Effect of Dilute Polymer Solutions on Bulk Turbulence

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Outline

Experimental observations of both *Eulerian* and *Lagrangian* statistics, including:

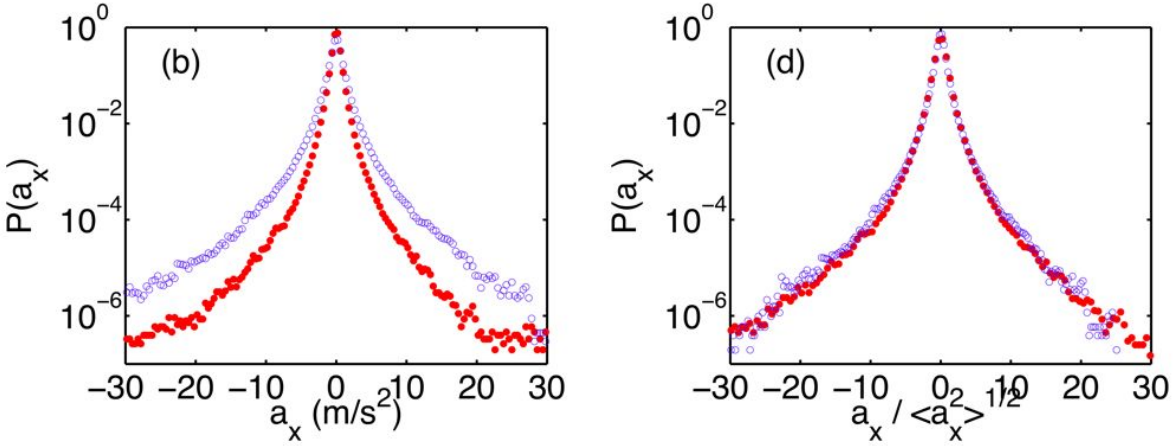
- Reynolds number (Wi) effect on:
  Velocity structure functions

- Polymer concentration effect on:
  Velocity structure functions
  Flow structures (Q-R PDF)
  Acceleration correlations

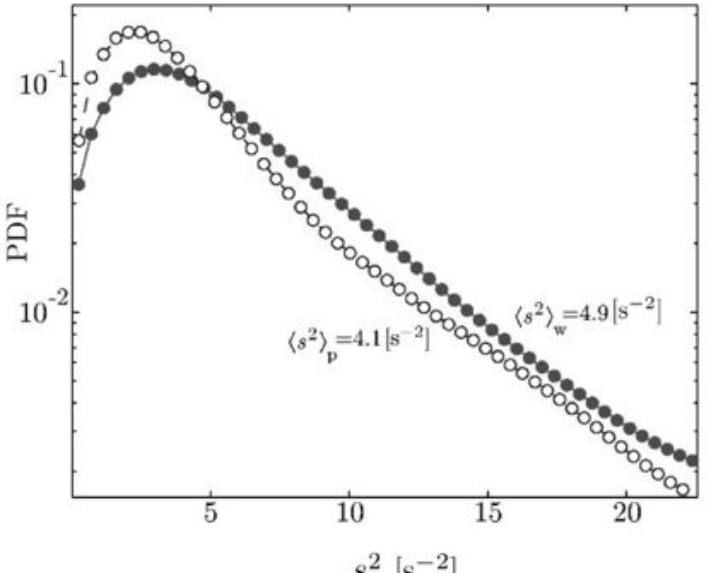
Previous Work

Very few experimental measurements of polymer effect on turbulence far away from boundaries:

Crawford et al (2002 & 2008)
0-10 ppm PAM
R_λ = 140 - 485



Liberzon et al (2005 & 2006)
20 ppm PEO, R_λ = 50



Polymers

Polyacrylamide (PAM): 18 x 10^6 amu

$$R_G = 0.5\mu m, \quad R_{\max} = 77\mu m, \quad \tau_p = 43ms$$

Turbulence property at R_λ = 350:

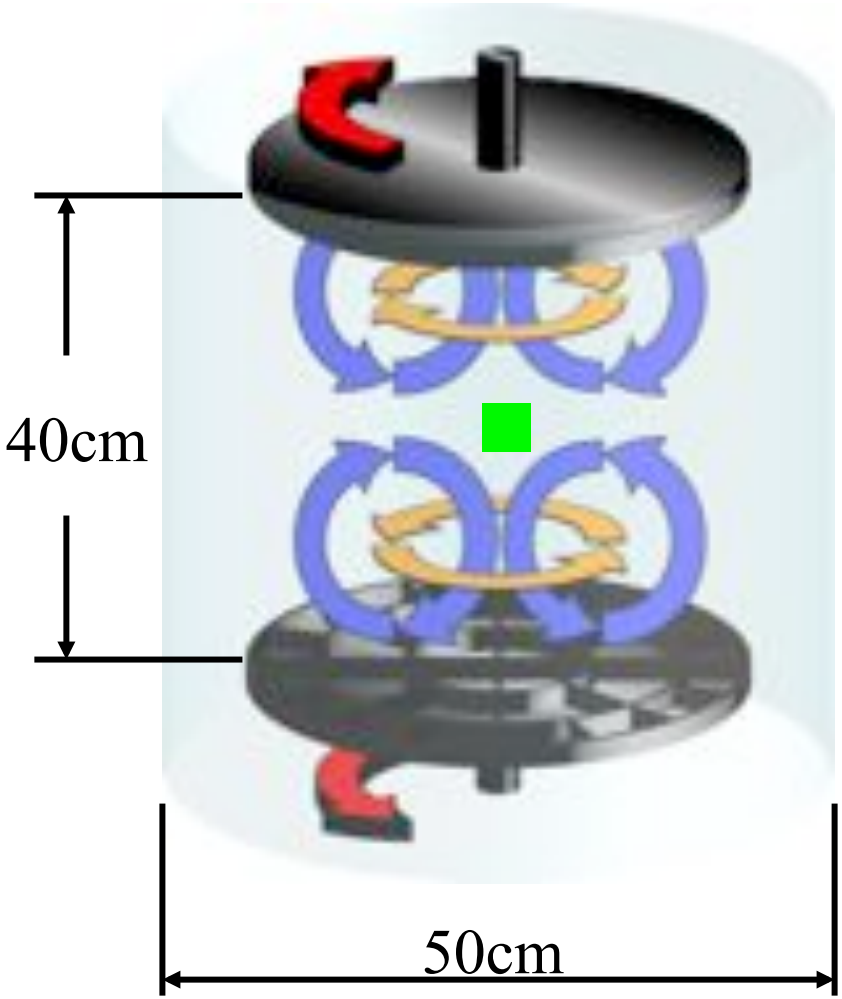
$$\eta = 84\mu m, \quad \tau_\eta = 7.1ms$$

$$Wi = \frac{\tau_p}{\tau_\eta} = 6$$

Change Reynolds number R_λ from 200 to 415,
concentration from 0 to 20 wppm.

Parameters of Experiments

von Karman swirling flow between counter-rotating disks



$$R_\lambda = 200 - 415$$

$$\eta = 190 - 64\ \mu\mathrm{m}$$

$$\tau_\eta = 37 - 4.1\ \mathrm{ms}$$

$$L \equiv \frac{u'^3}{\epsilon} \approx 7\ \mathrm{cm}$$

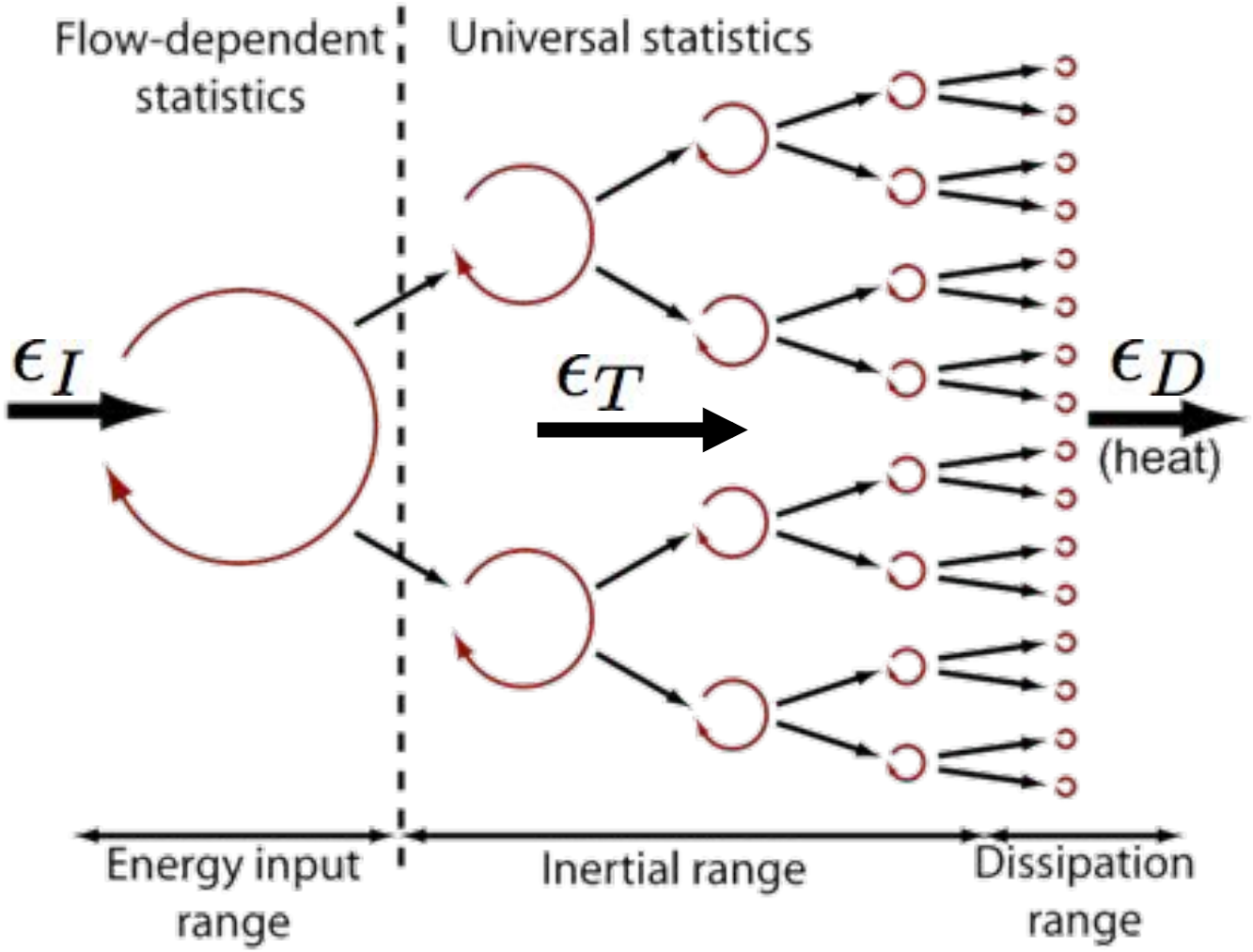
Size of the meas. vol. $(5\mathrm{cm})^3$

Tracer particles: nearly neutrally buoyant fluorescent polystyrene spheres.

