EURO-MAGHREB CONFERENCE IN MATHEMATICS

LEVICO TERME, OCTOBER 2-6 2023

MONDAY, OCTOBER 2

Amandine Véber - MAP5, CNRS and Paris Cité University.

A multitype growth-fragmentation process to model the growth of a filamentous fungus.

Abstract. Filamentous fungi form a large family of species playing an important role in the functioning of many ecosystems. They develop in space thanks to the growth and multiplication of filaments (also called hyphae) which allow the absorption and sharing of nutrients and other molecules. In this talk, we shall present a toy model for the development of a hyphal network, whose main aim is to identify a small number of key parameters describing the growth of the fungus in homogeneous conditions (in particular, in lab conditions) and to understand and quantify the impact of different forms of stress on this growth. The results presented are joint work with Vincent Bansaye (Ecole Polytechnique), Lena Kuwata (Univ. Paris Cité) and Milica Tomasevic (CNRS and Ecole Polytechnique) on the maths side, and Cécilia Bobée, Florence Chapeland-Leclerc, Thibault Chassereau, Pascal David, Eric Herbert, Christophe Lalanne, Clara Ledoux, Gwenaël Ruprich-Robert, all at LIED (Univ. Paris Cité) on the biology and physics side.

Ali Moussaoui-University of Tlemcen, Department of Mathematics, Algeria.

Mathematics for Sustainable Development: Bio Economic Models of Fisheries.

Abstract. This is a work based on [1]-[2]. We introduce two mathematical models designed for the management of fisheries. In the initial part, we present a mathematical model for managing artificial pelagic multisite fisheries. This model operates as a dynamic system, considering the interaction between fish stocks and fishing efforts within a fishery that is divided into artificial fishing sites, such as fish-aggregating devices (FADs) or artificial habitats (AHs). The aim of this study is to explore how the number of fishing sites impacts the overall activity of the fishery. Our findings reveal the presence of an optimal number of fishing sites that maximizes the total catch at equilibrium. In the second part of our presentation, we propose a bio-economic model for fisheries, which encompasses the fluctuations in fish stock, fishing effort, and the resource's market price, assumed to be influenced by supply and demand dynamics. In certain circumstances, we can observe a steady-state in fisheries management that we refer to as a "catastrophic" equilibrium, characterized by fish extinction and a sharp increase in prices. We are looking for the conditions that allow to avoid the extinction of the fished species and the establishment of a sustainable fishery.

References.

- [1] A. Moussaoui, P.Auger, C. Lett: Optimal number of sites in multi-site fisheries with fish stock dependent migrations. *Mathematical Biosciences and Engineering*, 8: 769–783, 2011.
- [2] A.Moussaoui, A. Ducrot, A. Moulai-Khatir, P. Auger, A model of a fishery with fish storage and variable price involving delay equations, *Mathematical Biosciences*, 362, 2023, 109022.

María Ángeles Rodriguez-Bellido - Dpto. Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla, Spain.

Results on optimal control problems associated to chemotaxis models.

Abstract. Chemotaxis models try to reproduce the biological process in which the spatial transport of the density of a living organism is modified by the presence of a chemical substance. Production and/or consumption of chemical by cells, degradation of chemical or logistic reaction for living organisms are other interactions that can also be taken into account in the mathematical modeling. Introducing a control over the system can be suitable for several purposes. The optimal control problem and the obtention of first-order optimality conditions for local optimal solutions are also some interesting subjects to be addressed in this talk.

This is a work based on [1], [2], [3] and [4] (in collaboration with P. Braz e Silva, F. Guillén-González, E. Mallea-Zepeda, C. F. Perusato and E.J. Villamizar-Roa).

References.

- [1] Bellomo, N.; Bellouquid, A.; Tao, Y.; Winkler, M. Toward a mathematical theory of Keller-Segel models of pattern formation in biological tissues. *Math. Models Methods Appl. Sci.* **25**, no. 9, 1663–1763, 2015.
- [2] Braz e Silva, P., Guillén-González, F., Perusato, C. F., Rodríguez-Bellido, M.A. Bilinear optimal control for weak solutions of the Keller-Segel logistic model in 2*D* domains. *Appl. Math. Optim.* **87**, no. 3: Paper No. 55, 20 pp., 2023. DOI: 10.1007/s00245-023-09988-y
- [3] Guillén-González, F., Mallea-Zepeda, E., Rodríguez-Bellido, M.A. Optimal bilinear control problem related to a chemo-repulsion system in 2D domains. *ESAIM Control Optim. Calc. Var.* **26**, Paper No. 29, 21 pp., 2020. DOI:10.1051/cocv/2019012.
- [4] Guillén-González, F.; Mallea-Zepeda, E.; Rodríguez-Bellido, M.A. A Regularity Criterion for a 3D Chemo-Repulsion System and Its Application to a Bilinear Optimal Control Problem. *SIAM J. Control and Optimization*, **58**, no. 3, 1457-1490, 2020.

Fatma Zohra-Nouri - Mathematical Modeling and Numerical Simulation Research Laboratory, Badji Mokhtar University-Annaba, Algeria.

A Study of a Problem involving a Fluid in Motion.

Abstract. This work consists of carrying out a mathematical and numerical study of a spectral method based on Legendre polynomials using the Galerkin approach for solving a problem of the mechanics and dynamics of a fluid with high variable density. This real problem comes from a phenomenon related to the environment, where the ocean is in motion by atmospheric pressure. This is a work based on [1, 2, 3,4].

References.

- [1] C.Canuto, M. Y. Hussaini, A.Quarteroni and T.A. Zang. Spectral Methods. *Springer Berlin, Heidelberg*, 2006.
- [2] N. Djedaidi, S. Gasmi and F.Z. Nouri. Interface dynamics for a bi-phasic problem in heterogeneous porous media. *Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications & Algorithms* 30: 21-33, 2023.
- [3] F.Z. Nouri. A New Approach for a Multiphase Flow Problem. *J. Applied Maths and computation* Volume 30, Issue Number 3: 32-42, 2019.
- [4] Roger K. Smith. Introductory lectures on fluid dynamics. *Fluid Dynamics*, Version: June 13, 2008.

Marwa Kchaou -ESPRIT-School of Engineering, Tunisie.

