Large solutions for biharmonic maps in four dimensions.

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We investigate the existence of large solutions for biharmonic maps from a 4-dimensional Euclidean domain Ω into S^4.
Introducing the notion of topological degree for Sobolev maps from $\mathbb{R}^4$ to S^4, we show that there exists locally minimizing extrinsic biharmonic maps u^* of topological degree -1 and 1. The proof is based upon P.L. Lions' *concentration compactness* principle. This allows us to exclude the phenomena of concentration and vanishing at infinity, for minimizinig sequences for the Hessian energy with prescribed topological degree -1 or 1, up to rescalings and translations. We infer that the degree is preserved in the limit.
Then, for $\Omega = B_1$ unit ball in $\mathbb{R}^4$, we show the existence of two non homotopic biharmonic maps for certain Dirichlet boundary data. The key step is a "*sphere attaching lemma*" stating the existence of a map u, non homotopic to the absolute minimizer $\underline{u}$ of the Dirichlet problem, having less energy than the sum of the energies of $\underline{u}$ and u^*. Thus, we can exclude bubbling of minimizing sequences in the considered homotopy class in order to conclude compactness.