$$d_p = 33\mu m,\ \rho_p/\rho_w = 1.06$$

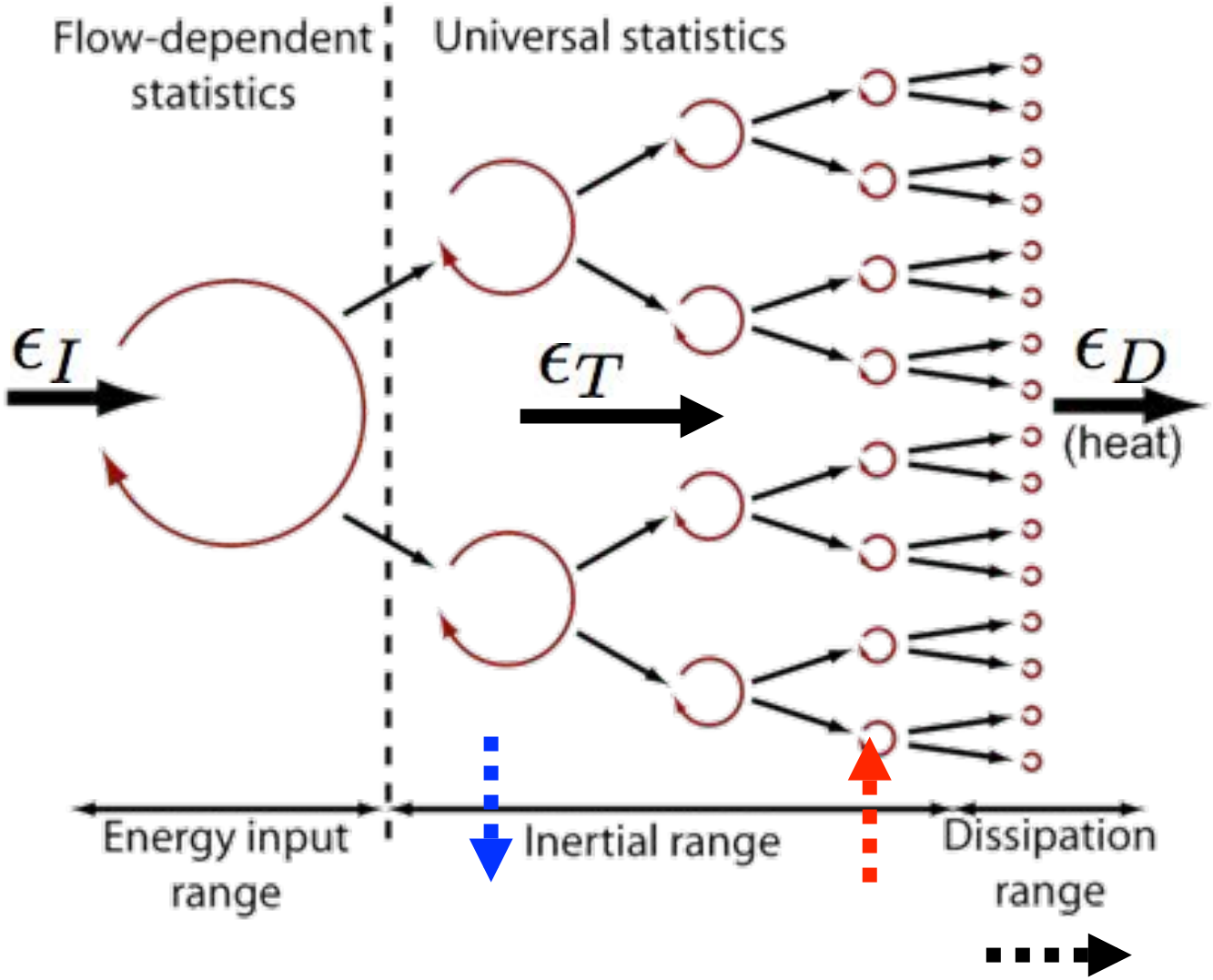
Reynolds (Wi) Effect on Velocity Structure Functions Keep $\phi = 5$ ppm

Turbulence Energy Cascade



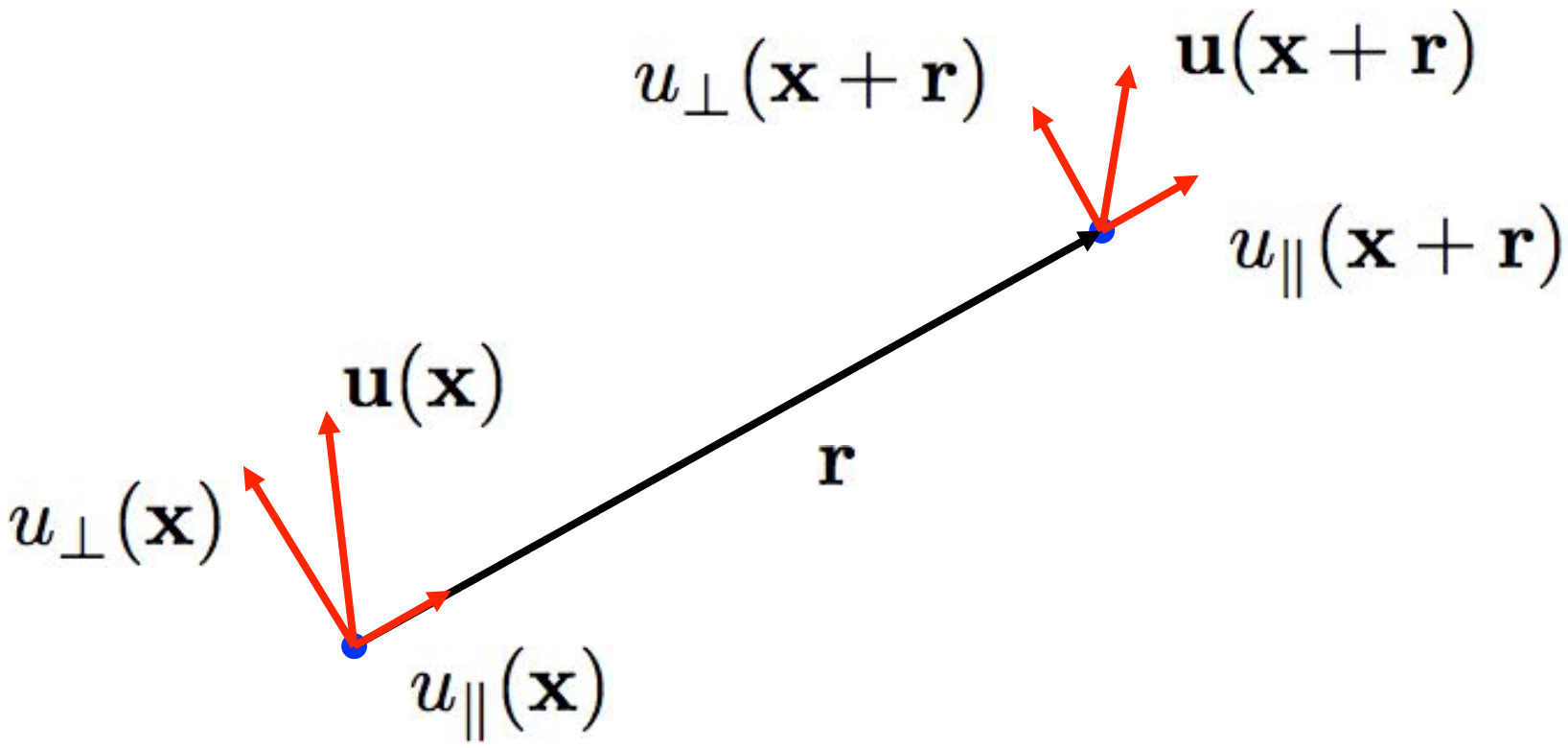
$$\epsilon_I = \epsilon_T = \epsilon_D$$

Turbulence Energy Cascade



$$\epsilon_I \overset{?}{=} \epsilon_T \overset{?}{=} \epsilon_D$$

Eulerian Structure Functions

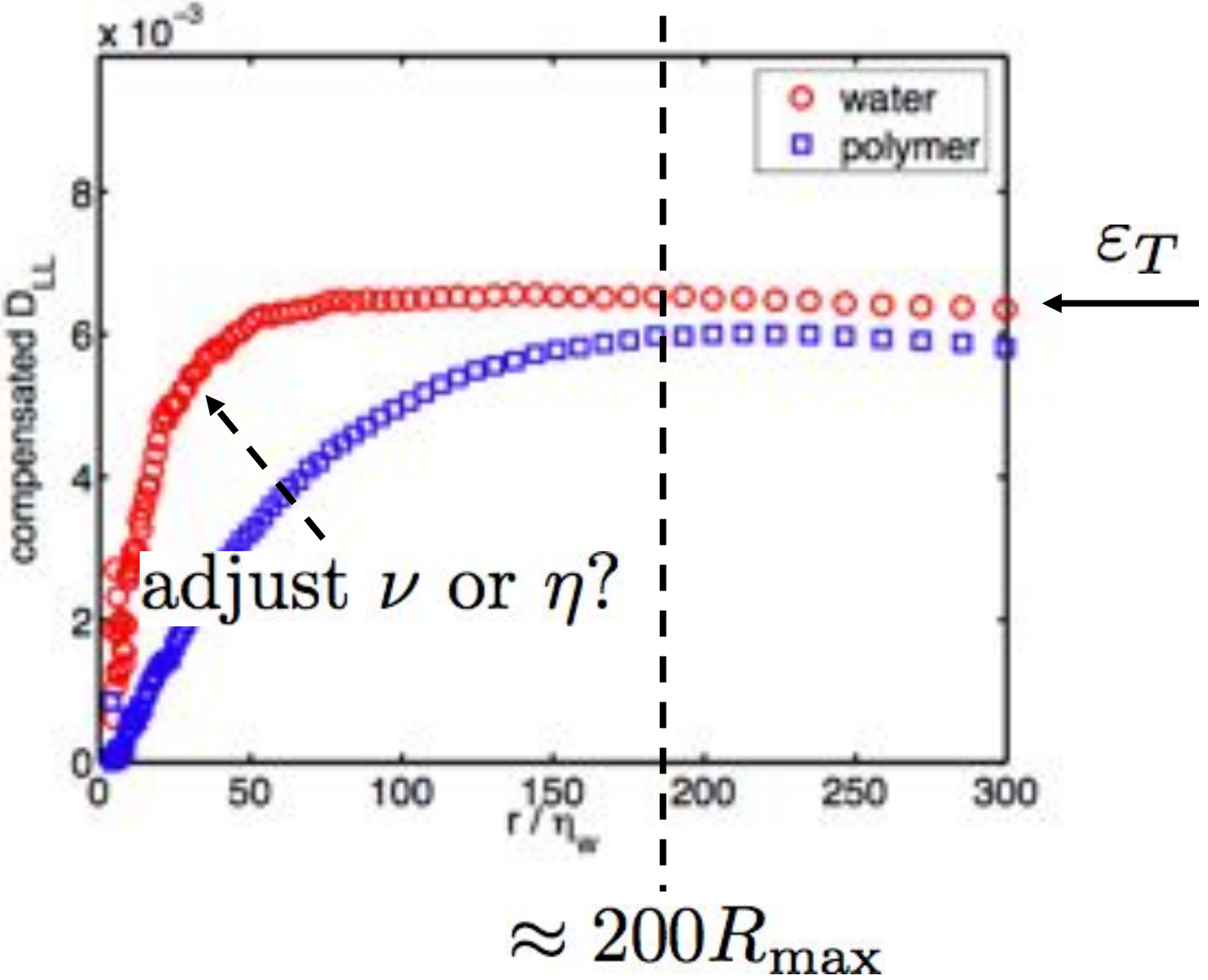


$$D_{NN}(r) = \langle |u_\perp(\mathbf{x}+\mathbf{r}) - u_\perp(\mathbf{x})|^2 \rangle$$

$$D_{LL}(r) = \langle |u_\parallel(\mathbf{x}+\mathbf{r}) - u_\parallel(\mathbf{x})|^2 \rangle$$

