FORMULAIRE d'INTÉGRATION

Dans ce qui suit "c" est une constante réelle.

PRIMITIVES connues en terminale

$$\int a\ dx = ax + c$$

$$\int x\ dx = \frac{x^2}{2} + c$$

$$\int x^m\ dx = \frac{x^{m+1}}{m+1} + c \qquad m \in N$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + c \qquad x \neq 0$$

$$\int \frac{dx}{2\sqrt{x}} = \sqrt{x} + c \qquad x \neq 0$$

$$\int x^\alpha\ dx = \frac{x^{\alpha+1}}{\alpha+1} + c \qquad \alpha \in Q - (-1)\ \ et\ \ x \neq 0\ si\ \alpha < 0$$

$$\int \frac{dx}{x} = \ln|x| + c \qquad x \neq 0$$

$$\int e^x\ dx = e^x + c$$

$$\int a^x\ dx = \frac{a^x}{\ln a} + c \qquad a > 0\ \ et\ \ a \neq 1$$

$$\int \sin x\ dx = -\cos x + c$$

$$\int \cos x\ dx = \sin x + c$$

$$\int \frac{dx}{\cos^2 x} = \int (1 + \tan^2 x)\ dx = \tan x + c \qquad x \neq \frac{\pi}{2} + k\pi \quad k \in Z$$

$$\int \frac{dx}{\sin^2 x} = -\mathrm{cotan} x + c \qquad x \neq k\pi \quad k \in Z$$

PRIMITIVES usuelles

$$\int \mathrm{sh} x\ dx = \mathrm{ch} x + c$$

$$\int \mathrm{ch} x\ dx = \mathrm{sh} x + c$$

$$\int \mathrm{th} x\ dx = \ln(\mathrm{ch} x) + c$$

$$\int \frac{dx}{\mathrm{ch}^2 x} = \int (1 - \mathrm{th}^2 x)\ dx = \mathrm{th} x + c$$

$$\int \cos^2 x\ dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$\int \sin^2 x\ dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\int \tan x\ dx = -\ln|\cos x| + c \qquad x \neq \frac{\pi}{2} + k\pi\ et\ k \in Z$$

$$\int \ln x\ dx = x\ \ln x - x + c \qquad x > 0$$

PRIMITIVES usuelles (suite)

$$\int \frac{dx}{\sin x} = \ln|\tan(\frac{x}{2})| + c \qquad x \neq (2k+1)\pi \ et \ k \in Z$$

$$\int \frac{dx}{\cos x} = \ln|\tan(\frac{x}{2} + \frac{\pi}{4})| + c \quad x \neq \frac{\pi}{2} + 2k\pi \ et \ k \in Z$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \ \ln\left|\frac{1+x}{1-x}\right| \ + c \qquad |x| \neq 1 \quad (\text{ ou Argthx si } x \in]-1,1[\ )$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \text{ Arcsin x } = -\text{Arccos x } + c \qquad |x| < 1$$

$$\int \frac{dx}{1+x^2} = \text{ Arctan}x \ + c$$

$$\int \frac{dx}{\sqrt{x^2+1}} = \ \ln|x + \sqrt{x^2+1}| + c \quad (\text{ ou Argshx })$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \ \ln|x + \sqrt{x^2-1}| + c \quad |x| > 1 \ (\text{ ou Argchx si } x > 1)$$

$$\int \frac{dx}{\sqrt{x^2+h}} = \ \ln|x + \sqrt{x^2+h}| + c \quad x^2 + h > 0$$

$$\int \frac{dx}{\text{sh}x} = \ln|\text{th}(\frac{x}{2})| + c \quad x > 0 \quad ou \quad x < 0$$

$$\int \frac{dx}{\text{ch}x} = 2\text{Arctan}(e^x) + c$$

$$\int \frac{dx}{\text{th}x} = \ln|\text{sh}x| + c \quad x > 0 \quad ou \quad x < 0$$

INTEGRATION par PARTIES

$$\int u(x)\ v'(x)\ dx \ = \ u(x)\ v(x) \ - \ \int u'(x)\ v(x)dx$$ u et v différentiables

$$\boxed{\int_a^b u(x)\ v'(x)\ dx \ = [\ u(x)\ v(x)\ ]_a^b \ - \ \int_a^b u'(x)\ v(x)dx}$$

CHANGEMENT de variable

cas affine: $(\ a \neq 0)$

$$\int f'(ax+b)\ dx \ = \ \frac{1}{a}\ f(ax+b) + c$$

$$\int (ax+b)^\alpha dx = \frac{1}{a}\ \frac{(ax+b)^{\alpha+1}}{\alpha+1} + c \quad \alpha \neq -1 \quad (ax+b \neq 0 \ si \ \alpha < 0)$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ \ln|ax+b| + c \quad ax+b \neq 0$$

cas général:

$$\int f'[u(x)]\ u'(x)\ dx = \ f[u(x)] + c$$

$$\int [u(x)]^\alpha\ u'(x)\ dx = \frac{[u(x)]^{\alpha+1}}{\alpha+1} + c \quad \alpha \neq -1 \quad (u(x) \neq 0 \ si \ \alpha < 0)$$

$$\int \frac{u'(x)}{u(x)}\ dx = \ \ln|u(x)| + c \quad u(x) \neq 0$$

$$\int_a^b f(x)dx \ = \int_{t_a}^{t_b} f[\phi(t)]\ \phi'(t)\ dt$$ avec ϕ monotone et différentiable sur $[t_a, t_b]$ et $a = \phi(t_a)$, $b = \phi(t_b)$