element of it, which corresponds to $p_{k,t}$, can be expressed as

$$p_{k,t}=\binom{t}{k}\frac{\prod_{i=0}^{k-1}(\gamma i+1)\prod_{j=0}^{t-1-k}(N-1+\gamma j)}{\prod_{m=0}^{t-1}(\gamma m+N)}\,, \tag{6}$$

for $k\leqslant t$, and $p_{k,t}=0$ for $k>t$, and where we have defined $\prod_{i=0}^{-1}(\ldots)=1$, and $\binom{t}{k}$ refers to the combinatorial number $t!/[(t-k)!k!]$.

Equation (6) is the exact solution of eq. (2) for the initial condition $p_{k,t=0}=\delta_{k,0}$. Therefore, this is the analytical expression of the degree distribution of the *bottom* nodes when sequential attachment is applied.

In the limit of $\gamma=0$ eq. (6) reduces to

$$p_{k,t}=\binom{t}{k}\left(\frac{1}{N}\right)^k\left(1-\frac{1}{N}\right)^{t-k}, \tag{7}$$

for $k\leqslant t$, and $p_{k,t}=0$ for $k>t$. Equation (7) is the solution of the sequential problem when pure random attachment is applied. For pure preferential attachment, *i.e.*, in the limit of $\gamma\to\infty$, the degree distribution takes the form

$$p_{k,t}=(1-1/N)\delta_{k,0}+1/N\delta_{k,t}\,. \tag{8}$$

Parallel attachment. – In this section, we focus on parallel attachment, *i.e.*, when more than one edge are added per time step. We do not aim to derive an exact analytical expression for the degree distribution of this problem but provide a reasonable approximation. Recall that for parallel attachment t refers to the event of incorporating a new *top* node. We assume that $\mu\ll N$ and expect eq. (2) to be a good approximation of the process after replacing $A(k,t)$ with $A_p(k,t)$. We define $A_p(k,t)$ as

$$A_p(k,t)=\begin{cases}\frac{(\gamma k+1)\mu}{\gamma\mu t+N} & \text{for } k\leqslant\mu t,\\ 0 & \text{for } k>\mu t,\end{cases} \tag{9}$$

for $t>0$ while for $t=0$, $A_p(k,t)=(\mu/N)\delta_{k,0}$. The term μ appears in the denominator of eq. (9) for $k\leqslant\mu t$ because in this case the total degree of the *bottom* nodes at any point in time is μt rather than t as in eq. (3). The numerator contains a μ because at each time step there are μ edges that are being incorporated into the network rather than a single edge.

It is important to mention here that eq. (2) cannot exactly represent the stochastic parallel attachment because it explicitly assumes that in one time step a node of degree k can only get converted to a node of degree $k+1$. Clearly, the incorporation of μ edges in parallel allows the possibility for a node of degree k to get converted to a node of degree $k+\mu$. The correct expression for the evolution of $p_{k,t}$ reads

$$p_{k,t+1}=\left(1-\sum_{i=1}^{\mu}\widehat{A}(k,i,t)\right)p_{k,t}+\sum_{i=1}^{\mu}\widehat{A}(k-i,i,t)p_{k-i,t}\,, \tag{10}$$

where $\widehat{A}(k,i,t)$ represents the probability at time t of a node of degree k of receiving i new edges in the next time step. We expect eq. (2) to be a good approximation of eq. (10) when $\widehat{A}(k,1,t)\gg\widehat{A}(k,i,t)$, where $i>1$. The solution of eq. (2) with the attachment kernel given by eq. (9) reads

$$p_{k,t}=\binom{t}{k}\frac{\prod_{i=0}^{k-1}(\gamma i+1)\prod_{j=0}^{t-1-k}\left(\frac{N}{\mu}-1+\gamma j\right)}{\prod_{m=0}^{t-1}\left(\gamma m+\frac{N}{\mu}\right)}\,. \tag{11}$$

We expect eq. (11) to approximate the degree distribution of the stochastic process with parallel attachment for $\mu\ll N$. This means that we cannot expect the approximation to hold for large values of γ or μ/N.

In the limit of random attachment, *i.e.*, $\gamma=0$, eq. (11) becomes $p_{k,t}=\binom{t}{k}(\frac{\mu}{N})^k(1-\frac{\mu}{N})^{t-k}$.

Figures 2(a)-(c) and 3(a)-(b) show a comparison between stochastic simulations and eq. (11), which reveal that eq. (11) is a good approximation for $\mu\ll N$ and relatively low values of γ. For large values of γ, as mentioned above, the approximation fails. However, for $\mu=1$, eq. (11) reduces to eq. (6), which in this case is the exact solution and the theory works for all values of γ (see figs. 2(d) and 3(c)).