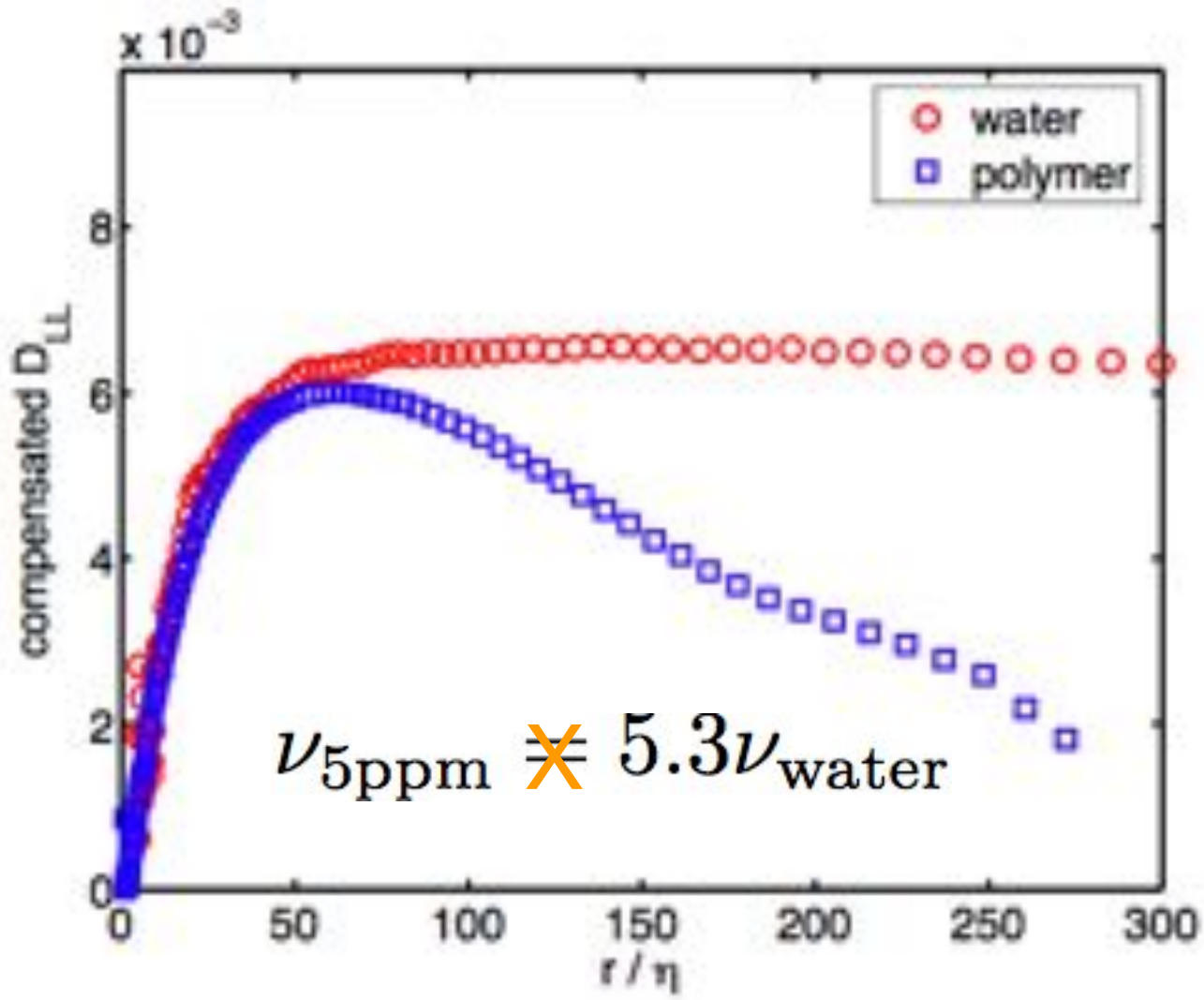
Polymer Effect on Structure Function

$$R_\lambda = 290, \quad \phi = 5 \text{ ppm}$$

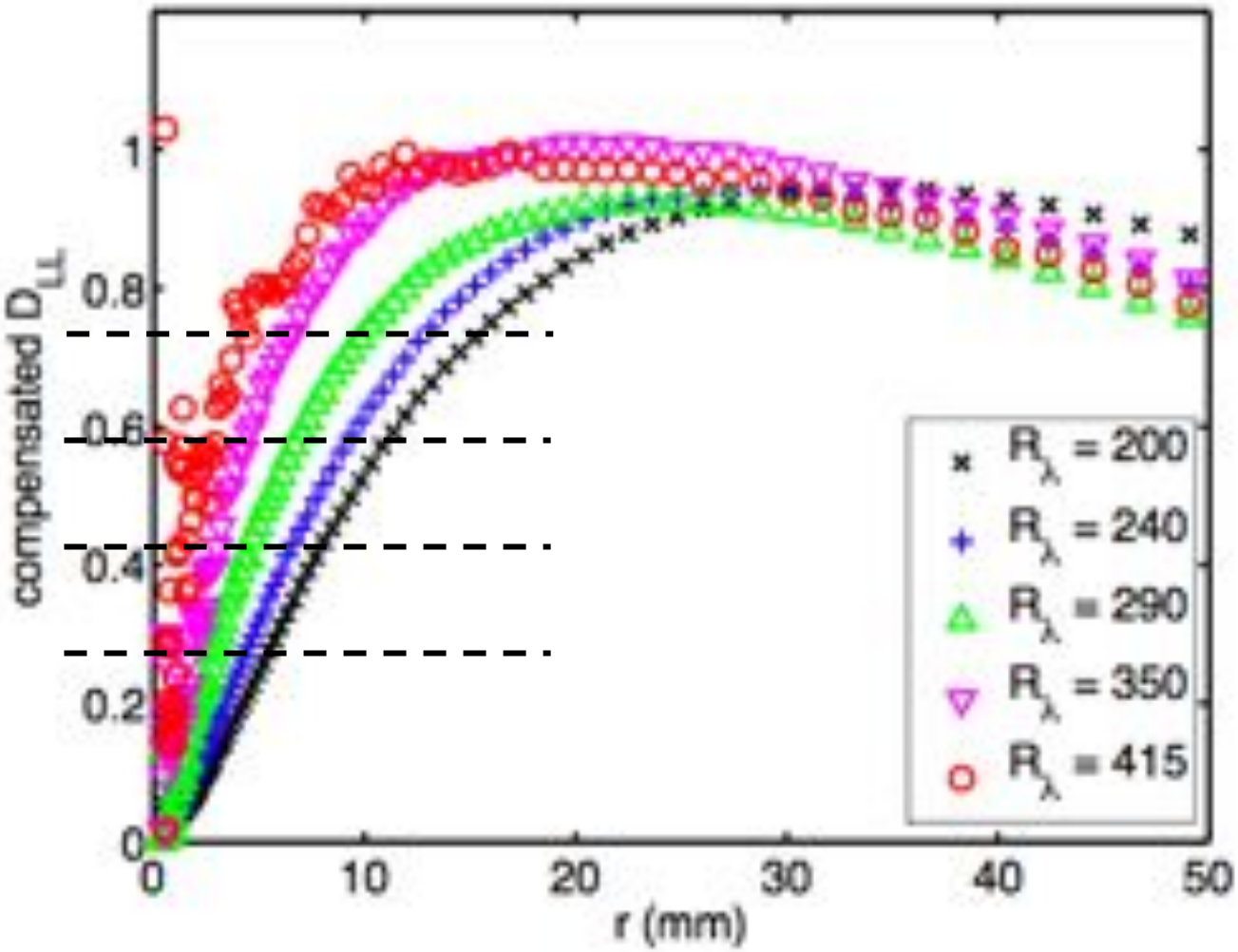


Viscosity Effect?