Homogenization theory for the derivation of second order macroscopic models of diffusion MRI in permeable and impermeable media.

Abstract. Starting from a reference partial differential equation model of the complex transverse water proton magnetization in a voxel due to diffusion-encoding magnetic field gradient pulses, one can use periodic homogenization theory to establish macroscopic models. A previous work introduced asymptotic models that accounted for impermeable and permeable interfaces in the imaging medium. So, we formulate higher order asymptotic models to treat the case of impermeable and permeable media. We explicitly solved these new asymptotic models to obtain a system of ordinary differential equations that can model the diffusion MRI signal and we present numerical results showing the improved accuracy of the new models in the regime of lower and higher permeability.

This is a work based on [ref1:Marwa Kchaou, Houssem Haddar and Maher Moakher, The derivation of homogenized diffusion kurtosis models for diffusion MRI, Journal of Magnetic Resonance 298 (2019), 48-57.] and [ref2: M. Kchaou and Jing-Rebecca Li, A second order asymptotic model for diffusion MRI in permeable media, ESAIM: M2AN, 4(2023), 1953-1980.].

References.

- [1] Marwa Kchaou, Houssem Haddar and Maher Moakher. The derivation of homogenized diffusion kurtosis models for diffusion MRI.*Journal of Magnetic Resonance*, 298: 48-57, 2019.
- [2]Marwa Kchaou and Jing-Rebecca Li. A second order asymptotic model for diffusion MRI in permeable media.*ESAIM: M2AN*, 4: 1953-1980, 2023.

Ahlem Abdelouahab-University of Tlemcen Algeria.

Analysis of a multiphase free boundary problem for a tumor growth.

Abstract. We study the existence of solutions of the following free boundary problem

$$\begin{cases} \Delta u &= \lambda\Big(\varepsilon H(u-\mu_1)+(1-\varepsilon)H(u-\mu_2)\Big) & \text{in } \Omega(t) \\ u &= \overline{u}_\infty & \text{on } \partial\Omega(t) \end{cases} \tag{1}$$

where $\Omega(t) \subset R^3$ a regular domain at $t > 0$, ε, $\overline{u}_\infty$, λ, μ_1, μ_2 are positive parameters and H is the Heaviside step function.

The problem (1) has two free boundaries: the outer boundary of $\Omega(t)$ and the inner boundary whose evolution is implicit generated by the discontinuous nonlinearity H. The problem (1) arise in tumor growth models. First, we show the existence of radial solutions of problem (1) where $\Omega(t)$ is a spherical domain.

Secondly, using the perturbation technic combining to local methods, we prove the existence of solutions and characterize the free boundaries of problem (1) near the corresponding radial solutions.

This is a work based on [1] and [2].

References.

- [1] S. Bensid, Perturbation of the free boundary in elliptic problems with discontinuities, *Electronic Journal of Differential Equations*, **132**, (2016), 1-14.
- [2] H. M. Byrne, M. A. J. Chaplain, Growth of necrotic tumours in the presence and absence of inhibitors. *Math Biosci*, **131**, (1995), 187-216.

Cherifa Guezzen - University of Tlemcen.

The impact of imperfect quarantine in controlling infectious diseases.

Abstract. This is a work based on [1] and [2].
In this work,we investigated the effect of the quarantine on an epidemic in a heterogeneous environment using a general reaction-diffusion SIQR epidemic model. We begin by determining the expression of the basic reproduction number $\mathscr{R}_0$ as in [ref1].For $R_0 \leq 1$, the disease will die out, which is guaranteed by the global asymptotic stability of the disease-free steady state, and for $R_0 > 1$, the disease will persist. Next, we determined the required environment to contain the epidemic by providing a comparative analysis between the classical SIR epidemic model and the SIQR epidemic model.finally, from the numerical results we deduce that for suitable values of quarantine and relapse rates, infectious disease can be contained for varying total population.

References.

- [1] W. Wang, X.-Q. Zhao. Basic reproduction number for reaction-diffusion epidemic models. *SIAM J. Appl. Dyn. Syst*, 1652:1673, 2012.
- [2] L. Duan, L. Huang, and C. Huang. Spatial dynamics of a diffusive SIRI model with distinct dispersal rates and heterogeneous environment. *Communications on Pure Applied Analysis* , 3539:3560, 2021.

TUESDAY, OCTOBER 3

Abdennasser Chekroun - Laboratoire d'Analyse Non Linéaire et Mathématiques Appliquées, Tlemcen University, Tlemcen, Algeria.

Reaction-diffusion PDE coupled to integral equation with nonlocal dispersal term and time delay.

Abstract. This talk concerns two mathematical problems. At first, we consider a class of biological models represented by a system composed of reaction diffusion PDE coupled with difference equations (renewal equations) in n-dimensional space, with nonlocal dispersal terms and implicit time delays. The difference equation generally arises, by means of the method of characteristics, from an age-structured partial differential system. Using upper and lower solutions, we study the existence of monotonic planar traveling wave fronts connecting the extinction state to the uniform positive state. The corresponding minimum wave speed is also obtained. The second problem is an SIR epidemic model with infection age and spatial diffusion in the case of homogeneous Neumann/Dirichlet boundary conditions. This model is a generalization of the model studied in Magal P, McCluskey CC, Webb GF, 2010, to the spatially heterogeneous system. By using the method of characteristics, we reformulate the model into a system of a reaction–diffusion equation and a Volterra integral equation. For the reformulated system (in both cases Neumann/Dirichlet), we define the

basic reproduction number R_0 by the spectral radius of the next generation operator, and show that this rate provides the global threshold dynamics of the infection. I finish my talk by providing numerical examples that support our theoretical results. We conclude that the intensity of the disease spread is affected by the shape of the spatial domain in the case of Dirichlet boundary condition. This is in contrast to the case of homogeneous Neumann boundary condition with spatially homogeneous parameters in which the basic reproduction number R_0 is explicitly given by a positive constant and independent on the shape of the spatial domain.

References.

- [1] M. Adimy, A. Chekroun, and B. Kazmierczak. Traveling waves for reaction-diffusion PDE coupled to difference equation with nonlocal dispersal term and time delay. *Math. Model. Nat. Phenom.* , 17:1-31, 2022.
- [2] A. Chekroun, and T. Kuniya. Global threshold dynamics of an infection age-structured SIR epidemic model with diffusion under the Dirichlet boundary condition. *Journal of Differential Equations*, 269: 117-148, 2020.

