Workshop "Duality and More"
May 16-20, Université Côte d'Azur
Laboratoire J.A. Dieudonné, Parc Valrose, Nice, France

This workshop is placed within the thematic contours coming out of Mai Gehrke's DuaLL ERC project:
Duality in Formal Languages and Logic — a unifying approach to complexity and semantics.

We hope that this exchange of ideas can lead to future workshops too.

Talks will be in the Salle de Conférences, except on Tuesday and Thursday afternoons when they will be in Salle 2.

Coffee breaks will be served between the talks in the morning and afternoon.

Program

Monday May 16

9:30 - 10:30 – Rafał Stefański : *Single-use Automata for Infinite Alphabets*
11:00 - 12:00 – Luigi Santocanale : *Unitless Frobenius quantales*
lunch break
14:00 - 15:00 – Sam van Gool : *Preserving joins at primes: duality for automata and domains*
15:30 - 16:30 – Silvio Ghilardi : *Uniform Interpolation in First-Order Theories*

Tuesday May 17

9:30 - 10:30 – Samson Abramsky : *From Kochen-Specker to Feder-Vardi*
11:00 - 12:00 – Maria Manuel Clementino : *How algebraic are ordered (abelian) groups?*
lunch break
14:00 - 15:00 – Wesley Fussner : *Metalogical Properties via Finitely Subdirectly Irreducible Algebras*
15:30 - 16:30 – Graham Manuell : *Characterising congruence frames*

Wednesday May 18

9:30 - 10:30 – Jean-Eric Pin : *Pervin spaces*
11:00 - 12:00 – Jorge Picado : *Continuity in pointfree topology and beyond*

lunch break
14:00 - 15:00 – Célia Borlido : *Difference-restriction algebras of partial functions with operators: discrete duality*
15:30 - 16:30 – Brett McLean : *Difference–restriction algebras of partial functions with operators: discrete duality – part II*

20:00 — Workshop Dinner : "Le Galet"
Choice of vegetarian, fish or meat

Thursday May 19

9:30 - 10:30 – Fred Wehrung : *Projective classes as images of accessible functors*
11:00 - 12:00 – Tomáš Jakl : *Kleisli and Eilenberg-Moore Laws in the Setting of Game Comonads*
lunch break
14:00 - 15:00 – Thomas Colcombet
15:30 - 16:30 – Vincent Moreau : *Higher-order automata and profiniteness*

Friday May 20

9:30 - 10:30 – Luca Reggio : *Arboreal categories and homomorphism preservation theorems*
11:00 - 12:00 – Anna Laura Suarez : *Canonical extensions and sublocales of the frame of filters*
lunch break
14:00 - 15:00 – Paul-André Melliès : *A functorial excursion between algebraic geometry and linear logic*
15:30 - 16:30 – Jérémie Marquès : *Interpolation and duality in model theory*

Abstracts

Single-use Automata for Infinite Alphabets

Rafał Stefański University of Warsaw

Register automata (as defined by Kaminski & Frances) are a well-established model for recognizing languages over infinite alphabets. They define a class of languages which shares many desirable properties of regular languages, but their definition is not as stable as the one of finite automata – for example, all the following variants of register automata define pairwise nonequivalent classes of languages: deterministic one-way, deterministic two-way, nondeterministic one-way, and nondeterministic two-way. In this talk, I am going to introduce the single-use restriction for register automata which states that every read access to a register should have the side effect of destroying that register's content. I am going to argue that the single-use version of register automata defines a robust class of languages. In particular, with the single-use restriction in place, one-way and two-way deterministic register automata recognize the same class of languages. The talk is based on a joint research with Mikołaj Bojańczyk and Nathan Lhote.