$$R_\lambda = 290, \quad \phi = 5 \text{ ppm}$$



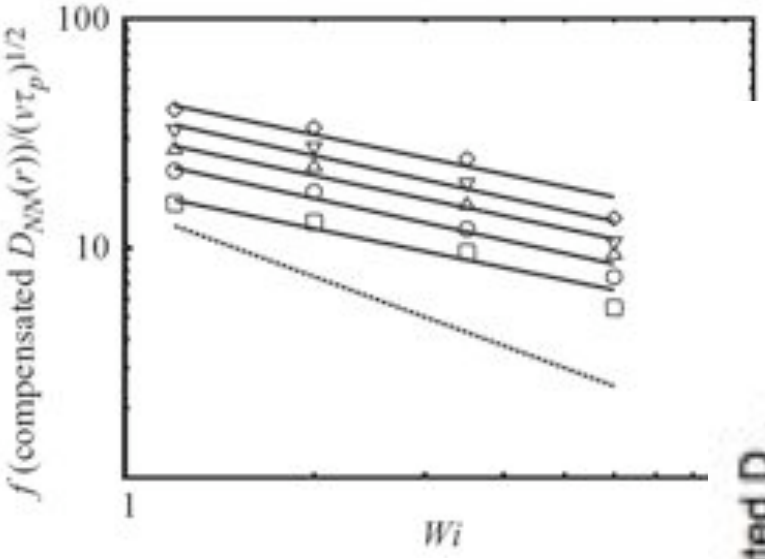
R_λ (Wi) Dependence



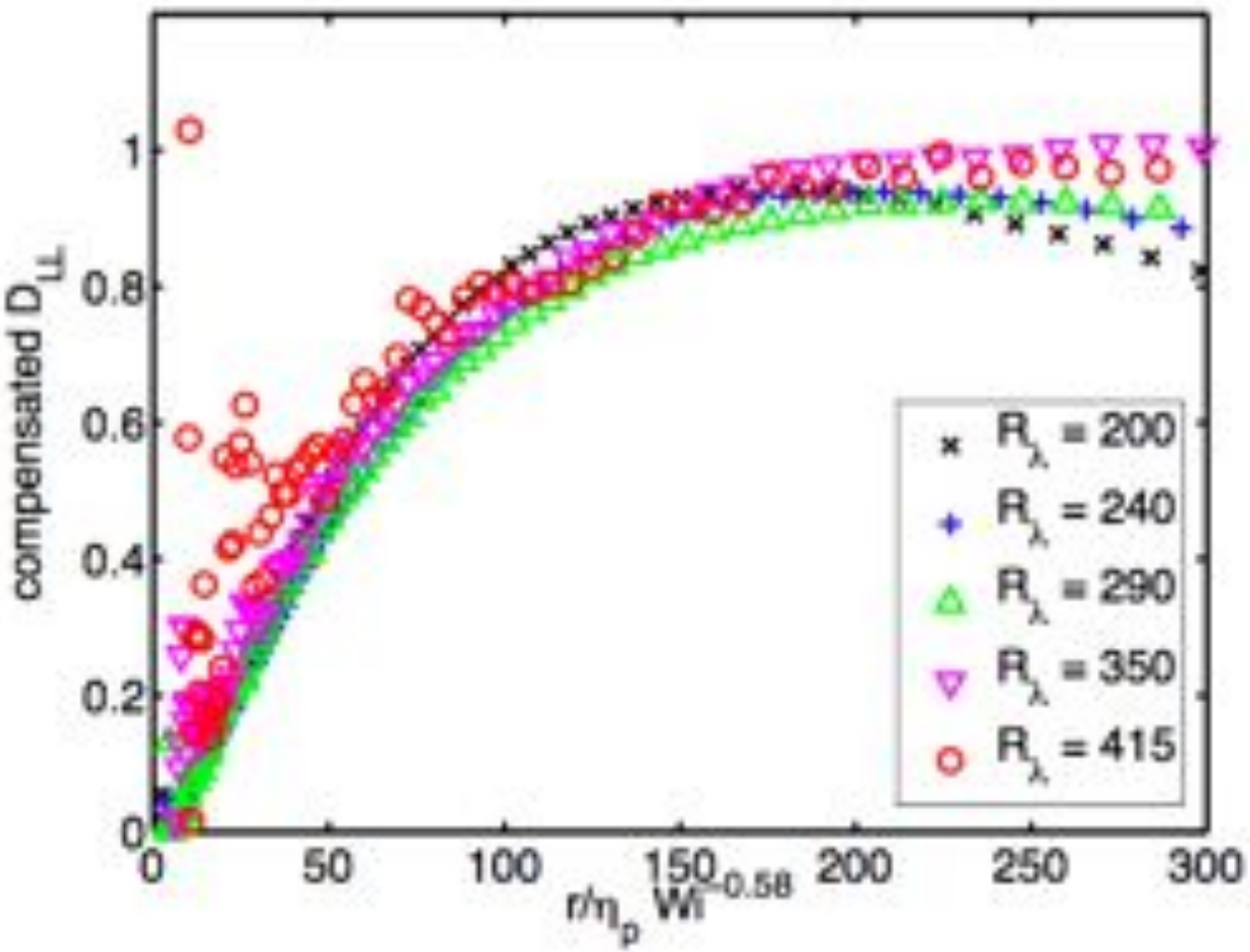
$$r_p \sim \eta_p W i^n$$

$$\eta_p \equiv (\nu \tau_p)^{1/2}$$

Length Scale

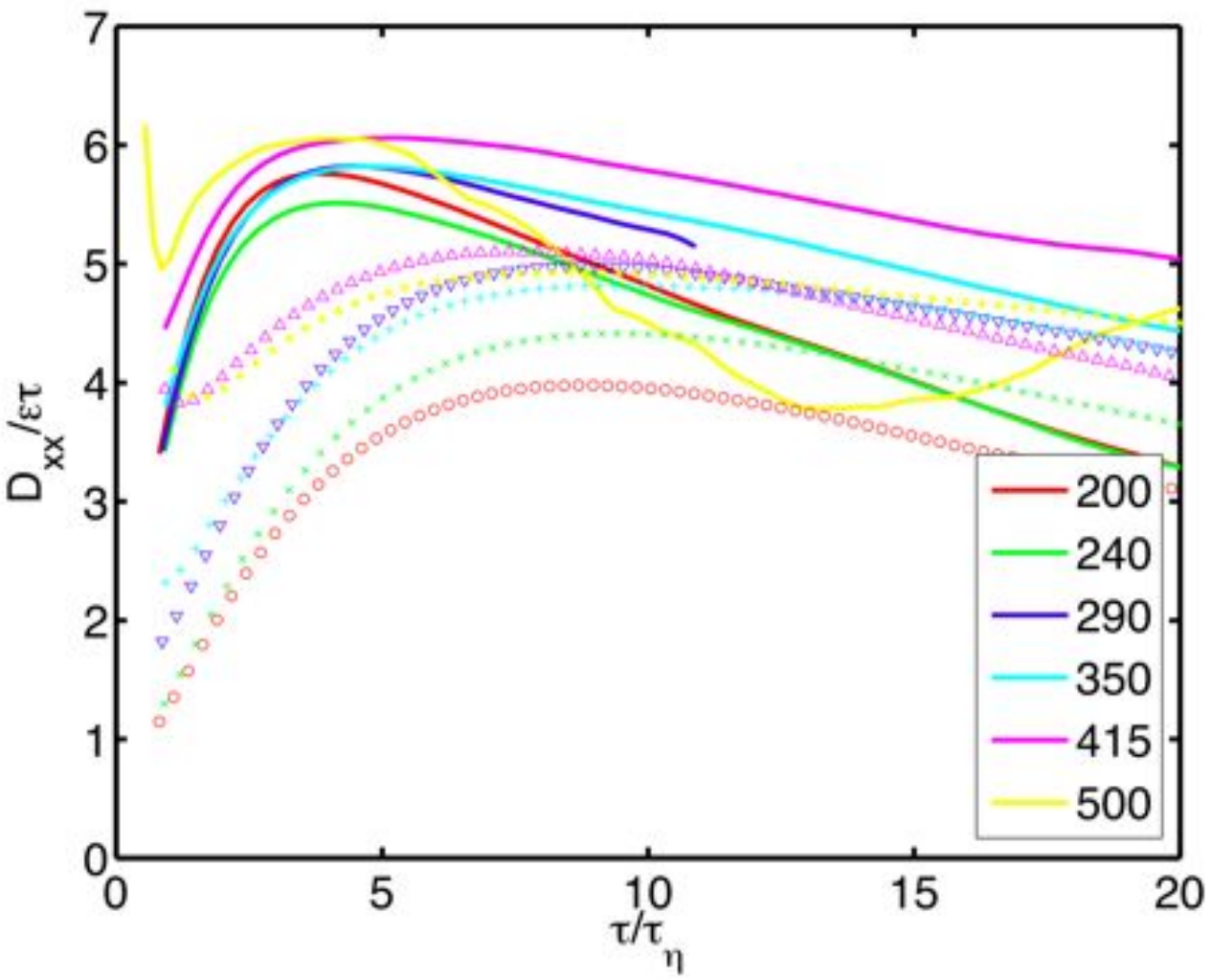


$$r_p \sim \eta_p Wi^{-0.58}$$

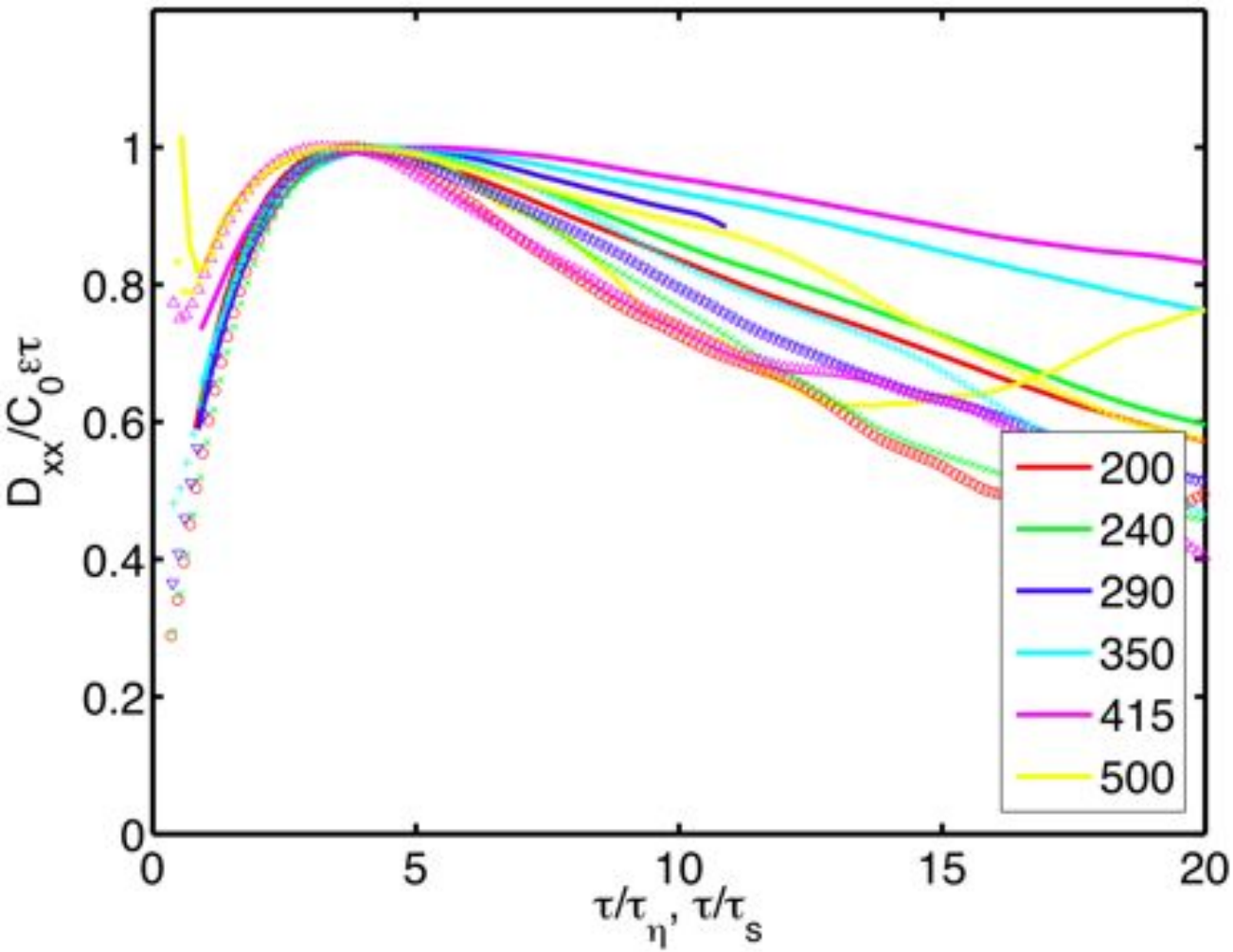


Ouellette, Xu, Bodenschatz (**JFM**, 2009)

