

HOMOTOPICAL DAYS/ JOURNEES HOMOTOPIQUES

Rencontre dédiée à Clemens Berger, pour ses 60 ans

Laboratoire J.-A. Dieudonné, Université Côte d'Azur

Du 7 au 9 Décembre 2022

Titles and abstracts

Michael Batanin

Title: Grothendieck homotopy theory for polynomial monads.

Abstract. Grothendieck homotopy theory of small categories can be considered as a theory of ∞ -groupoids but it has a strong combinatorial flavour very useful in numerous applications. The goal of my talk is to show that many fundamental constructions of Grothendieck homotopy theory can be extended from small categories and presheaves over them to finitary polynomial monads and their algebras. This includes Grothendieck construction and Quillen theorem A, algebraic homotopy Kan extensions and the theory of homotopy cofinal morphisms.

Dominique Bourn, Université du Littoral, Calais

Title: On the concept of Algebraic Crystallography

Abstract. Category Theory provides us with a clear notion of what is an internal structure. This will allow us to focus our attention on a certain type of relationship between context and structure. With the notion of unital category, we have to face a context in which on an object X , there is at most one structure of internal monoid, and with the notion of Mal'tsev category to a context in which on an object X , there is at most one Mal'tsev operation (i.e. ternary operation $p : X \times X \times X \rightarrow X$ satisfying the Mal'tsev identities $p(x, y, y) = x = p(y, y, x)$). From these observations (and many others which will be recalled in the talk) we shall introduce the following type of relationship:

Definition 0.0.1. A finitely complete category E is said to be crystallographic with respect to a given algebraic structure S when, on any object X in E , there is at most one internal algebraic structure of this kind.

This terminology is chosen because it emphasizes that, in such a setting, the algebraic structure in question is, punctually, growing so scarce.

The aim of this talk will be to produce examples and to establish the very first properties and general questionings about this notion.

This will lead to two unexpected applications :

1) with an Eckmann-Hilton flavour, the fact that the fundamental group of some very simple topological structures is trivial;

2) a rather spectacular outcome with an example of a variety H which is crystallographic for the structure of abelian group and in which the punctual scarcity of abelian group objects seems to be offset by a kind of multiplication of this structure inside it. Indeed, the category AbH of abelian objects in this variety H will appear :

i) to fully faithfully embed the category Ab of abelian groups by a functor $h : Ab \rightarrow AbH$, and in an independent way

ii) to faithfully contain any category $K\text{-Vect}$ of K -vector spaces, provided that the field K is not of characteristic 2, by a functor $w_K : K\text{-Vect} \rightarrow AbH$.

In this way, the abelian group $(V, +)$ underlying a K -vector space V will be represented by two distinct objects $h(V, +)$ and $w_K(V, +)$ in the abelian category AbH .

We shall give an example of the same kind of multiplication process in a non-pointed context.

Carles Casacuberta, Universitat de Barcelona

Title: Localizations of models of theories with arities

Abstract: We give a necessary and sufficient condition for the existence of liftings of enriched localizations and colocalizations on a bicomplete closed symmetric monoidal category V to models of algebraic theories enriched in V with arbitrary arities. This condition is automatically fulfilled for single-sorted finite-product theories if V is additive and generated by multiples of the monoidal unit. This is joint work with Aina Ferrà.

Benoit Fresse

Title: "Cochain models of operads. The realization problem for E_n -operads"

Abstract: In the first part of my talk, I will report on the result of a recent research with Lorenzo Guerra about the definition of cochain models for the p -complete homotopy theory of operads. This model is based on my joint work with Clemens Berger "Combinatorial operad actions on cochains". Then I will explain the definition of graph complex operads in this model, and the relationship between this construction of graph operads and the problem of studying the topological realizations of the n -Poisson operads in p -complete homotopy theory (the realization problem for E_n -operads).

Kathryn Hess

Title: The universal Hochschild shadow from bicategories to $(\infty, 2)$ -categories

Abstract: (joint with Nima Rasekh)

The theory of shadows, first introduced by Ponto, is an axiomatic, bicategorical framework that generalizes (topological) Hochschild homology and satisfies analogous important properties, such as Morita invariance. I'll explain how to generalize Berman's extension of Hochschild homology to bicategories in order to prove that there is an equivalence between functors out of the Hochschild homology of a bicategory and shadows on that

bicategory, from which it follows that Hochschild homology of bicategories actually provides a universal shadow on bicategories and which enables us to formulate Morita invariance functorially. I'll then describe the infinity categorical generalization of this story, parts of which are still conjectural.

Ralph Kaufmann

Title: Graphs and moduli spaces from a Feynman categorical perspective

Abstract: Feynman categories are a powerful and convenient tool to study various questions in algebra, geometry and physics.

In this talk, we will give recent results obtained with C. Berger on these subjects.

The first is combinatorial. We establish that the Borisov-Manin category of graphs is a Feynman category and together with its full Feynman subcategory of aggregates forms a category internal to Feynman categories.

The second ties together cyclic, non-sigma cyclic, non-genus modular, modular, non-sigma modular operads and their non-connected versions via their corepresenting Feynman categories and push-forwards of functors.