From random to preferential attachment. – Figure 2 shows that there is a clear transition from random to preferential attachment. At $\gamma=0$ (see fig. 2(a)) we observe that $p_{k,t}$ is centered around the maximum

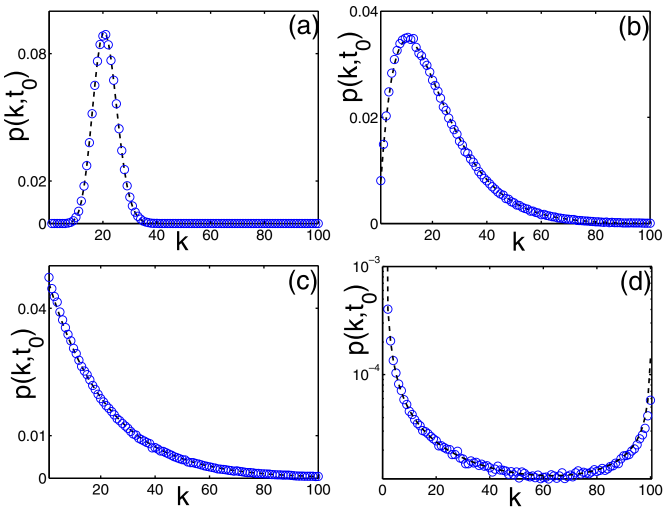

Fig. 2: (Colour on-line) The four possible degree distributions depending on γ. Symbols represent average over 5000, in (a)-(c), and 50000, in (d), stochastic simulations. The dashed curve is the theory given by eq. (11). From (a) to (c), $t_0 = 1000$, $N = 1000$ and $\mu = 20$. (a) at $\gamma = 0$, $p(k,t)$ becomes a binomial distribution. (b) $\gamma = 0.5$, the distribution exhibits a maximum which shifts with time for $0 \leqslant \gamma < 1$. (c) $\gamma = 1$, $p(k,t)$ does no longer exhibit a shifting maximum and the distribution is a monotonically decreasing function of k for $1 \leqslant \gamma \leqslant (N/\mu) - 1$. (d) $\gamma = 2500$, $t_0 = 100$, $N = 1000$ and $\mu = 1$. $p(k,t)$ becomes an u-shaped curve for $\gamma > (N/\mu) - 1$.

(mode of the distribution) which shifts with time at a speed of μ/N per time step, while the width of the distribution also spreads with time. This behavior corresponds to a situation in which all the *bottom* nodes receive roughly the same number of edges with time. The well-defined maximum tells us about the average number of edges each *bottom* node has, while the variance of the distribution indicates the presence of fluctuations around that mean value which increases with time.

For $0 < \gamma < 1$ the distribution is no longer symmetric (see fig. 2(b)). *Bottom* nodes having small degree rarely receive an edge, and so $p_{k,t}$ decays slowly for small value of k. However, the distribution still exhibits a maximum —the mode of the distribution— which shifts with time (see fig. 3(a)).

For $1 \leqslant \gamma \leqslant (N/\mu) - 1$ the distribution loses the (local) maximum and is monotonically decreasing (see fig. 2(c)). We can always find a *bottom* node with small-degree because small-degree nodes hardly get an edge. On the other hand, there are very few nodes with high degree, and these ones receive almost all the incoming edges. The temporal evolution of the distribution for this range of γ is shown in fig. 3(b).

For $\gamma > (N/\mu) - 1$ the distribution described by eq. (11) exhibits a u-shape. As mentioned above, we cannot expect eq. (11) to approximate the stochastic process for such large values of γ. Stochastic simulations performed in this range of γ for $\mu > 1$ are very noisy and the u-shape