Lagrangian Structure Functions



Lagrangian Structure Functions

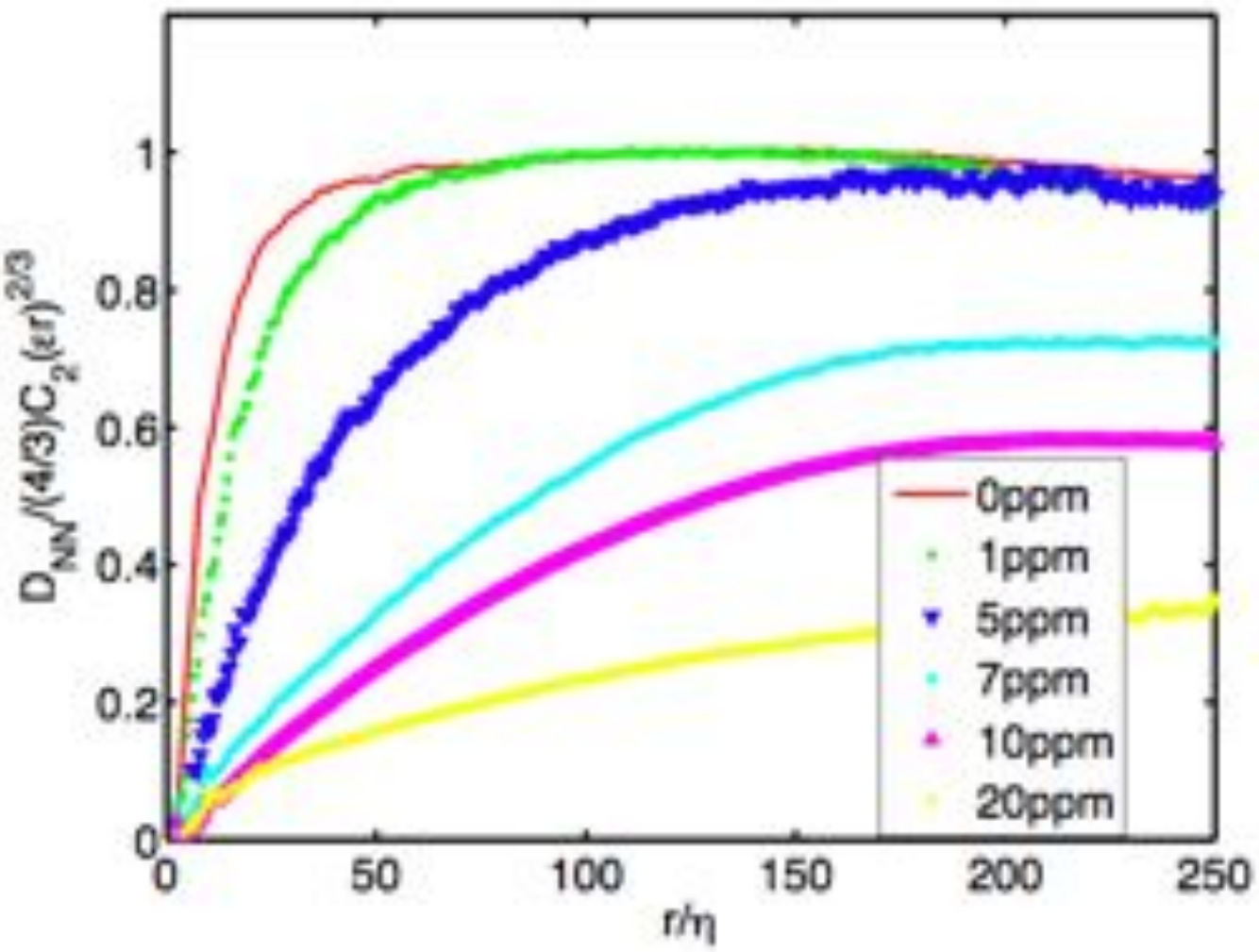


$$\tau_s = (r_p^2/\nu)^{1/2}$$

Concentration Effect on Velocity Structure Functions

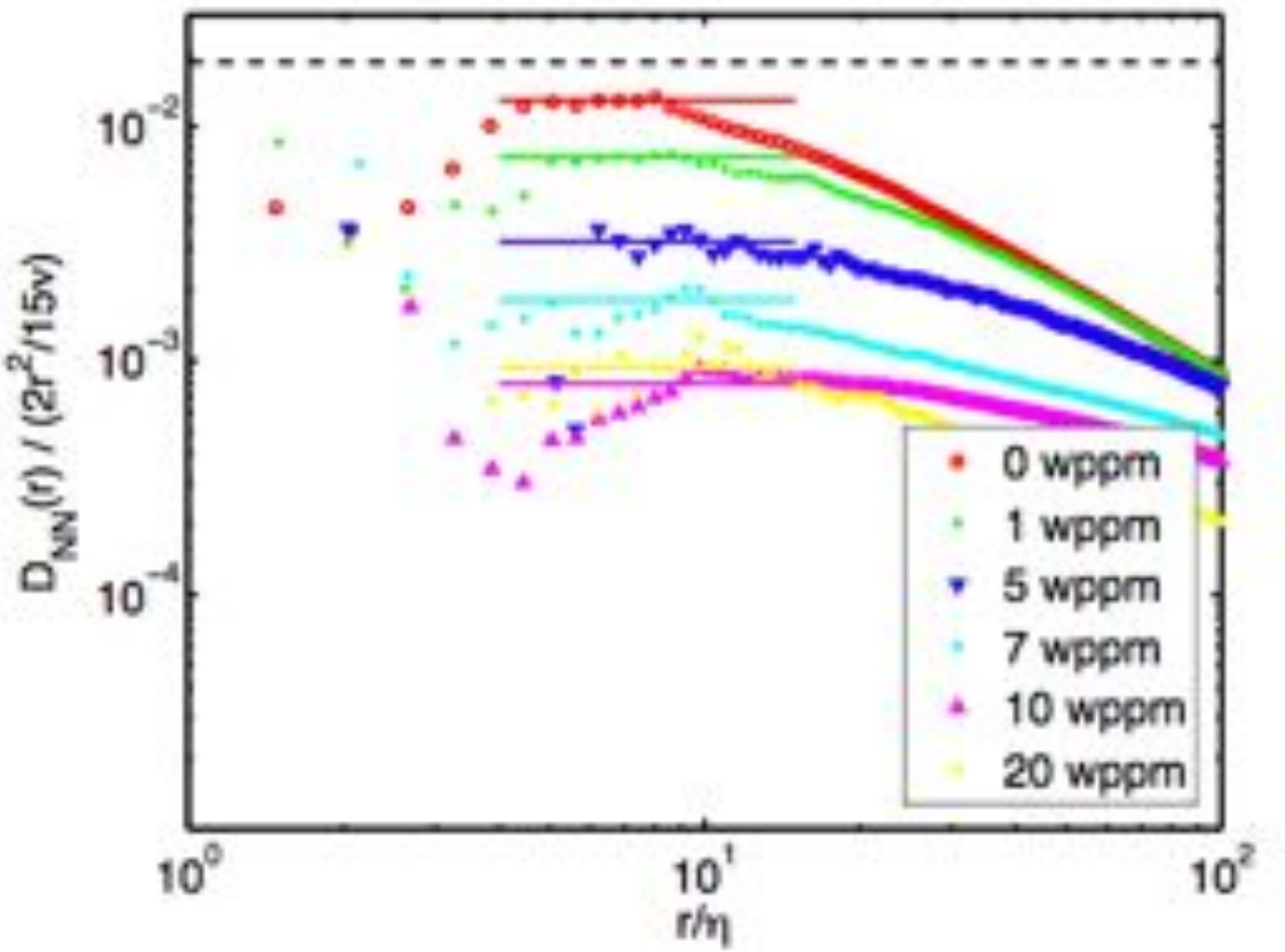
Keep $R_\lambda = 350$

Eulerian Structure Functions



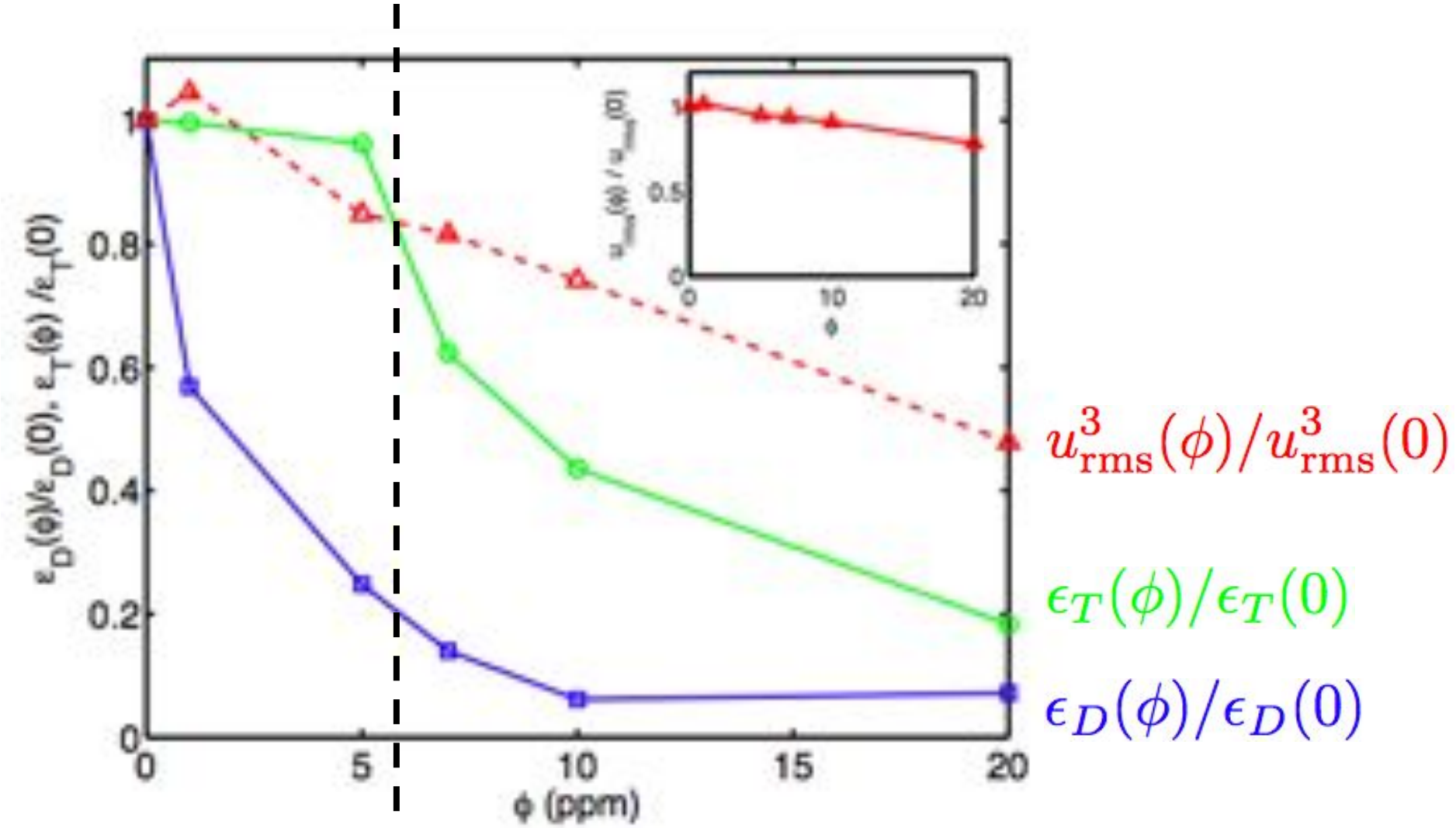
$$D_{NN}(r) = \frac{4}{3} C_2 (\epsilon_T r)^{2/3}; \quad (\eta \ll r \ll L)$$

Eulerian Structure Functions



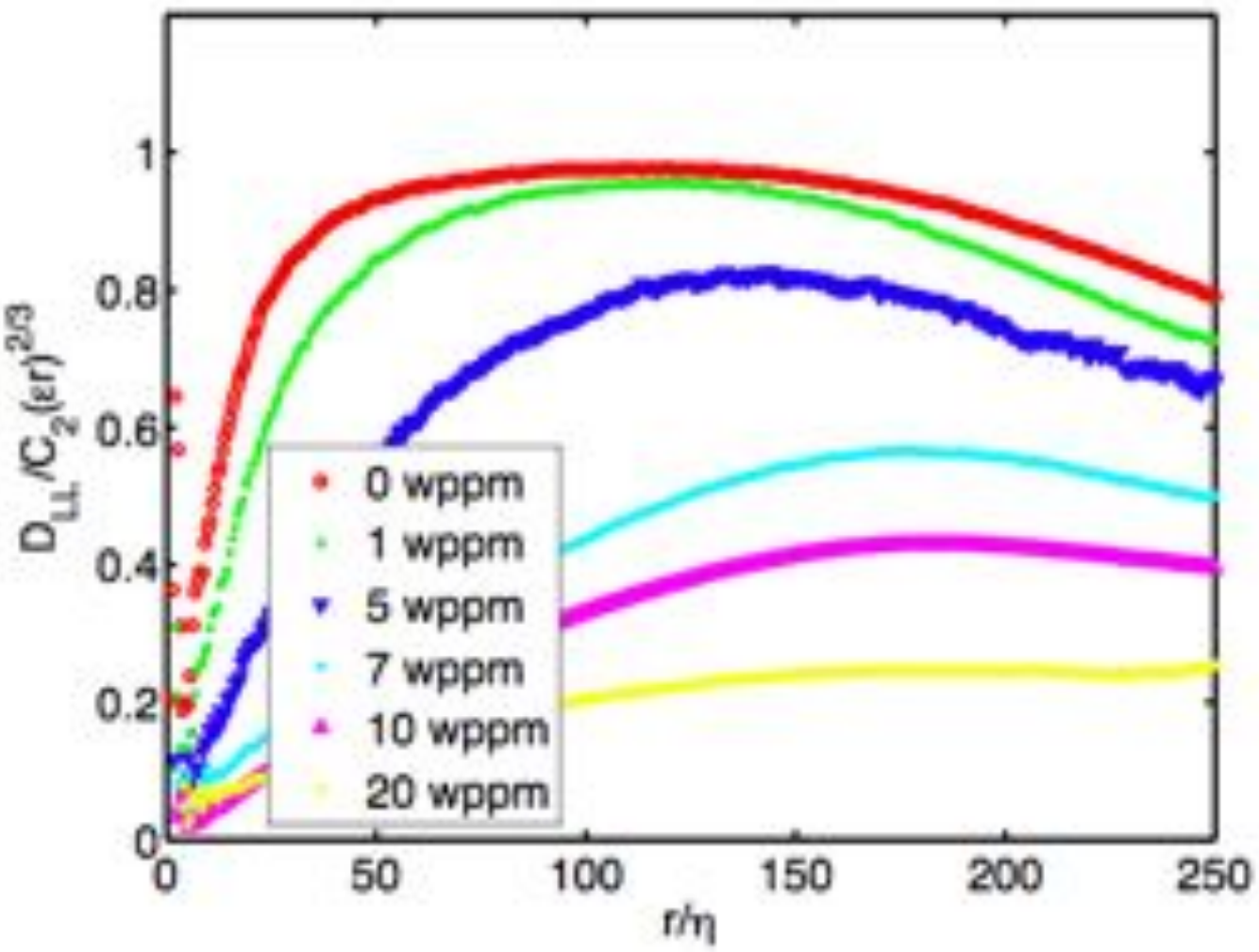
$$D_{NN}(r) = \frac{2\epsilon_D}{15\nu} r^2; \quad (r \ll \eta)$$

Energy Transfer and Dissipation Rates



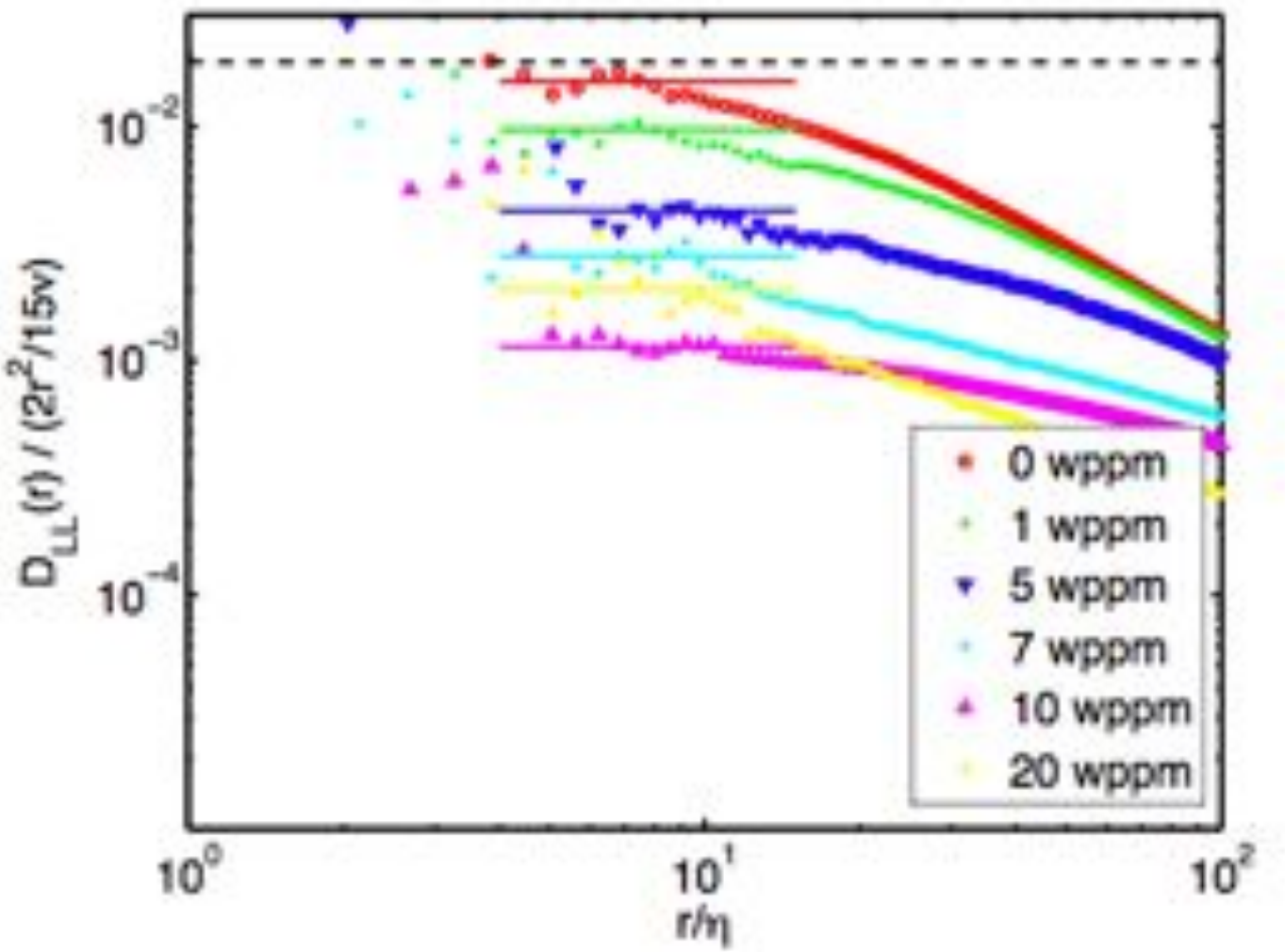
Change of ϵ_T can not be explained by $u_{\rm rms}$ alone.