Maurizia Rossi - University of Milano-Bicocca.

The nodal length of random spherical harmonics.

Abstract. We investigate the behavior of the “typical" Laplacian eigenfunction of a compact smooth Riemannian manifold. In particular, motivated by both Yau’s conjecture on nodal sets and Berry’s ansatz on planar random waves, we consider Gaussian eigenfunctions on the sphere and study the distribution of the length of their nodal lines in the high energy limit. The results we obtain raise several questions regarding both the distribution of other geometric functionals and the behavior of nodal statistics of random eigenfunctions on a “generic" manifold. In this talk, mainly based on [1], we answer some of these questions, relying on recent developments in the theory of Normal approximations for Wiener chaos [2].

References.

- [1] Domenico Marinucci, Maurizia Rossi, and Igor Wigman. The asymptotic equivalence of the sample trispectrum and the nodal length for random spherical harmonics. *Ann. Inst. H. Poincaré Probab. Statist.*, 56(1): 374-390, 2020.
- [2] Ivan Nourdin, and Giovanni Peccati. *Normal Approximations Using Malliavin Calculus: from Stein’s Method to Universality.* Cambridge Tracts in Mathematics. Cambridge University Press, 2012.

Fahd Karami- Laboratoire MIMSC, Ecole Supérieure de Technologie d’Essaouira -Université Cadi Ayyad- Essaouira Maroc.

On Some Nonlinear Equations and Systems applied to image restoration.

Abstract. In this work, we present a class of nonlinear equations and systems applied to the restoration of degraded images. These models are obtained by regularizing of the well known Perona-Malik and Total variation problems. We investigate the well posedness of the proposed models and we provide illustrative results through numerical tests and simulations.

Joint work with A. Atlas, K. Sadik, D. Meskine and O. Oubbih

Hugo Lavenant - Bocconi University.

Lifting functionals defined on maps to measure-valued maps via optimal transport.

Abstract. How can one lift a functional defined on maps from a space X to a space Y into a functional defined on maps from X into $\mathscr{P}(Y)$ the space of probability distributions over Y? Looking at measure-valued maps can be interpreted as knowing a classical map with uncertainty, and from an optimization point of view the main gain is the convexification of Y into $\mathscr{P}(Y)$. We will explain why trying to single out the largest *convex* lifting amounts to solve an optimal transport problem with an infinity of marginals which can be interesting by itself. Moreover we will show that, to recover previously proposed liftings for functionals depending on the Jacobian of the map, one needs to add a restriction of additivity to the lifted functional.

Abderrahmane Habbal - Université Côte d'Azur.

A game theoretic viewpoint on boundary data recovery coupled to shape identification problems.

Abstract. Inverse problems such as data recovery, obstacle detection, or the identification of coefficients in distributed systems are known to be severely ill-posed in the sense of Hadamard. They are also known to be resistant to numerical approximation, which explains why the majority of current research in this field focuses on the development and study of efficient and stable algorithms for noisy data. In this presentation, not very far from the well-trodden paths of regularization and decomposition, we investigate the capabilities of game theory in addressing such problems and demonstrate that this formalism is capable of effectively addressing these problems, including those that were previously inaccessible due to their coupled ill-posed inverse problems nature. This work is based on approaches developped in [1, 2, 3, 4].

References.

- [1] Ouni, Marwa, Abderrahmane Habbal, and Moez Kallel. "A three-player Nash game for point-wise source identification in Cauchy–Stokes problems." *Journal of Computational and Applied Mathematics* 417 (2023): 114613.
- [2] Chamekh, R., Habbal, A., Kallel, M. and Zemzemi, N. A Nash game algorithm for the solution of coupled conductivity identification and data completion in cardiac electrophysiology, *Mathematical Modelling of Natural Phenomena* 14, 2, February 2019, 15 pages.
- [3] A. Habbal, M. Kallel, and M. Ouni, Nash strategies for the inverse inclusion Cauchy-Stokes problem, *Inverse Problems and Imaging (IPI)* 13 , 4, 2019, pp. 827-862.
- [4] A. Habbal and M. Kallel. Neumann-Dirichlet Nash strategies for the solution of elliptic Cauchy problems, *SIAM Journal on Control and Optimization*, 2013, vol. 51, no 5, pp. 4066-4083.

Oumaima Ben Fraj.

Inverse problems of the stable determination of time-dependent coefficients appearing in some PDE's from partial observations.

Abstract. This work concerns the study of inverse problems of the stable recovery of time-and-space dependent coefficients appearing in different partial differential equations: the wave equation, the magnetic Schrödinger equation and the convection-diffusion equation, by only considering arbitrary boundary measurements. This is a work based on [1] and [2].

References.

- [1] M. Bellassoued, D. Jellali, and M. Yamamoto. Title: Stability estimate for the hyperbolic inverse boundary value problem by local Dirichlet-to-Neumann map *Journal of mathematical analysis and applications*, 2, 1036–1046, (2008).
- [2] Y. Kian, E. Soccorsi Title: Hölder stably determining the time-dependent electromagnetic potential of the Schrödinger equation *SIAM*, 2, 627–647, (2019).

Aroua Nesrine - University of Tunis El Manar, National Engineering School Of Tunis, ENIT-LAMSIN.

Stable determination of a second order perturbation of the polyharmonic operator by boundary measurements.

Abstract. In this talk, We consider the inverse problem for the polyharmonic operator, which can be used in many fields such as medical imaging, geophysics, elasticity theory, etc. We prove that the second order perturbations of the polyharmonic operator are uniquely determined by the corresponding Dirichlet to Neumann map. More precisely, we show in dimension $n \geq 3$, a logarithmic type stability estimate for the inverse problem under consideration.
This is a work based on [1].

References.

- [1] Nesrine Aroua and Mourad Bellassoued. Stable determination of a second order perturbation of the polyharmonic operator by boundary measurements. *Journal of Mathematical Analysis and Applications*, Volume 522, Issue 2, 2023.

Siham Boukarabila - University of Tlemcen, Algeria.