Some of these results reestablish the calculations for the push-forward (modular envelope) of the cyclic associative operad obtained by Costello, K-Penner, Markl, Lazarev-Chuang and the authors.

The new results are an extension to the non-connected case, a tower of intermediate coverings and the calculation via a Kan extension. The latter allows us to precisely connect the calculation to Igusa's categories of ribbon graph contractions.

The third result is about 2d TFTs. As a consequence of the adjunction of push-forward and pull-back there is an adjunction of algebras. In one form this recovers the familiar statement that a closed 2d TFT is given by a commutative Frobenius algebra and an open 2d TFT by a Frobenius algebra. We make the setting precise here, since we do not use cobordisms. One of the new features is that we disentangle units, dualization and composition, giving very precise statements in all cases. This is made possible by a new general definition of algebras over functors from Feynman categories.

Combining the first and the third statement, allows us to give the correlation functions underlying algebraic string topology from first principles via pull-backs. This includes the Chas Sullivan product, bracket, as well as the Goresky-Hingston coproduct and many other operations. These now follow purely from adjunctions.

If time permits, we will comment on how combining these results with the W -construction, which exists for all cubical Feynman categories, realizes moduli spaces and their spines are naturally as cubical complexes.

Tom Leinster

Title: Magnitude homology

Abstract: Magnitude is a numerical invariant of enriched categories. It unifies topological Euler characteristic, groupoid cardinality, and invariants of geometric measure such as volume, surface area and fractal dimension. It can also be categorified: there is a graded homology theory of enriched categories called magnitude homology, whose Euler characteristic is (sometimes) magnitude. Thus, magnitude homology categorifies

magnitude in the same sense that Khovanov homology categorifies the Jones polynomial.

It is particularly interesting in the case of metric spaces. For example, while topological homology detects the existence of holes, magnitude homology detects how big the holes are. I will give an overview, mentioning the original magnitude homology for graphs introduced by Hepworth and Willerton, its generalization to enriched categories by Shulman and myself, and the many results in metric geometry found by a host of others.

Muriel Livernet

Title: Homotopy theory of spectral sequences

In this talk we will present different notions of weak equivalences on spectral sequences, their relations and their connection with model structures on the category of filtered complexes and multicomplexes. This is a joint work with Sarah Whitehouse.

Martin Markl

Title: Operads and the blob complex (joint work with M. Batanin)

Abstract: I will show that the Morrison-Walker blob complex appearing in Topological Quantum Field Theory is an operadic bar resolution of a certain operad composed of fields and local relations. My talk will be based on the draft "Operads, operadic categories and the blob complex" available at <https://users.math.cas.cz/~markl/>.

Paul-André Mellies, CNRS, Université Paris Cité, INRIA

Title: Template games and the Gray tensor product of 2-categories

Abstract: In this talk, I will describe an unexpected connection between template game models of linear logic, and the Gray tensor product of 2-categories. I will start by defining template games as specific 2-categories with positions as 0-cells, trajectories as 1-cells and reshufflings as 2-cells. As we will see, one technical difficulty in the construction of the game model is that the category $S = 2\text{-Cat}$ of small 2-categories equipped with the Gray tensor product is monoidal, and not cartesian. This prompts us to extend the original framework of template games based on spans and formulated in a category S with finite limits, to a more general framework based on bicomodules and formulated in a monoidal category S with coreflexive equalizers, preserved by the tensor product componentwise. We obtain in this way a monoidal bicategory of template games, where every multiplicative additive formula A of linear logic is interpreted as a template game, defined as 2-category equipped with a Gray comonoid structure, and every proof $A \multimap B$ is interpreted as a strategy between A and B , defined as a Gray A - B -bicomodule.

Ieke Moerdijk

Title: The complete graph operad

Abstract: The "complete graph operad" was introduced by Clemens Berger quite a while ago, and was soon followed by the introduction of a variant by Brun, Fiedorowicz and Vogt. However, the relation of these operads to each other and to other well-known operads left something to be clarified. In this talk, I will show that these operads are equivalent, and in fact play a central role among the family of E_n -operads. (The talk is based on joint work with Andre Beuckelmann.)

Bruno Vallette, Université Sorbonne Paris Nord

Title : Deformation theory of Cohomological Field Theories

Abstract : In this talk, I will develop the deformation theory of morphisms of modular operads with focus on the special case of Cohomological Field Theories (CohFTs), that is algebras over the moduli spaces of stable curves with marked points. This will lead to two new natural extensions of the notion of a CohFT: homotopical (necessary to structure chain-level Gromov--Witten invariants) and quantum (with examples found in the works of Buryak--Rossi on integrable systems). I will introduce a new version of Kontsevich's graph complex, enriched with tautological classes, which will lead to a new universal deformation group acting on the moduli spaces of quantum homotopy CohFTs. This group of symmetries will be shown to contain both the pronipotent Grothendieck--Teichmüller group and the Givental group. (Joint work with Vladimir Dotsenko, Sergey Shadrin, Arkady Vaintrob available at arxiv.org/abs/2006.01649.)