Eulerian Structure Functions



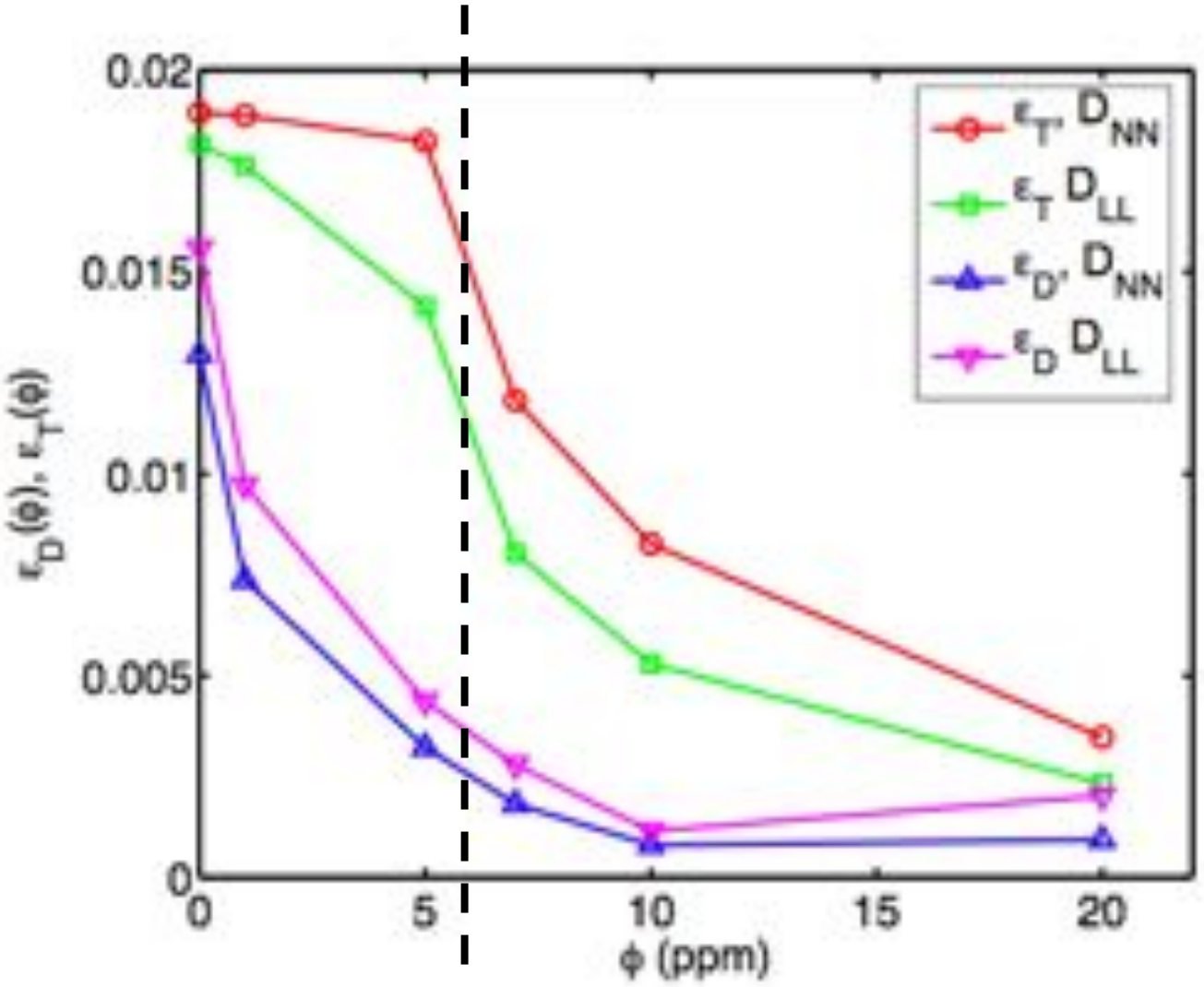
$$D_{LL}(r) = C_2(\epsilon_T r)^{2/3}; \quad (\eta \ll r \ll L)$$

Eulerian Structure Functions



$$D_{LL}(r) = \frac{\epsilon_D}{15\nu} r^2; \quad (r \ll \eta)$$

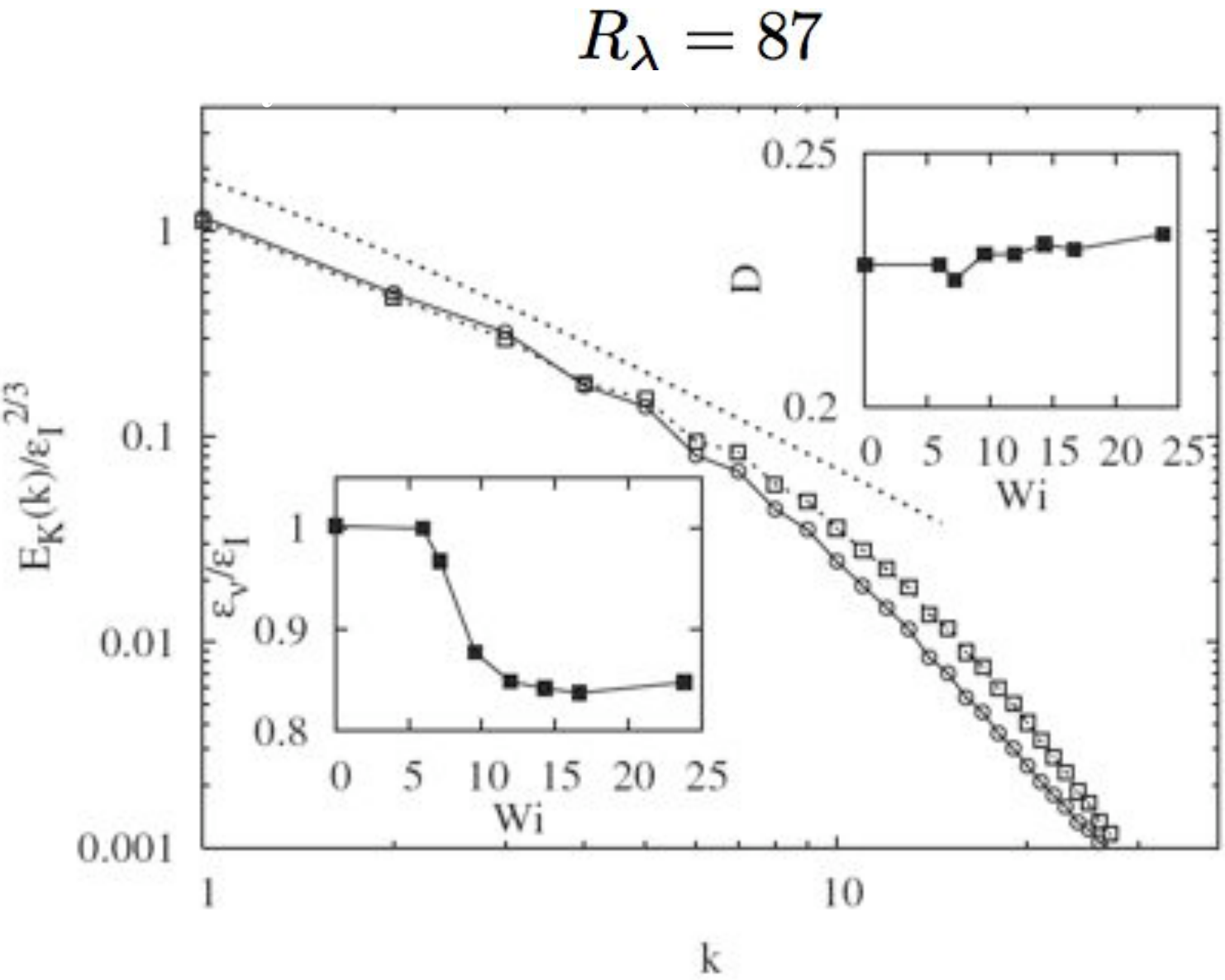
Energy Transfer and Dissipation Rates



A transition in ϵ_T between 5 and 7 ppm.

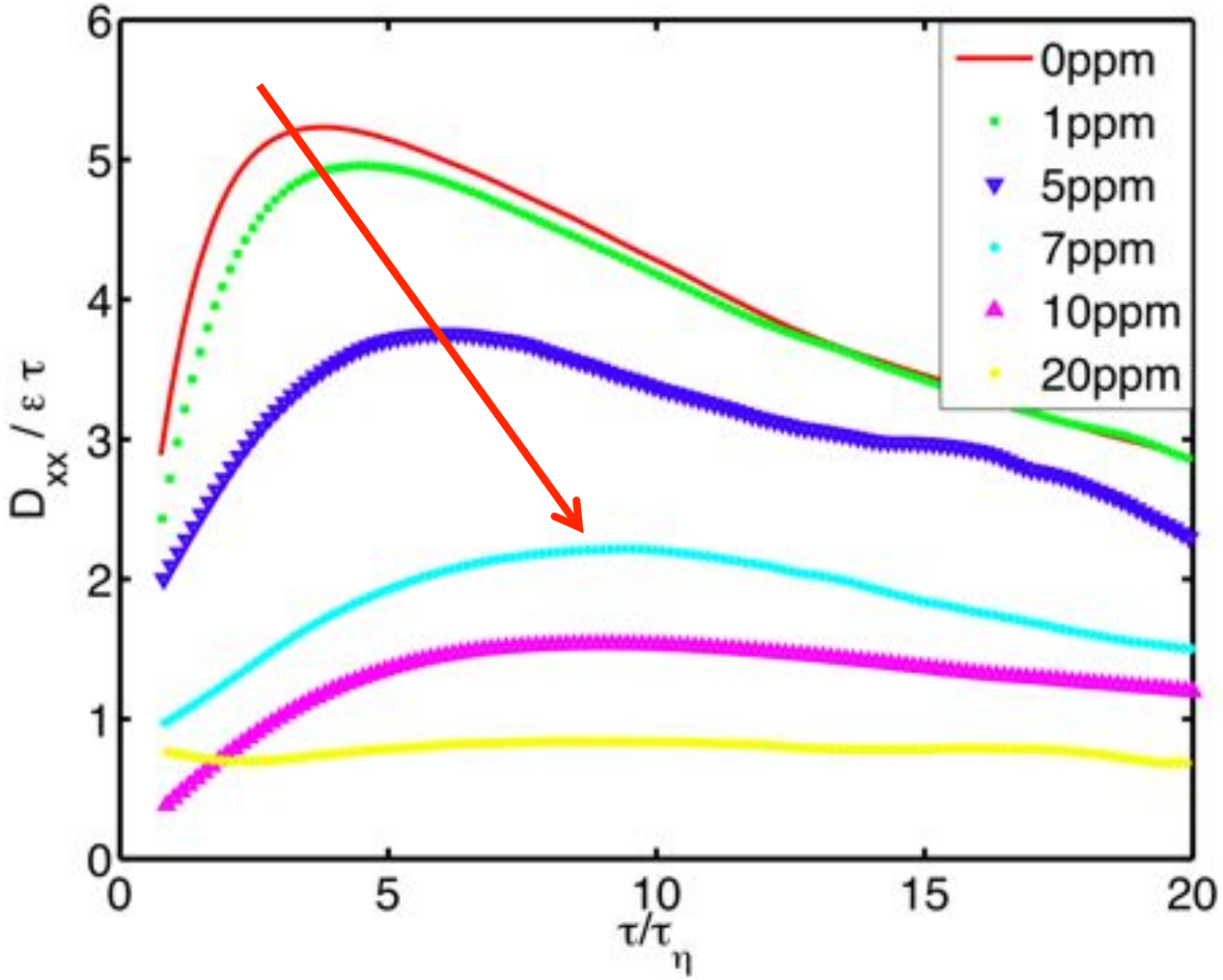
Ouellette, Xu, Bodenschatz (**JFM**, 2009)

Wi (ϕ) Dependence from DNS



Berti et al. (EPL, 2006): linear model for polymers, Wi ←→ ϕ

Lagrangian Structure Functions



Peak shifts to larger time lags. Related to ϵ_D or ϵ_I ?

Conclusions

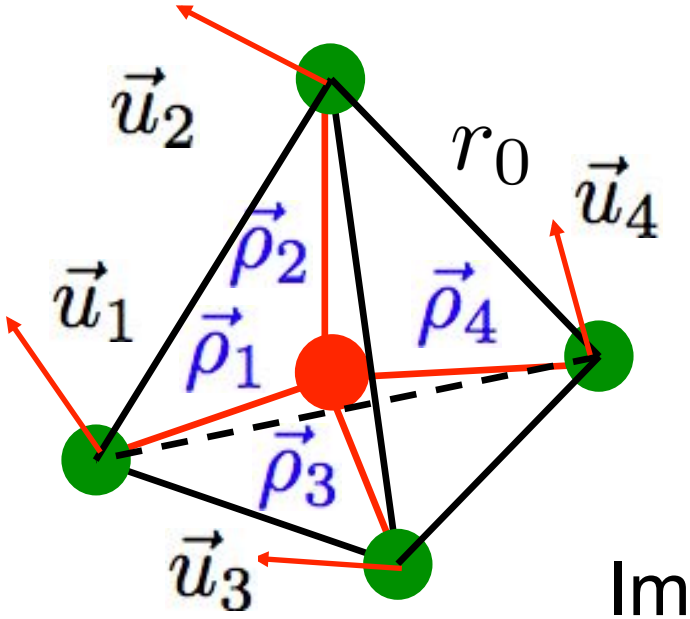
- Effect of polymers at varying Wi (by varying R_λ) on Eulerian velocity structure functions may be normalized with re-scaled separation. The length-scale has a (weak?) Wi dependence.