Unitless Frobenius quantales

Luigi Santocanale LIS, Aix-Marseille University
(work in collaboration with C. de Lacroix)

It is often stated that a Frobenius quantale is necessarily unital. I'll show how, modulo a slight change in the definition of these structures, we can have Frobenius quantales that are not unital. Unitless Frobenius quantales allow a natural theory, such as phase semantics and a representation theorem via phase quantales. Important examples of these structures arise from Wille's tensor product of complete lattices and related notions: tight endomaps of a complete lattice always form a Girard quantale which is unital if and only if the lattice is completely distributive. I'll give a characterisation (and enumeration) of tight endomaps of the diamond lattices M_n. An analogous example (via the notion of nuclear map) arise from trace class operators of a possibly infinite dimensional Hilbert space: the collection of its closed

subspaces always is a Girard quantale which happens to have a unit if and only if the Hilbert space is finite dimensional. Curiously, we cannot coherently add units to Frobenius quantales that are not unital: such an extension necessarily destroys negations.

Preserving joins at primes: duality for automata and domains

Sam van Gool IRIF, Université Paris Cité

In this talk, on joint work with Mai Gehrke, I will show how a lattice-theoretic notion that we call "preserving joins at primes" appears in the duality-theoretic study of both automata theory and domain theory, thus revealing a new connection between the two fields.

Uniform Interpolation in First-Order Theories

Silvio Ghilardi Università degli Studi di Milano

We report some recent results concerning interpolation and uniform interpolation in first-order theories, focusing on combination aspects and on their semantic counterparts. This is joint work with D. Calvanese, A. Gianola, M. Montali, A. Rivkin.

From Kochen-Specker to Feder-Vardi

Samson Abramsky University College London
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The seminal work of Kochen and Specker [11] showed that quantum mechanics is fundamentally *contextual*: the properties of a quantum system must be considered relative to the context in which they are measured. There is no consistent way of assigning values to all the observables. In [3, 4, 2], contextuality was studied from a sheaf-theoretic point of view, and sheaf cohomology was used to characterise the obstructions to having a consistent

global assignment to all the variables. One could say that cohomology detects the *holes* which prevent there being a consistent picture of a global *whole*.

Constraint satisfaction is an important algorithmic paradigm which allows the application of structural methods to central questions of complexity theory. The "non-uniform" version CSP(B) for a fixed finite σ-structure B, where σ is a finite relational vocabulary, asks for an instance given by a finite σ-structure A whether there is a σ-homomorphism $A \to B$. The celebrated Feder-Vardi Dichotomy Conjecture [8] asked whether for every B, CSP(B) is either polynomial-time solvable, or NP-complete. This conjecture was recently proved by Bulatov and Zhuk [5, 12].

Recently, Adam Ó Conghaile has pointed out surprisingly close connections between these two, prima facie completely unrelated topics [7], further developed in [1].

- The idea of k-consistency in constraint satisfaction, an approximation method which yields exact results in a wide range of cases, is naturally represented as the coflasquification (dual to the well-known Godement construction [9]) of a sheaf of partial homomorphisms.

- These representations take the same form as the sheaf-theoretic representations of contextuality in [3]. This in turn allows the cohomological criteria for contextuality introduced in [4, 2] to be used to give a computationally efficient refinement of k-consistency.

- The results in [4, 2] can be leveraged to show that this refined version of k-consistency gives exact results for all linear templates, which form one of the main classes for which the standard k-consistency algorithm fails.

- Current work is aimed at determining the exact power of the cohomological refinement of k-consistency.

- The same ideas can be adapted to give a very similar analysis for the widely studied Weisfeiler-Leman equivalences [10], which give polynomial-time approximations to graph and structure isomorphism. Cohomological refinements of these equivalences can then be introduced, and are shown in [7] to defeat various families of counter-examples based on the Cai-Furer-Immerman construction [6], which is paradigmatic in finite model theory.

[1] Samson Abramsky, *Notes on cohomological width and presheaf representations*, 2022, Technical Report.

[2] Samson Abramsky, Rui Soares Barbosa, Kohei Kishida, Raymond Lal, and Shane Mansfield, *Contextuality, cohomology and paradox*, 24th EACSL Annual Conference on Computer Science Logic, CSL 2015, September 7-10, 2015, Berlin, Germany (Stephan Kreutzer, ed.), LIPIcs, vol. 41, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2015, pp. 211–228.

[3] Samson Abramsky and Adam Brandenburger, *The sheaf-theoretic structure of non-locality and contextuality*, New Journal of Physics **13** (2011), no. 11, 113036.