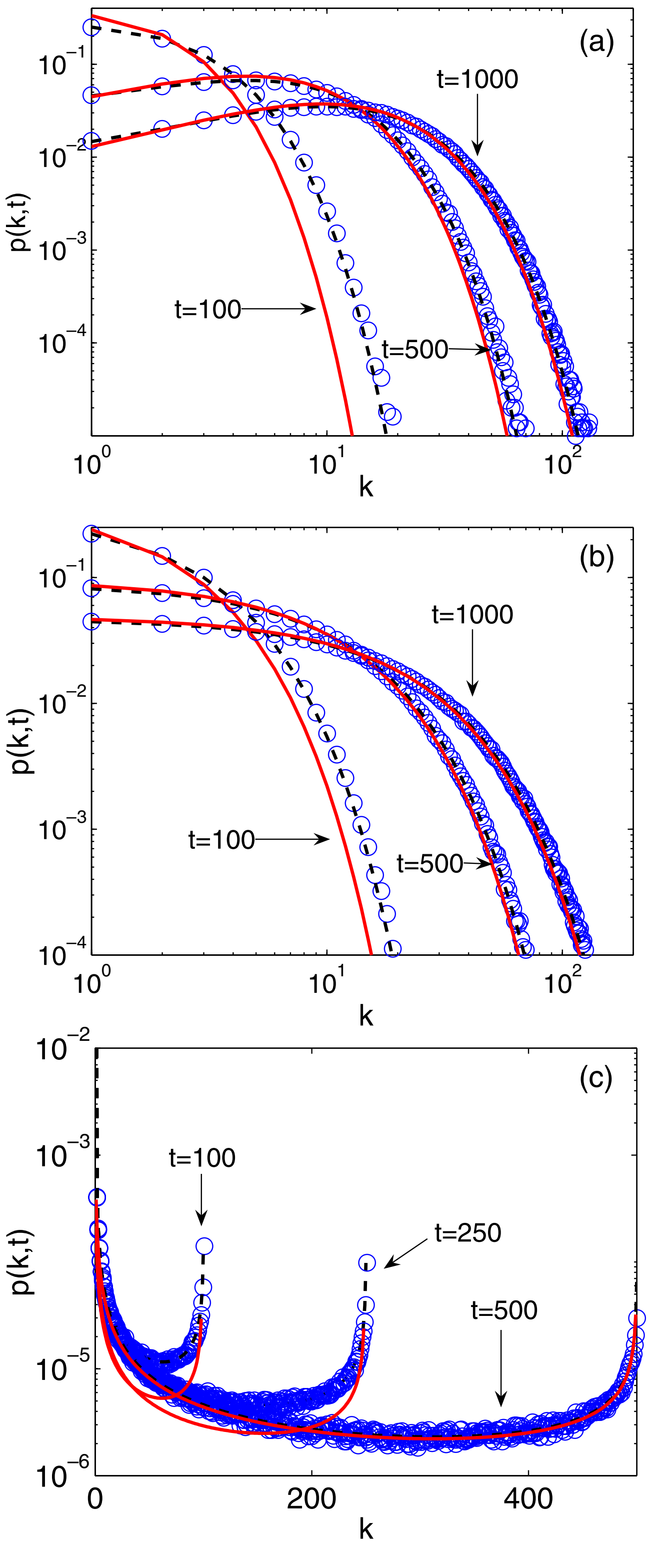

Fig. 3: (Colour on-line) Temporal evolution of $p(k,t)$ for various values of γ. Symbols represent average over 5000, in (a)-(b), and 50000, in (c), stochastic simulations. The black dashed curve is the theory, represented through eq. (11). The red solid curves correspond to the approximation given by the beta-distribution, eq. (12). (a) $\gamma = 0.5$, and (b) $\gamma = 1$. $N = 1000$ and $\mu = 20$. Compare with fig. 2(b) and (c). (c) $\gamma = 2500$, $N = 1000$ and $\mu = 1$, see fig. 2(d).

cannot be obtained by averaging over few simulations. However, for $\mu = 1$ eq. (6) is the exact solution and the u-shaped distribution emerges in a clear way, as shown in fig. 2(d). As $\gamma \to \infty$ an effect similar to "winner takes all" is observed. The node that gets the first incoming edge becomes dominant and attracts the majority of the subsequent incoming edges. The degree distribution in this case exhibits two peaks, one located at $k = 0$ and the other one at $k = t$ (see figs. 2(d) and 3(c)). The existence of the u-shaped distribution indicates that there is one node which can compete against all the other nodes in getting the new incoming edge; *i.e.*, the probability for this node of getting the incoming edge (let us label the dominant node 1, and refer to this probability as P_1) is greater than the sum of the probabilities of all the other nodes (let us refer to this probability as P_r). To get a crude understanding of the emergence of the u-shaped distribution for large values of γ, we have to find out under which circumstances $P_1 > P_r$. For this purpose, let us focus on the growth model and particularly on eq. (1). If at $t = 0$ all *bottom* nodes have degree 0, then $\tilde{A}(k_i^0) = 1/N$, where $1 \leqslant i \leqslant N$. Let us assume that in the next step the *bottom* node which gets the edge is the one we label as 1. The probability for node 1 getting an edge in the next step is $\tilde{A}(k_1^0) = (\gamma + 1)/(\gamma + N)$ (and P_1 is simply $P_1 = \tilde{A}(k_1^0)$), while for the rest of the nodes that probability is $\tilde{A}(k_i^0) = 1/(\gamma + N)$, where $i > 1$ (and so, P_r is $P_r = (N-1)/(\gamma + N)$). The condition we are looking for, translates simply into $\gamma > N - 2$ (below we show this simplified reasoning offers a slightly underestimated value of this critical γ). Assuming that subsequent incoming edges fall on node 1, it holds for the next time steps that $P_1 > P_r$. However it is important to notice that eventually, for any finite γ, edges will land also on the other nodes and because of this, the distribution is smooth and the dominance of node 1 decays with time. For $\gamma \to \infty$ this "dispersion" effect vanishes and the distribution takes the form given by eq. (8).