- The effect of polymer concentration on *apparent* energy transfer rate and energy dissipation rate is different.

- The effect of both Wi (R_λ) and concentration may be explained from De Gennes' elastic theory (Xi et al, APS/DFD, 2011).

Concentration Effect on Flow Structure

Q-R Projection of Velocity Gradient



Velocity gradient *perceived* by tetrads:

$$\mathbf{M}: \quad \mathbf{M}\vec{\rho}_i = \vec{u}_i - \frac{1}{4}\sum_i \vec{u}_i$$

Impose incompressibility: $\mathrm{tr}(\mathbf{M}) = 0$

Nontrivial invariants:

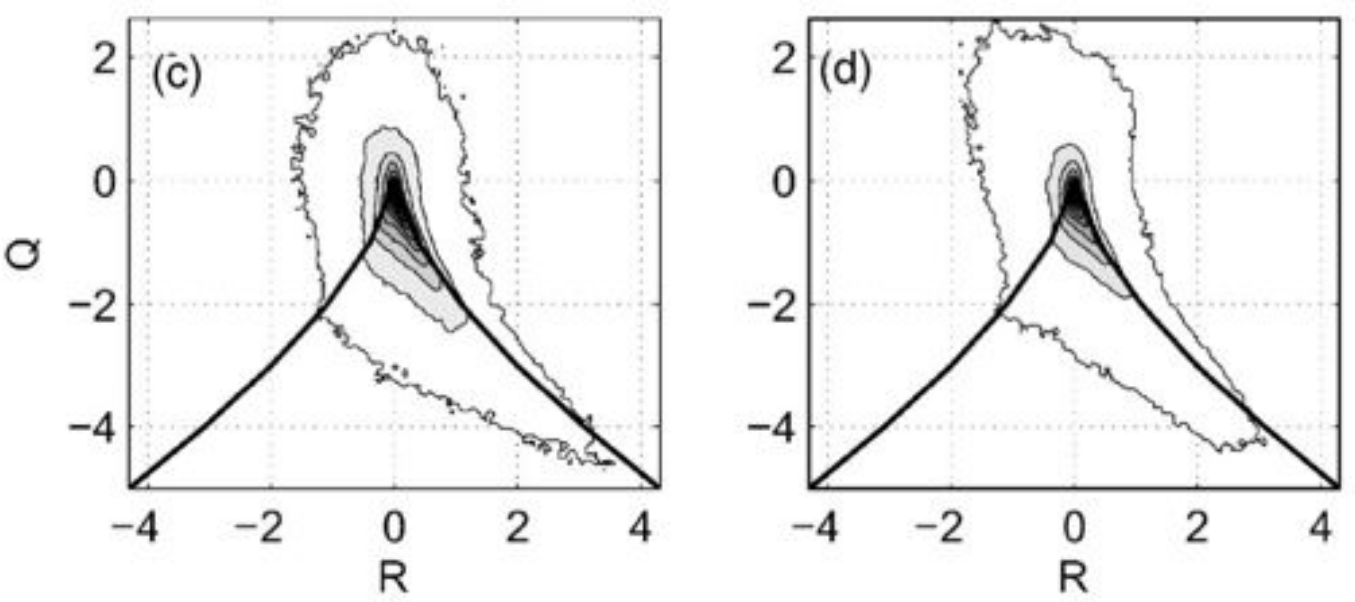
$$Q = -\frac{1}{2}\mathrm{tr}(\mathbf{M}^2) \qquad R = -\frac{1}{3}\mathrm{tr}(\mathbf{M}^3)$$

M reduces to *true* velocity gradient when $r_0 < \eta$.

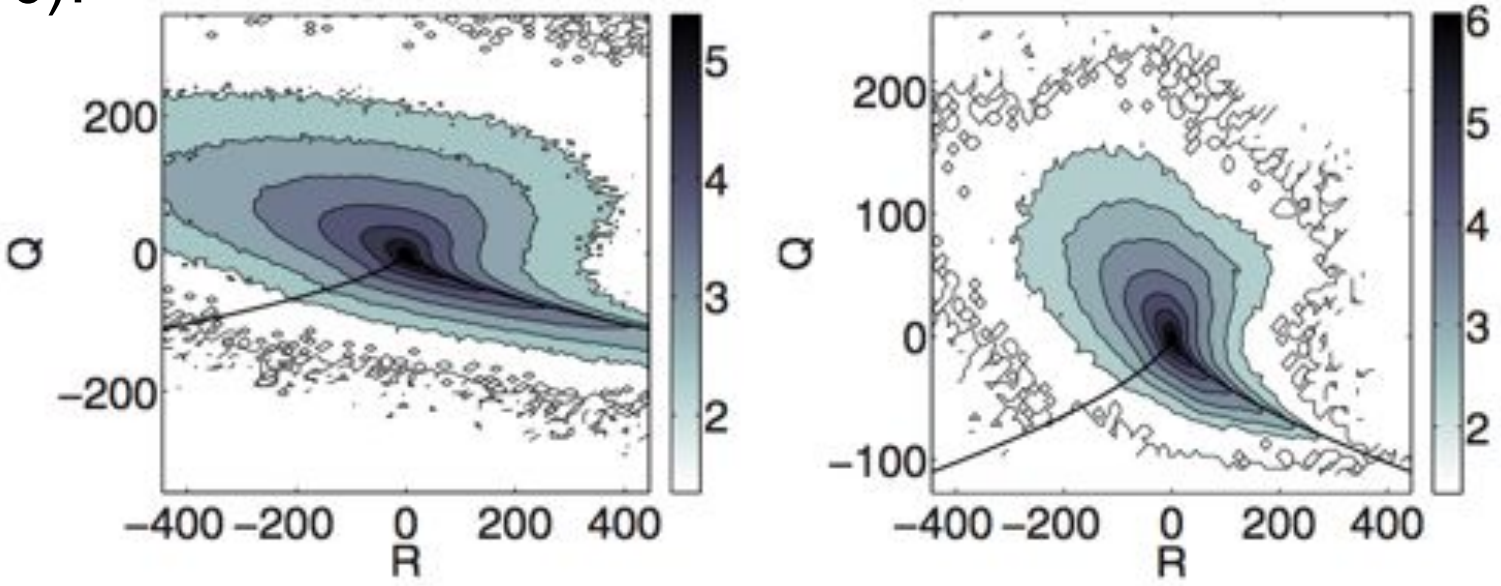
Chertkov, Pumir, Shraiman (**Phys. Fluids**, 1999)

Previous Observations (Dissipative Scale)

Liberzon et al (2006):
Experiment
20 ppm PEO, $R_\lambda = 50$

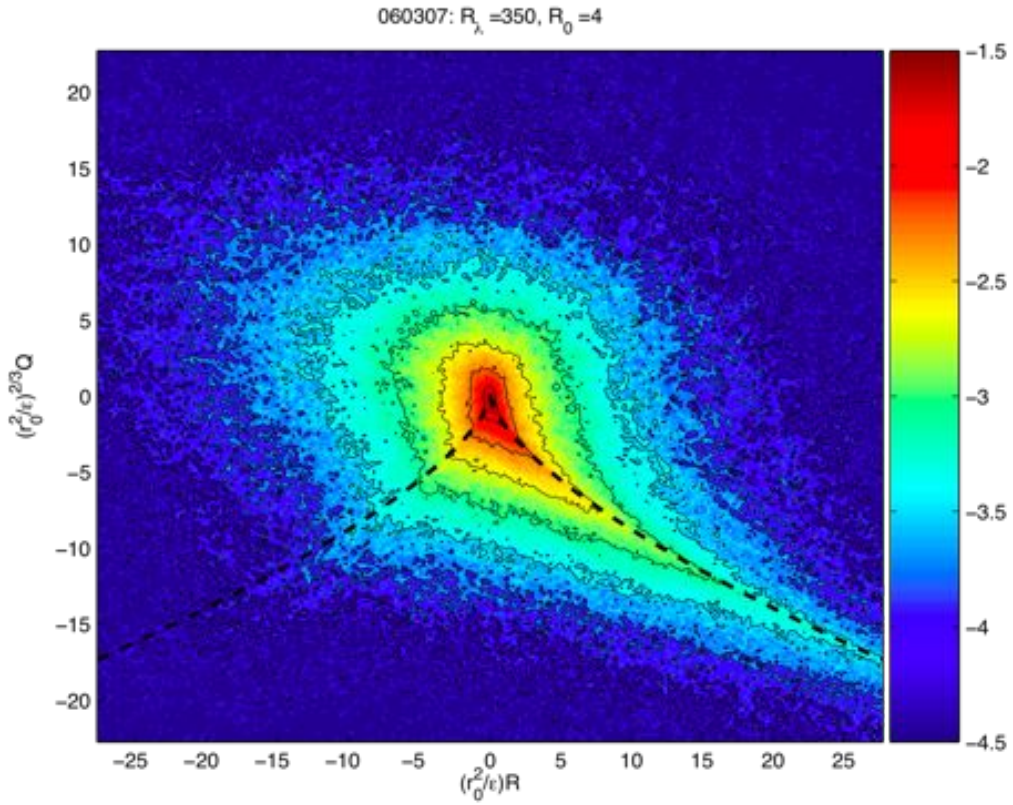


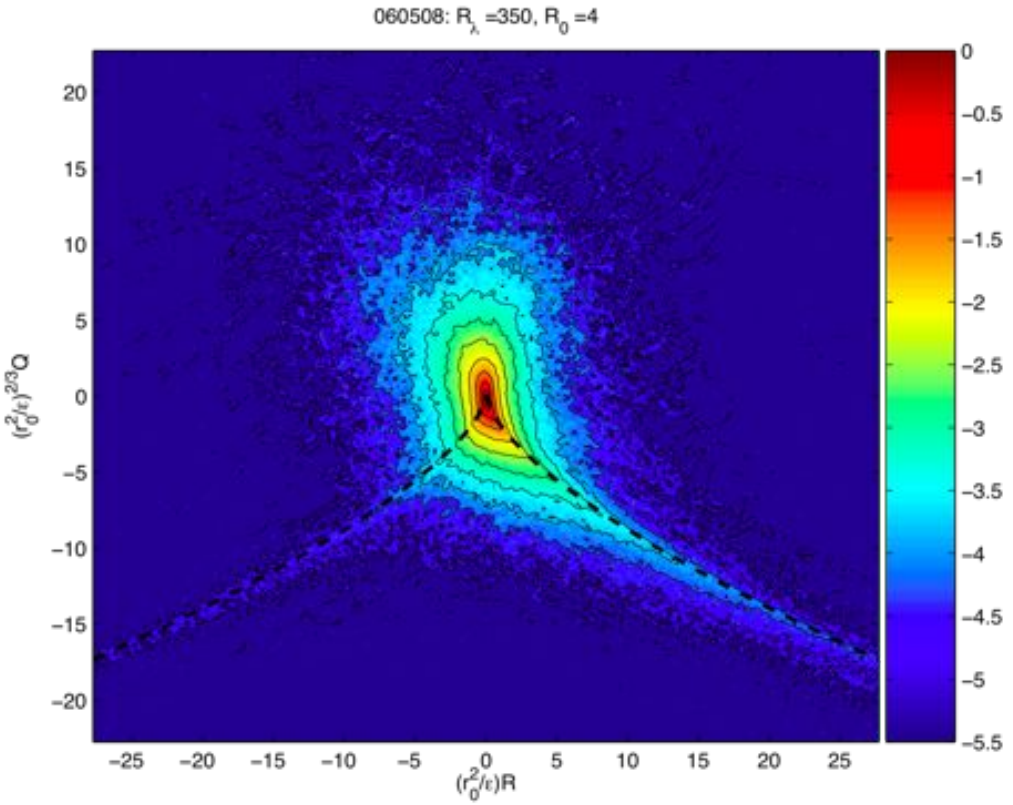
Perlekar et al (2010):
DNS, $R_\lambda = 80$