Nonlinear Elliptic Equation Involving a Superquadratic Gradient Term.

Abstract. We show the existence of solution to the following problem:

$$\begin{cases} -\Delta u &= \dfrac{|\nabla u|^q}{|u|^\alpha} + \lambda f \quad \text{in} \quad \Omega \\ u &= 0 \qquad\qquad\quad \text{on} \quad \partial\Omega \end{cases} \tag{2}$$

where $N > 2$, Ω is a bounded regular open subset of $\mathbb{R}^N$, $q > 2$, $\alpha < \frac{q}{2}$, $\lambda > 0$ and f is a nonnegative function belonging to a suitable Lebesgue space.

In ordre to solve problem (2), we established a Sobolev regularity for $v^{1-\beta}$, where v is the unique solution for the Poisson problem and $\beta < \frac{1}{2}$.

Under additional hypotheses on the data, we were able to show that our solution is unique.
These results take part from the following paper [1].

References.

- [1] B. Abdellaoui. Multiplicity Result for Quasilinear Elliptic Problems with General Growth in the Gradient. *Adv. Nonlinear Stud.* **8**,no. 2: 289-301, (2008).
- [2] S.Boukarabila, A. Primo, and A. Younes. Singular elliptic problem with super-quadratic growth in the gradient. *Complex Variables and Elliptic Equations*, (2023),
  DOI: 10.1080/17476933.2023.2250732.
- [3] H. Brezis, X. Cabré. Some simple nonlinear PDE's without solution. Boll. Unione. Mat. Ital. Sez. B, 8: 223-262,(1998).
- [4] Z.-Q. Chen and R. Song. Estimates on Green functions and Poisson kernels for symmetric stable processes. Math. Ann.,312(3): 465-501, (1998).

Mohammed Louriki - Cadi Ayyad University.

Lévy Bridges with Random Length for Modelling of the Financial Information.

Abstract. The main purpose of this talk is to extend the information-based approach of Bedini-Buckdahn-Engelbert to a more general set-up. Instead of using only a Brownian bridge as an information process, we consider another important type of information process. To model the flow of information concerning the time of the bankruptcy of a company (or a state) arriving on the market, we introduce Lévy bridges with random length, generalizing the Brownian bridge and gamma bridge information processes. Our first goal is to rigorously define a Lévy bridge with random length. Our second task is to establish the Markov property with respect to its completed natural filtration and thus with respect to the usual augmentation of the latter. The resulting conclusion is the right-continuity of completed natural filtration. Certain examples of such a process are considered. This is a work based on [1].

References.

- [1] Erraoui Mohamed, Hilbert Astrid , and Louriki Mohammed. On a Lévy process pinned at random time. *Forum Math.*, 397–417, 2021.

WEDNESDAY, OCTOBER 4

David Poyato - University of Granada.

Mean-field limit for non-exchangeable multi-agent systems.

Abstract. In this talk I will discuss on a recent derivation of the mean-field limit for multi-agent systems on a large class of sparse graphs. More specifically, the case of non-exchangeable multi-agent systems consisting of non-identical agents is addressed, where the heterogeneous distribution of connectivities in the network is known to have critical effects on the collective dynamics. As a result, we obtain a Vlasov equation for the continuum of agents, which still captures the heterogeneous distribution of weights at the macroscopic scale in terms of the so-called extended graphons. Our method of proof does not only involve PDEs and stochastic analysis, but also graph theory through a novel concept of limits of sparse graphs (extended graphons) for the structure of the network, which can be regarded as a new non-trivial extension of the seminal works by L. Lovasz and B. Szegedy for dense graph limits. Our proof allows removing some of the main restrictive hypotheses in the previous literature on the connectivities between agents (dense graphs) and the cooperation between them (symmetric interactions).

If time allows, I will also discuss on a recent gradient flow formulation, which is valid for a variety of kinetic equations with heterogeneities arising in collective dynamics, in particular the above class of Vlasov equations over graphons. Our approach is based on the study of the so-called fibered Wasserstein space, which we show it exhibits similar features to the classical Wasserstein space, and we are able to extend Otto calculus and the well-known theory of gradient flows on metric spaces by Ambrosio, Gigli, Savaré to treat systems with heterogeneous variables.

This is based on joint works with P.-E. Jabin (Penn State University), J. Soler (University of Granada) [1] and J. Peszek (University of Warsaw) [2].

References.

- [1] P.-E. Jabin, D. Poyato, and J. Soler. Mean-field limit of non-exchangeable systems. *arXiv:2112.15406*.
- [2] J. Peszek, and D. Poyato. Heterogeneous gradient flows in the topology of fibered optimal transport. *arXiv:2203.08104*.

Imane Boussetouan - National Higher School of Engineering and Technology, Annaba, Algeria.

Existence and regularity results for Stokes and Navier-Stokes equations with Dirichlet and Navier boundary conditions.

Abstract. We consider two types of mixed boundary conditions associated to the Stokes and Navier-Stokes equations. We study the Stokes equation with Dirichlet boundary condition on some part of the boundary and Navier-type boundary condition on the remaining part. We prove the existence and uniqueness of weak and strong solutions of the corresponding problem in the Hilbert setting and then in L^p-theory for any $1 < p < +\infty$. Next, we assume that the full Navier boundary condition is prescribed on one part of the boundary and Dirichlet boundary condition on the other part. We investigate the generalized and strong solutions for the given system by taking into account the regularity of the friction coefficient, considered as a function. Finally, we extend the obtained results to the stationary Navier-Stokes system by using some classical arguments. This is a work based on [1,2].

This is a joint work with C. Amrouche (Pau and Pays de l'Adour University, France)

References.

- [1] C. Amrouche, I. Boussetouan. Vector potentials with mixed boundary conditions. Application to the Stokes problem with pressure and Navier-type boundary conditions. *SIAM J. Math. Anal*, Vol 53, N 2: 1745–1784, 2021.
- [2] C. Amrouche, A. Ghosh. Stokes and Navier-Stokes equations with Navier boundary condition. *J. Diff. Equa*, Vol 285: 258–320, 2021.

Andrea Marchese - Trento, Italy.

The singularities of soap films.