[4] Samson Abramsky, Shane Mansfield, and Rui Soares Barbosa, *The cohomology of non-locality and contextuality*, Proceedings 8th International Workshop on Quantum Physics and Logic, QPL 2011, Nijmegen, Netherlands, October 27-29, 2011 (Bart Jacobs, Peter Selinger, and Bas Spitters, eds.), EPTCS, vol. 95, 2011, pp. 1–14.

[5] Andrei A Bulatov, *A dichotomy theorem for nonuniform CSPs*, 2017 IEEE 58th Annual Symposium on Foundations of Computer Science (FOCS), IEEE, 2017, pp. 319–330.

[6] Jin-Yi Cai, Martin Fürer, and Neil Immerman, *An optimal lower bound on the number of variables for graph identification*, Combinatorica **12** (1992), no. 4, 389–410.

[7] Adam Ó Conghaile, *Cohomological k-consistency*, 2021, Technical Report.

[8] Tomás Feder and Moshe Y Vardi, *The computational structure of monotone monadic SNP and constraint satisfaction: A study through Datalog and group theory*, SIAM Journal on Computing **28** (1998), no. 1, 57–104.

[9] Roger Godement, *Topologie algébrique et théorie des faisceaux*, Hermann, 1958.

[10] Sandra Kiefer, *The Weisfeiler-Leman algorithm: an exploration of its power*, ACM SIGLOG News **7** (2020), no. 3, 5–27.

[11] Simon Kochen and Ernst P. Specker, *The problem of hidden variables in quantum mechanics*, Journal of Mathematics and Mechanics **17** (1967), no. 1, 59–87.

[12] Dmitriy Zhuk, *A proof of the CSP dichotomy conjecture*, Journal of the ACM (JACM) **67** (2020), no. 5, 1–78.

How algebraic are ordered (abelian) groups?

Maria Manuel Clementino CMUC, Universidade de Coimbra

This talk consists of two parts. In the first part we present an overview of the 1-dimensional categorical behaviour of the category OrdGrp of (pre)ordered groups, based mostly on the results of [1]. We will show in particular that OrdGrp is far from being protomodular, a property of the category of groups that has been used very successfully in the past three decades. Having at hand ordered structures, one may ask whether one can enrich this category in Ord in a convenient way. This question leads to the second part of the talk, which is based in [2], where we discuss an Ord-enriched version of protomodularity, and in particular show that, as Ord-enriched categories, whilst OrdGrp behaves badly the category OrdAb of ordered abelian groups is Ord-protomodular.

[1] M.M. Clementino, N. Martins-Ferreira, A. Montoli, On the categorical behaviour of preordered groups, J. Pure Appl. Algebra 223 (2019), 4226-4245.

[2] M.M. Clementino, A. Montoli, D. Rodelo, On Ord-protomodularity, in preparation.

Metalogical Properties via Finitely Subdirectly Irreducible Algebras

Wesley Fussner Universität Bern

Often one may establish that a deductive system $\vdash$ has a given metalogical property by proving that an associated equational class K of algebraic models enjoys a corresponding algebraic property. Such logic-to-algebra "bridge theorems" provide a powerful technique for studying metalogical properties of deductive systems, provided that there is some means of establishing the requisite algebraic properties. In this talk, I will present a host of tools for lifting logically-relevant algebraic properties to an equational class K from its subclass of so-called finitely subdirectly irreducible members (i.e., quotients by completely meet-irreducible congruences). In particular, we will see that a congruence-distributive equational class has the congruence extension property if and only if its subclass of finitely subdirectly irreducibles does. I will also exhibit similar results for the amalgamation property, strong amalgamation property, and transferable injections property. Under suitable technical hypotheses, we will also see that there are effective algorithms to decide whether an equational class has each of these properties. This is joint work with George Metcalfe.

Characterising congruence frames

Graham Manuell CMUC, Universidade de Coimbra

Frames provide an algebraic approach to the study of topology. Remarkably, the lattice of congruences on a frame is itself a frame. It is natural to ask precisely which frames occur in this way. While congruence frames have been extensively studied, the problem of giving a complete internal characterisation of them remained open for over four decades. Taking a bitopological perspective allows us to provide such a characterisation in terms of elements which play the role of 'smallest dense sublocales'. I will also briefly mention a second characterisation in terms of bicompleteness with respect to a certain quasi-uniformity.