Beta-distribution. – In this section we offer a quantitative analysis of the above-mentioned transitions by showing that for $\gamma > 0$, $p_{k,t}$ approaches a beta-distribution asymptotically with time.

For $t \gg \eta$, where $\eta = N/(\gamma\mu)$, we can approximate the products in eq. (11) by gamma-functions and apply Stirling's approximation. After some algebra we obtain

$$p_{k,t} \simeq C^{-1} \, (k/t)^{\gamma^{-1}-1} \, (1 - k/t)^{\eta - \gamma^{-1} - 1} , \qquad (12)$$

where C is a normalization constant defined by $C = \int_0^t (k'/t)^{\gamma^{-1}-1} (1 - k'/t)^{\eta-\gamma^{-1}-1} \mathrm{d}k'$.

Figure 3 shows a comparison between stochastic simulations (circles), the theoretical solution given by eq. (11) (black dashed curve), and the approximation given by eq. (12) (red solid curve) for three different values of γ at various times. It can be observed that $p_{k,t}$ approaches asymptotically to eq. (12) (compare the black dashed and the red solid curves). Notice that eq. (12) does not have any fitting parameter and represents a beta-distribution $f(x; \alpha, \beta)$ of the variable $x = k/t$ and fixed parameters $\alpha = \gamma^{-1}$ and $\beta = \eta - \gamma^{-1}$.

For $0 < \gamma < 1$, $\alpha > 1$ and $\beta > 1$ the mode of the distribution is given by $(\alpha - 1)/(\alpha + \beta - 2) = ((\gamma^{-1}) - 1)/(\eta - 2)$. This can be easily verified by taking the first derivative of eq. (12) equal to zero. From this we learn that in this range of γ the maximum of the distribution k_{max} is located at $k_{max} = t((\gamma^{-1}) - 1)/(\eta - 2)$ (see fig. 3(a)). In the limit of $\gamma \to 0$ we retrieve the behavior of k_{max} observed for pure random attachment, *i.e.*, $k_{max} = t(\mu/N)$.

At $\gamma = 1$, $\alpha = 1$ and $\beta > 0$, the moving peak is no longer found, *i.e.*, the mode of the distribution is always located at $k_{max} = 0$ (see fig. 3(b)). This condition also holds for $1 < \gamma \leqslant (N/\mu) - 1$.

For $\gamma > (N/\mu) - 1$, there is a different form of the degree distribution that emerges. For $\alpha < 1$ and $\beta < 1$, $p_{k,t}$ becomes u-shaped with a peak fixed at $k = 0$ and the other one shifting with t. For $\mu = 1$ the additional peak is located at $k = t$ (see fig. 3(c)).

Concluding remarks. – We have studied the growth of bipartite networks in which the number of nodes in the *bottom* set is kept fixed. We have considered random and preferential node attachment as well as a combination of both. We have derived the degree distribution evolution equation for sequential and parallel attachment of nodes. For sequential attachment we have provided the exact analytical solution of the problem. For parallel attachment we have obtained an approximate expression of the degree distribution, and through simulations we have provided numerical evidence which shows that this approximation is reasonable when $\mu \ll N$ and γ is small.

Finally, we have shown that for both, sequential and parallel attachment, the degree-distribution falls into one of the four possible regimes: a) $\gamma = 0$, a binomial distribution whose mode shifts with time, b) $0 < \gamma < 1$, a skewed distribution which exhibits a mode that shifts with time, c) $1 \leqslant \gamma \leqslant (N/\mu) - 1$, a monotonically decreasing distribution with the mode frozen at $k = 0$, and d) $\gamma > (N/\mu) - 1$, an u-shaped distribution with peaks at $k = 0$ and $k = t$.

Our results might be useful in explaining the dynamical growth of various systems, as for example, the speech sound inventories of the world's languages, which can be rendered a bipartite structure as explained through the phoneme-language network in [21,22]. A detailed study of the parameter γ, which controls the degree distribution of the network, can then shed some light on the amount of randomness/preference that has gone into shaping the evolution of such systems.

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