Abstract. Mass minimizing Integral currents mod p are minimal surfaces exhibiting a geometric structure which is observed in soap films but it is not present in classical solutions of the Plateau's problem. I will discuss recent results on their singularities, allowing a posteriori a formulation of the problem as a free boundary problem. Based on joint works with Camillo De Lellis, Jonas Hirsch, Luca Spolaor, and Salvatore Stuvard.

Mustapha Mokhtar-Kharroubi - Université de Franche-Comté Besançon, France.

On growth-fragmentation equations with mass-loss or death.

Abstract. We are concerned with well posedness and time asymptotics of evolution equations of the form

$$\begin{aligned} &\frac{\partial}{\partial t}u(x,t)+\frac{\partial}{\partial x}\left(r(x)u(x,t)\right)+\left(a(x)+d(x)\right)u(x,t)\\ =\ &\int_x^{+\infty}a(y)b(x,y)u(y,t)dy,\ \ u(x,0)=u_0(x),\ \ x,t>0 \end{aligned}$$

under the structural assumptions

$$\begin{cases} \int_0^y xb(x,y)dx = y\left(1-\eta(y)\right), \quad 0\le\eta(y)\le 1\ (y\ge 0)\\ \int_0^1 \frac{1}{r(\tau)}d\tau<+\infty,\ \ \int_1^\infty \frac{1}{r(\tau)}d\tau=+\infty. \end{cases}$$

The equation is considered in the physical space

$$X_{0,1}:=L^1(\mathbb{R}_+;\ (1+x)\,dx)$$

of "*finite mass and finite agregate number*".

We give general conditions insuring the existence of a C_0-semigroup $(V(t))_{t\geq 0}$ governing this equation in $X_{0,1}$ and having a spectral gap, i.e.

$$r_{ess}(V(t)) < r_{\sigma}(V(t))$$

(r_{ess} refers to the essential spectral radius); this implies the so-called asynchronous exponential growth property, i.e.

$$\left\| e^{-\lambda t} V(t) - P \right\|_{\mathscr{L}(X_{0,1})} = O(e^{-\varepsilon t})$$

(for some $\varepsilon > 0$) where λ is the leading eigenvalue of the generator and P is the corresponding spectral projection.

References.

- [1] M. M-K. On spectral gaps of growth-fragmentation semigroups with mass loss or death. *Commun. Pure Appl. Anal*, no. 4:1293–1327, 2022.
- [2] M. M-K , J. Banasiak. On spectral gaps of growth-fragmentation semigroups in higher moment spaces. *Kinet. Relat. Models*, no. 2: 147–185, 2022.

Mohammed Nor Friouri, National Higher School of Mathematics Sidi Abdellah, and Laboratoire d'Analyse Non Linéaire et Mathématiques Appliquées, University of Tlemcen, Algeria.

Stability analysis of an age-structured epidemic model.

Abstract. The aim of this work is to make a mathematical study of an age-structured epidemic model with a general class of nonlinear incidence rate.

We are interested to prove the global asymptotic stability of equilibria where the condition of stability is given by the basic reproduction number R_0. We show that the endemic equilibrium is globally asymptotically stable (EE) when $R_0 > 1$ and the disease-free equilibrium is globally asymptotically stable (DFE) when $R_0 < 1$.

Finally, we propose some numerical simulations to illustrate our theoretical results. This is a work based on [1] and [2].

References.

- [1] M. N. Frioui, T. M. Touaoula and B. Ainseba. Global Dynamics of an age-structured model with relapse. *Discrete and Continuous Dynamical Systems- series B* pp. 2245-2270, Volume 25, Number 6, June 2020.
- [2] A. Chekroun, M. N. Frioui, T. Kuniya and T. M. Touaoula. Global stability of an age-structured epidemic model with general Lyapunov functional. *Mathematical Biosciences and Engineering*, 16(3): number:1525–1553, Feb 2019.

Mohamed Lamine Mostefai - École Normale Supérieure, Alger, Algeria.

Stability result of a one-dimentional swelling porous-elastic soils system with thermal effect and distributed delay term.

Abstract. In this work, we consider a swelling porous thermoelastic system with the presence of a temperature effact and distributed delay terms. The coupling gives new contributions to the theory associated with asymptotic behaviors of swelling porous elastic soils. We will established the well-posedness and the general decay result is proved by the multiplier method. . This is work based on [1], [2], [3] and [4].

References.

- [1] A. Choucha, D. Ouchenane, and Kh. Zennir, General decay of solutions in one-dimensional porous-elastic with memory and distributed delay term, (2020), In press..
- [2] A.C. Eringen, A continuum theory of swelling porous elastic soils. Int. J. Eng. Sci. 32(8), 1337–1349 (1994).
- [3] A. S. Nicaise and C. Pignotti. Stabilization of the wave equation with boundary or internal distributed delay. Diff. Int. Equs., 21(9-10):935-958, (2008).
- [4] Zuazua E. Controllability and Observability of Partial Differential Equations: Some results and open problems In Handbook of Differential Equations:Evolutionary Differential Equations, vol 3, C. M. Dafermos and E. Feireisl eds., Elsevier Science, pp. 527-621.

Sofiane Chalal- École Centrale-Supelec, Paris, France.

On the quantum mean-field feeback control.

Abstract. "In this talk, we will briefly review the main ingredients of the theory of Mean-Field Games (MFG) and Control (MFC). Drawing from [1], [2], and [3], we will explain the necessity to consider quantum filtering theory to build Quantum MFG and Quantum MFC. Lastly, based on [ref4], we will examine the principal equation for quantum mean-field feedback control, known as the MF-Belavkin equation. We will demonstrate the well-posedness of this equation and present its application in the stabilization of an N-Qubits system [4]." *Joint work with* : Nina Hadis AMINI (CNRS, L2S CentraleSupélec), Gaoyue GUO (MICS CentraleSupélec)

References.

- [1] Vassili Kolokoltsov. Quantum Mean-Field Games. *The Annals of Applied Probability*, 35, 2022.
- [2] Vassili Kolokoltsov. Dynamic Quantum Games. *Dynamic Games and Applications*, 22, 2021.
- [3] Vassili Kolokoltsov. The law of large numbers for quantum stochastic filtering and control of Many-particle systems. *Theoretical and Mathematical Physics*, 21, 2021.
- [4] Sofiane Chalal, Nina Hadis Amini, and Gaoyue Guo. On the mean-field Belavkin equation. *IEEE Control Systems Letters*, 6, 2023.