Pervin spaces

Jean-Eric Pin IRIF, Université Paris Cité

A Pervin space is a set equipped with a lattice of subsets, called its Pervin structure. The wealth of this notion stems from the fact that Pervin spaces can be looked on in at least four different ways: (a) as an algebraic object, that is, a set equipped with a lattice of subsets, (b) as a topological space, generated by the Pervin structure, (c) as a partially ordered set, equipped with the specialisation preorder, (d) as a quasi-uniform space, which was Pervin's original motivation.

I will survey this notion and show how duality ideas translate into this setting.

Continuity in pointfree topology and beyond

Jorge Picado CMUC, Universidade de Coimbra

Mending the contravariance of the natural pointfree representation of classical spaces and continuous maps one replaces the category of frames by its dual category Loc of locales. To make Loc a concrete category one then replaces frame homomorphisms by their right Galois adjoints (referred to as localic maps). This rather formal representation of generalized continuous maps turns out to be surprisingly geometrically satisfactory: we prove that a localic map can be characterized, among plain maps between underlying sets, in terms related to the continuity property of preserving closed and open subobjects by preimage, combined with some unavoidable additional condition that originates in the fact that now complements of closed subobjects have to be formed in the Heyting algebra of all sublocales of the locale and not set-theoretically in the Boolean algebra of all subsets of the space. All this holds in fact in the more general setting of implicative semilattices, that is, meet-semilattices with top element in which the unary meet operations have right adjoints. Locales and frames are just the complete implicative semilattices, and frame homomorphisms are nothing but the residuated semilattice homomorphisms preserving the top element. This setting provides generalizations of continuous and open maps between spaces to an algebraic (not

necessarily complete) pointfree setting. This is a joint work with Marcel Erné and Ales Pultr [1].

[1] M. Erné, J. Picado and A. Pultr, Adjoint maps between implicative semilattices and continuity of localic maps, Algebra Universalis 83 (2022) 1-23, Article number: 13.

Difference-restriction algebras of partial functions with operators: discrete duality

Célia Borlido CMUC, Universidade de Coimbra

The study of algebras of partial functions is an active area of research that investigates collections of partial functions and their interrelationships from an algebraic perspective. The partial functions are treated as abstract elements that may be combined algebraically using various natural operations. Many different selections of operations have been considered, each leading to a different class/category of abstract algebras. In this talk, we will consider algebras of partial functions for a foundational signature consisting of two operations, both binary: the standard set-theoretic *relative complement* operation and a *domain restriction* operation. We will exhibit an adjunction between the category whose objects are the atomic algebras representable as a collection of partial functions closed under relative complement and domain restriction, and whose morphisms are the complete homomorphisms, and a category of set quotients. This generalises the discrete adjunction between the atomic Boolean algebras and the category of sets. One can then show that, for a suitable notion of *complete*, this adjunction restricts to a duality on the *complete* atomic representable algebras, which generalizes the discrete duality between complete atomic Boolean algebras and sets.

This is based on joint work with Brett McLean.

Difference–restriction algebras of partial functions with operators: discrete duality – part II

Brett McLean University of Ghent

This is the second part of the previous talk. I will cover completeness, the duality obtained from our adjunction by restricting to complete algebras, and

the extension of these adjunction, completion, and duality results to algebras equipped with additional (completely additive and compatibility preserving) operators.

This is joint work with Célia Borlido

Projective classes as images of accessible functors

Friedrich Wehrung Université de Caen

Many naturally defined classes of structures are defined as classes of reducts of other structures in a richer language. Typical examples are the class of multiplicative groups of fields, the class of all partially ordered sets of finitely generated ideals in unital rings, the class of all lattices of principal ℓ-ideals in Abelian lattice-ordered groups, and many others. All those classes can be represented by existential second-order quantification over a first-order formula. Such classes are usually called (relatively) projective classes, in short PC. It turns out that PC classes are exactly the images of accessible functors on accessible categories. We introduce a method enabling us to prove that many of those PC classes are not co-PC (i.e., complements of PC), which gives a measure of intractability for those classes. The second and third example above (i.e., ideal lattices of rings and ℓ-ideal lattices of Abelian lattice-ordered groups, respectively) fall in that category. Our argument originates in earlier work by the author with Pierre Gillibert ("condensates") together with an interpolation theorem in extended first-order logic established in 1992 by Heikki Tuuri.