Yassine Tahraoui - Center for Mathematics and Applications (NovaMath), NOVA SST, Portugal.

On local pathwise solution for third grade fluid equations with a multiplicative noise.

Abstract. Most studies on fluid dynamics have been devoted to Newtonian fluids, which are characterized by the classical Newton's law of viscosity. However, there exist many real fluids with nonlinear viscoelastic behavior that does not obey Newton's law of viscosity. My aim is to present a recent result about the 2D/3D stochastic 3^{rd}-grade fluids in the presence of multiplicative noise driven by a Q-Wiener process. Namely, the strong (local) well-posedness, in PDEs and probabilistic senses, which corresponds to an addapted stochastic process with sample paths defined up to a certain positive stopping time, with values in the Sobolev space H^3. The approach combines a cut-off approximation scheme, a stochastic compactness arguments and a general version of Yamada-Watanabe theorem. This talk is based on [1].

References.

- [1] Yassine Tahraoui and Fernanda Cipriano. Local strong solutions to the stochastic third grade fluid equations with Navier boundary conditions. *Stochastic Partial Differential Equations: Analysis and Computations*, (to appear), 2023.

Slim Omri - Université Tunis El Manar, Faculté des Sciences de Tunis, Département de mathématiques.

Continuous Gabor transform and uncertainty principles.

Abstract. Uncertainty principles in harmonic analysis claim that a function f and its Fourier transform cannot be simultaneously and sharply localized that is, it is impossible for a nonzero function to be arbitrary small as well as its Fourier transform. There are many formulations of this general fact where the smallness and the sharpness are interpreted in different ways. In the sixties and in order to solve the frequency localization's problem of the classical Fourier transform, D. Gabor introduced the short-time Fourier transform also called Continuous Gabor transform which is defined for $f \in L^2(\mathbb{R}^d)$ by

$$\forall (x,\omega) \in \mathbb{R}^d \times \mathbb{R}^d,\ \widehat{\mathscr{V}_g}(f)(x,\omega) = \int_{\mathbb{R}^d} f(t)\overline{g(t-x)}e^{-i\langle t|\omega\rangle}dt,$$

where $g \in L^2(\mathbb{R}^d)$ is a nonzero function called a window function. In this work we prove many uncertainty principles related to the Short time Fourier transform. This talk is based on [1], [2], [3] and [4].

References.

- [1] H. Lamouchi and S. Omri, *Time-frequency localization for the short-time Fourier transform*, Integral Transforms Spec. Funct. **27** (2016), 43–54.
- [2] H. Lamouchi and S. Omri, *Quantitative uncertainty principles for the short time Fourier transform and the radar ambiguity function*, Indian J. Pure Appl. Math. **48** (2017), no. 1, 147–161.
- [3] S. Ghobber, S. Omri, and O. Oueslati, *Logvinenko-Sereda type theorems for the short-time Fourier transform.* J. Pseudo-Differ. Oper. Appl. **13** (2022), Issue 1, Art 2.
- [4] S. Ghobber, S. Omri, and O. Oueslati, *Inequalities in time-frequency analysis.* §Math. Inequal. Appl. **26** (2023), Issue 2, 377-400.

Anna Doubova - University of Sevilla.

Inverse problem concerning degenerate diffusion in a parabolic equation arising in a climate model.

Abstract. In the last few years, degenerate parabolic differential equations have received increasing attention in view of the significant related theoretical analysis and practical applications in particularly in climate science. We consider the inverse problem of the identification of the degenerate diffusion coefficient in a one-dimensional parabolic equation by means of some additional data. Several types of degeneracy are considered. We establish stability and uniqueness results under appropriate assumptions. The proofs are based on techniques widely used in control theory and inverse problems, such as energy estimates and global Carleman inequalities. Finally, the theoretical results are successfully verified by numerical experiments. The presented work has been carried out in collaboration with Piermarco Cannarsa and Masahiro Yamamoto based on [1] and [2].

References.

- [ref1] P. Cannarsa, A. Doubova and M. Yamamoto, Inverse problem of reconstruction of degenerate diffusion coefficient in a parabolic equation, *Inverse Problems*, 37, no. 12, 2021.
- [ref2] P. Cannarsa, A. Doubova and M. Yamamoto, Inverse problem of reconstruction of interior degeneracy in a parabolic equation, *preprint*.

Zouhour Rezig - Faculté des sciences de Tunis, LAMSIN-ENIT, UTM.

Stable determination of a vector field in a non-self-adjoint dynamical Schrödinger equation on Riemannian manifolds.

Abstract. In this work, we deal with an inverse problem for a non-self-adjoint Schrödinger equation on a compact Riemannian manifold. We aim to stably recover a real vector field from the dynamical Dirichlet-to-Nuemann map. We establish, in dimension $n \geq 2$, a Hölder type stability estimate for the inverse problem under study. The proof is mainly based on the reduction to an equivalent problem for an electro-magnetic Schrödinger equation, the use of the invertibility of the geodesic ray transform on simple manifolds and a Carleman estimate designed for elliptic operators.
This is a work based on [1].

References.

- [1] M. Bellassoued, I. Ben Aicha, and Z. Rezig. Stable determination of a vector field in a non-self-adjoint dynamical Schrödinger equation on Riemannian manifolds. *Mathematical Control and Related Fields*, 11 (2) : 403-431, 2021.

Bruno - Iannazzo - Università di Perugia, Italy.

Geometries on the cone of positive definite matrices and their relation to matrix means.

Abstract. The affine-invariant metric is perhaps the most celebrated non-Euclidean geometry on the cone of n-by-n positive definite matrices, denoted by $\mathscr{P}_n$. It has been discovered independently in different fields of theoretical and applied mathematics with important applications: for instance, the barycenter of a set of matrices in this geometry has been understood as the legitimate generalization of the geometric mean from numbers to matrices.

Observing that the affine-invariant geometry can be obtained from the Hessian of the logarithmic potential on $\mathscr{P}_n$

$$\varphi(X) = -\log\det(X),$$

we introduce a new family of non-Euclidean geometries on $\mathscr{P}_n$ obtained from the Hessian of the power potential

$$\varphi_\beta(X) = \frac{1-\det(X)^\beta}{\beta}$$

and provide explicit expressions for the related geodesics and distances.

The new geometries provide a definition of matrix power mean alternative to the existing ones. We discuss some properties of the proposed power mean and relate it with the geometric mean, theoretically and computationally. We show how these geometries can be used to accelerate the convergence of optimization algorithms for computing the matrix geometric mean.

We discuss also some possible applications: in practice, endowing a space with a geometry may allows one to identify the hidden structure of an object, such as a dataset, belonging to that space; on the other hand, geometrical quantities such as geodesics and distances can be used to provide solutions to problems of averaging and clustering, respectively.

This is a joint work with N. Chouaieb and M. Moakher, based on [1].

References.

- [1] N. Chouaieb, B. Iannazzo, M. Moakher. Geometries on the cone of positive-definite matrices derived from the power potential and their relation to the power means. *Linear Algebra Appl.*, number:pages, year.

Abdelhamid Gouasmia - Laboratoire d'equations aux dérivées partielles non linéaires et histoire des mathématiques, Ecole Normale Supérieure, Algiers, and Department of Mathematics and Computer Science, Larbi Ben M'Hidi University, Oum El-Bouaghi, Algeria.

A singular system involving mixed local and non-local operators.

Abstract. This work sets forth results on the existence, non-existence, uniqueness, and regularities properties, as well as boundary behavior of solutions for the following singular systems involving mixed local and non-local elliptic operators:

$$\begin{cases} \mathscr{L}_1 u = k_1(x)\, u^{-\alpha_1}\, v^{-\beta_1}, & u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \\ \mathscr{L}_2 v = k_2(x)\, v^{-\alpha_2}\, u^{-\beta_2}, & v > 0 \quad \text{in } \Omega; \quad v = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega. \end{cases} \tag{S}$$

Here $\mathscr{L}_i$, $i = 1,2$ is the mixed operator, defined as:

$$\mathscr{L}_i = -\Delta + (-\Delta)^{s_i}. \tag{3}$$

Here $0 < s_1 < 1$, $0 < s_2 < 1$, and $\alpha_1, \alpha_2, \beta_1, \beta_2 > 0$ are positive constants. Let $\Omega \subset \mathbb{R}^N$, $N \geq 3$, be an open bounded domain with $C^{1,1}$ boundary $\partial\Omega$. More precisely, we first establish a new weak comparison principle for a singular equation. Afterward, we can discuss the non-existence of positive classical solutions, as well as construct suitable ordered pairs of sub-solutions and super-solutions. This allows us to obtain the existence of a pair of positive weak solutions for System (S) by employing Schauder's fixed-point Theorem in the associated conical shell. Finally, we adapt a method due to Krasnosel'skiís argument to establish the uniqueness of such a positive pair of solutions.

References.

- [1] Di Nezza, Eleonora and Palatucci, Giampiero and Valdinoci, Enrico. Hitchhiker's guide to the fractional Sobolev spaces. *Bull. Sci. Math.*, 521–573, 2012.
- [2] Dipierro, Serena and Lippi, Edoardo Proietti and Valdinoci, Enrico. (Non) local logistic equations with Neumann conditions. *Annales de l'Institut Henri Poincaré C*, 2022.

Hariz Khaled - Paderborn University.

Higher order fractional variational integrator based on convolution quadrature methods.

Abstract. Fractional dissipation is a powerful tool to study non-local physical phenomena such as damping models. The design of geometric, in particular, variational integrators for the numerical simulation of such systems relies on a variational formulation of the model. In [1] a new approach is proposed to deal with dissipative systems in a variational way for both, the continuous and discrete setting. It is based on the doubling of variables and their fractional derivatives. In this contribution we derive higher-order fractional variational integrators based on convolution quadratures [2] and study the numerical properties of those integrators.

References.

- [1] F. Jiménez and S. Ober-Blöbaum. Fractional Damping Through Restricted Calculus of Variations. Title. *Journal of Nonlinear Science*, 31, 2021.
- [2] C. Lubich. Discretized fractional calculus. *SIAM J. Math. Anal.*, 17(3):704-719 (1983).

Hanaa Zitane - Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, Portugal.

Finite time stability of generalized proportional fractional equations with delays.

Abstract. We investigate the notion of finite time stability for generalized proportional fractional systems (GPFSs) with time delays and variable coefficients. Then, we examine some sufficient conditions that allow concluding the GPFSs stability in a finite time interval, which include the nonhomogeneous and the homogeneous delayed cases. We present two different approaches. The first one is based on Hölder's and Jensen's inequalities, while the second one concerns the Bellman–Grönwall method using the recently introduced proportional generalized Grönwall inequality. Finally, we provide two numerical examples to show the practicability of the developed procedures.

References.

- [1] R. Agarwal, S. Hristova and D. O'Regan, Lyapunov functions and stability of Caputo fractional differential equations with delays, *Differ. Equ. Dyn. Syst.* **30**, 3: 513–534, 2022.
- [2] J. Alzabut, T. Abdeljawad, F. Jarad and W. Sudsutad, A Gronwall inequality via the generalized proportional fractional derivative with applications, *J. Inequal. Appl.* **2019**, Paper No. 101, 12 pp, 2019.
- [3] F. Jarad, T. Abdeljawad and J. Alzabut, Generalized fractional derivatives generated by a class of local proportional derivatives, *Eur. Phys. J. Spec. Top.* **226**, 16: 3457–3471, 2017.
- [4] M. Kuczma, *An introduction to the theory of functional equations and inequalities*, *Birkhäuser Verlag*, Basel, 2009.

Mohamed Lajili.

Edge sketches for multi-modal image registration based on Blake-Zisserman type energy.