Kleisli and Eilenberg-Moore Laws in the Setting of Game Comonads

Tomáš Jakl University of Cambridge

In recent years we have seen a growing number of examples of model comparison games (such as pebble games or Ehrenfreucht-Fraisse games) being encoded semantically, as comonads on the class of graphs or relational structures. Apart from the connections with logic (via the model comparison games), game comonads can also express a range of important parameters in finite model theory and combinatorics, such as tree-width and tree-depth.

After a brief overview of the emerging theory of game comonads, I will present my joint work with Dan Marsden and Nihil Shah. Our main focus are the so-called Feferman–Vaught-Mostowski-type theorems, which express when an operation on models preserves equivalence in a given logic. Proving these theorems in the comonadic setting amounts to defining a Kleisli law and checking certain smoothness conditions. Surprisingly, the duals of these laws, the Eilenberg-Moore laws (which specialise to comonad morphisms) are more naturally suited for checking preservation of combinatorial parameters rather than equivalence in logic.

Higher-order automata and profiniteness

Vincent Moreau IRIF, Université Paris Cité

Profinite methods and Stone-type dualities are of great importance in automata theory as they enable a topological approach of the theory of regular languages. We present higher-order automata, which recognize simply-typed lambda terms using ideas coming from domain theory and linear logic, and explain how these generalize usual word and tree automata through Church encodings. We then define profinite lambda terms in a parametric way and show how these relate to profinite words.

This is joint work with Paul-André Melliès and Sam van Gool.

Arboreal categories and homomorphism preservation theorems

Luca Reggio University College London

Game comonads (see Tomáš Jakl's talk) capture a number of model comparison games in a purely categorical fashion. This leads to structural descriptions of various combinatorial parameters and (equivalences with respect to) logic fragments that play a central role in finite model theory. In this talk, I will present an axiomatic approach to the framework of game comonads, based on the notion of arboreal category. I will then show how (resource-sensitive) homomorphism preservation theorems can be recast and proved at this axiomatic level. This is joint work with Samson Abramsky.

Canonical extensions and sublocales of the frame of filters

Anna Laura Suarez Université Côte d'Azur

I will show that there are certain significant sublocales (=pointfree subspaces) of the frame of filters Filt(L) of a frame L, and that these correspond to some sublocales of L with natural descriptions. I will then talk about the theory of polarities and the recent work of Tomas Jakl on canonical extensions of frames. I will show how the sublocales of the frame Filt(L) connect to this theory, and in particular I will show that these sublocales can be seen as variations of the canonical extension of the frame L. This is work in progress with Tomas Jakl.

A functorial excursion between algebraic geometry and linear logic

Paul-André Melliès CNRS, Université Paris Cité

In this talk, I will use the functor of points approach to Algebraic Geometry to establish that every covariant presheaf X on the category of commutative rings — and in particular every scheme X — comes equipped "above it" with a symmetric monoidal closed category PshModX of presheaves of modules. This category PshModX defines moreover a model of intuitionistic linear logic, whose exponential modality is obtained by glueing together in an appropriate way the Sweedler dual construction on ring algebras. The purpose of this work is to explore the idea that linear logic is a logic of generalised vector bundles, in the same way as dependent type theory is understood today as a logic of spaces up to homotopy.

Interpolation and duality in model theory

Jérémie Marquès Université Côte d'Azur

In this talk, on joint work with Sam van Gool, we will see how interpolation and compactness can be understood as the source of various theorems

in model theory. Specifically, we use Joyal's polyadic spaces, also known as type space functors, to give duality proofs of Gödel completeness, Beth definability and omitting types theorem. We employ topological arguments that apply in the setting of compact ordered spaces, so that we have a uniform approach to these results for coherent, intuitionistic and classical logics.