Abstract. In this talk, we are interested in deformable registration models for multi-modality images. Indeed, usually a geometric deformation can occur during the steps of recording, reconstruction and transmission of images of the same object. Therefore, the images need to be geometrically aligned for better interpretation, especially in clinical diagnosis using medical images. In traditional image registration methods, the deformation is obtained by solving an optimization problem over a space of deformations based on similarity measure. This optimization problem is considered as a minimization of an energy which is composed of two parts

$$\min_{\mathbf{u}\in\mathscr{W}} \{\lambda_1 D\left[G_1(T(\varphi(\mathbf{x})), G_2(R(\mathbf{x}))\right] + \lambda_2 J(\mathbf{u})\}, \tag{4}$$

where the deformation field is defined via the displacement field, with $\varphi(\mathbf{x}) = \mathbf{x} + \mathbf{u}(\mathbf{x})$, $\mathscr{W}$ is a properly chosen functional space, G_1 and G_2 are two functions which depend on the two imaging modalities, R represents the fixed image which is called the reference image, and T represents the moving image which is called the template image.

In the first part of this talk we present a new similarity term for image registration which is based on the geometric information (edges and thin structures) extracted from the images using the Blake–Zisserman's energy. The latter is well suited for detecting discontinuities at different scales, i.e., of first and second order. We give a theoretical analysis of the proposed model. Then, we present the Gauss–Newton method and multilevel technique which are used to speed up the numerical computations for the solution of this model. We display in the second part of the talk some numerical results of the new approach and we compare them with those obtained by some existing methods. The experiments illustrate the efficiency and effectiveness of the proposed model.

This is a work based on [1] and [2].

References.

- [1] Theljani, A and Belhachmi , Z. A discrete approximation of Blake and Zisserman energy in image denoising with optimal choice of regularization parameters. *Mathematical Methods in the Applied Sciences*, 3857-3871, 2021.

- [2] Anis Theljani and Ke Chen. An augmented Lagrangian method for solving a new variational model based on gradients similarity measures and high order regularization for multimodality registration *Inverse Problems and Imaging*,13, 2019.

FRIDAY, OCTOBER 6

Abdelaziz Rhandi - University of Salerno.

Well-posedness and stability of a class of linear systems.

Abstract. The aim of this work is to provide a sufficient criterion for positivity and well-posedness of a class of infinite-dimensional linear systems. This criterion is based on an inverse estimate with respect to the Hille-Yosida Theorem. Indeed, we establish a generation result for perturbed positive operator semigroups, namely, for positive unbounded boundary perturbations. This unifies previous results available in the literature and that were established separately so far, see [1], [2], [3], [4]. We also prove that uniform exponential stability persists under unbounded boundary perturbations. Finally, applications to a Boltzmann equation with non-local boundary conditions on a finite network and size-dependent population system with a delayed birth process are also presented.

References.

- [1] J. Banasiak and P. Namayanja, Asymptotic behaviour of flows on reducible networks, *J. Networks Heterogeneous Media*, 9: 197–216, 2014.
- [2] G. Borgioli and S. Totaro, 3D-streaming operator with multiplying boundary conditions: Semigroup generation properties, *Semigroup Forum*, 55: 110-117, 1997.
- [3] K. Latrach and M. Mokhtar-Kharroubi, Spectral analysis and generation results for streaming operators with multiplying boundary conditions, *Positivity*, 3: 273-296, 1999.
- [4] S. Mancini and S. Totaro, Solutions of the Vlasov equation in a slab with source terms on the boundaries, *Riv. Math. Univ. Parma*, 2: 33-47, 1999.

Sofiane Zouaoui - ENSTA and laboratory of EDPNL&HM of ENS-Kouba (Algeria).

On subdivision schemes and their applications in image processing.

Abstract. Subdivision schemes and multiresolution techniques are utilized in various applications, such as image processing and computer-aided design.

In this presentation, we introduce several families of linear ternary subdivision schemes and a new family of non-linear ternary subdivision schemes designed to eliminate the Gibbs phenomenon. We analyze their convergence, stability, and various properties, including the reproduction polynomials of any m-arity subdivision scheme. Furthermore, we apply multiresolution techniques along with both linear and non-linear filters to denoise images. This work based on [1] and [2].

References.

- [1] S.Amat, S.Busquir, J.Ruiz, J.C.Trillo and S.Zouaoui. On some new variational problems for image denoising. *Math Meth Appl Sci*, 42 : 5881-5897, 2019.
- [2] S.Zouaoui, S. Amat, S. Busquier and MJ Legaz. Some New n-Point Ternary Subdivision Schemes without the Gibbs Phenomenon. *Mathematics*, 10: 2674, 2022.

Khadija Sadik - Ecole Supérieure de Technologie d'Essaouira, Cadi Ayyad university, Morocco.

Nonlocal total variation system for the restoration of textured images.

Abstract. The ultimate goal of image denoising is to extract important features such as noise from images. Many nonlocal models [1,2,3,4] have been proposed to denoise a corrupted images. In this current work, we study the nonlocal total variation reaction diffusion system for image denoising as well its discrete model. This model is obtained by analyzing the case $p \longrightarrow 1$ of the nonlocal reaction diffusion system proposed in [5], the authors used the decomposition approach of H^{-1} norm suggested by Meyer [6] more appropriate to represent the oscillatory patterns and small details in the textured images. The existence and uniqueness of the proposed model are established. Then, we present some numerical simulations and comparative experiments.

References.

- [1] A. Buades, B. Coll, J. M. Morel A review of image denoising algorithms, with a new one *Multiscale Model. Simul.,* 4(2):490–530, 2005.
- [2] G. Gilboa, S. Osher. Nonlocal operators with applications to image processing *Multiscale Model. Simul.*, 7(3):1005–1028, 2008.
- [3] A. Elmoataz, D. Xavier and O. Lezoray. Non-local morphological pdes and p-laplacian equation on graphs with applications in image processing and machine learning. *IEEE Journal of Selected Topics in Signal Processing,*, (6):764–779,2012.
- [4] F.Karami, K. Sadik, and L. Ziad. A variable exponent nonlocal p(x)-laplacian equation for image restoration. *Computers and Mathematics with Applications*, 75:534–546., 2018.
- [5] Fahd Karami, Driss Meskine and Khadija Sadik. A new nonlocal model for the restoration of textured images. *Journal of Applied Analysis and Computation.*, Vol.9, (6) :2070-2095, 2019.
- [6] Y. Meyer. Oscillating patterns in image processing and nonlinear evolution equations. volume 22 of University Lecture Series. American Mathematical Society, Providence, RI. The fifteenth Dean Jacqueline B. Lewis memorial lectures. 2001.