Experimental Observation

$$R_\lambda = 350, \quad r_0/\eta \approx 48$$



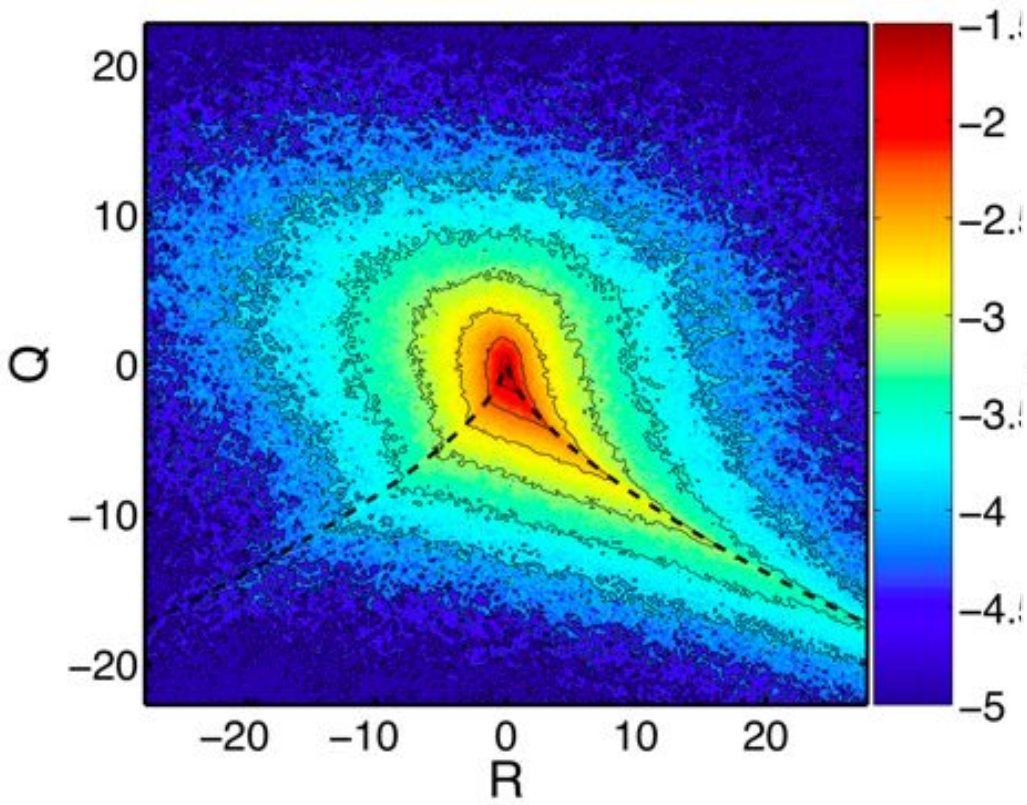


Water

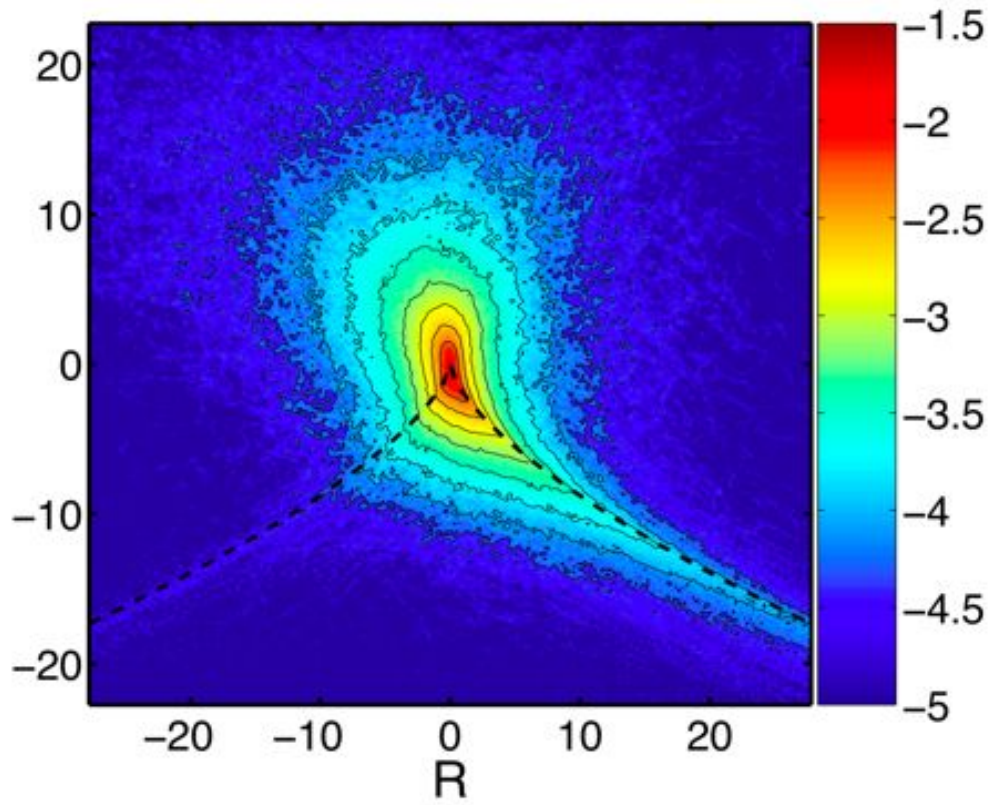
PAM, ϕ =10ppm

Experimental Observation

$$R_\lambda = 350, \quad r_0/\eta \approx 71$$



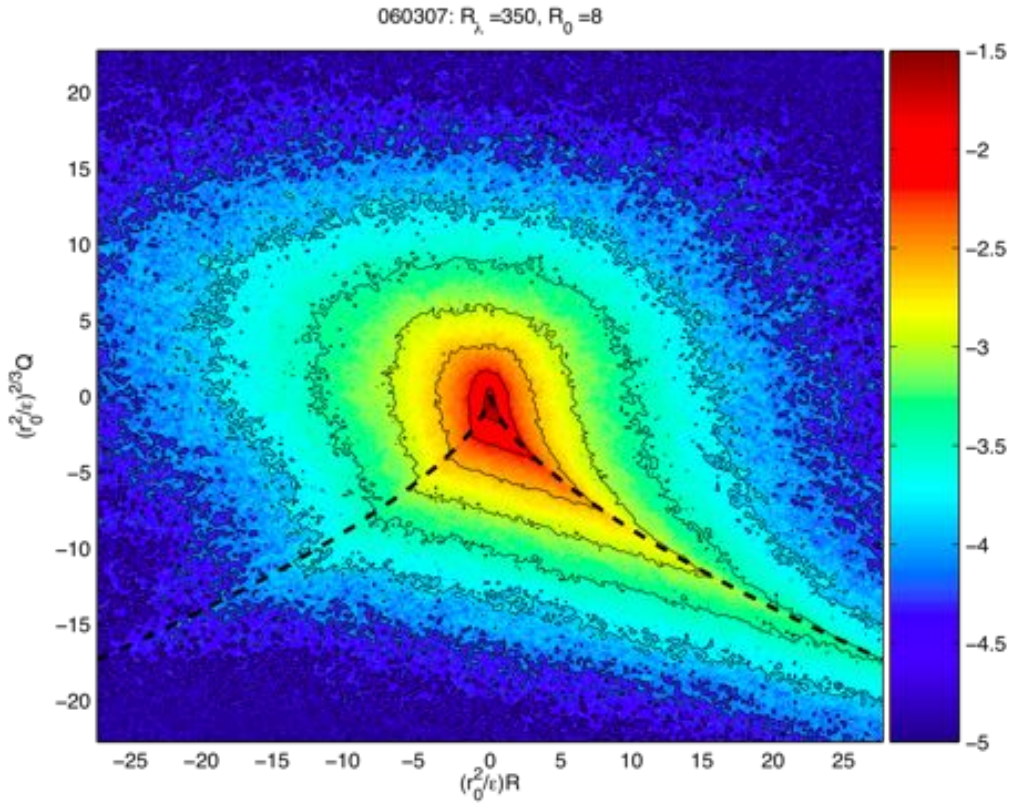
Water

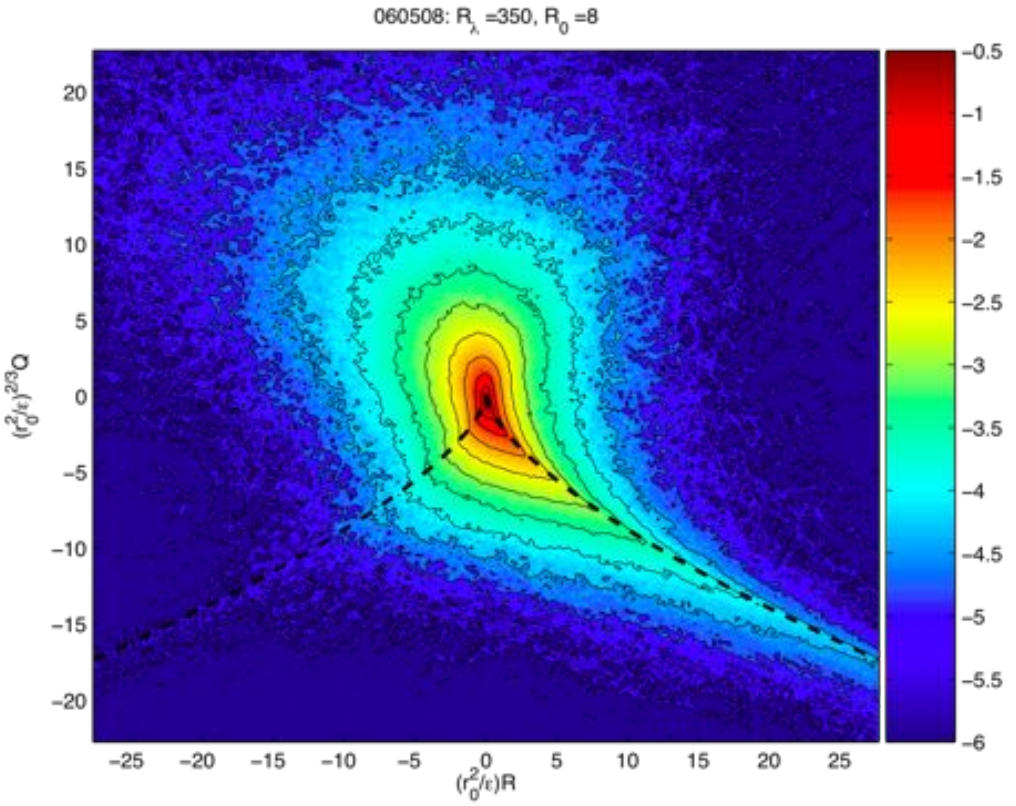


PAM, ϕ =10ppm

Experimental Observation

$$R_\lambda = 350, \quad r_0/\eta \approx 95$$



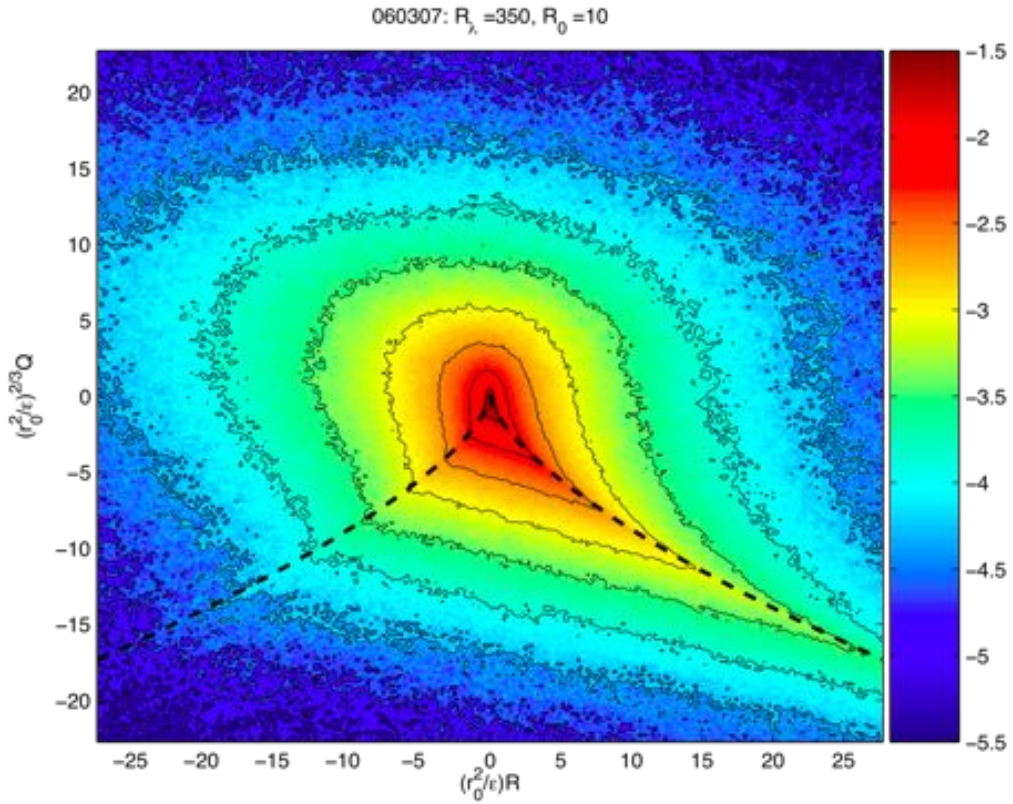


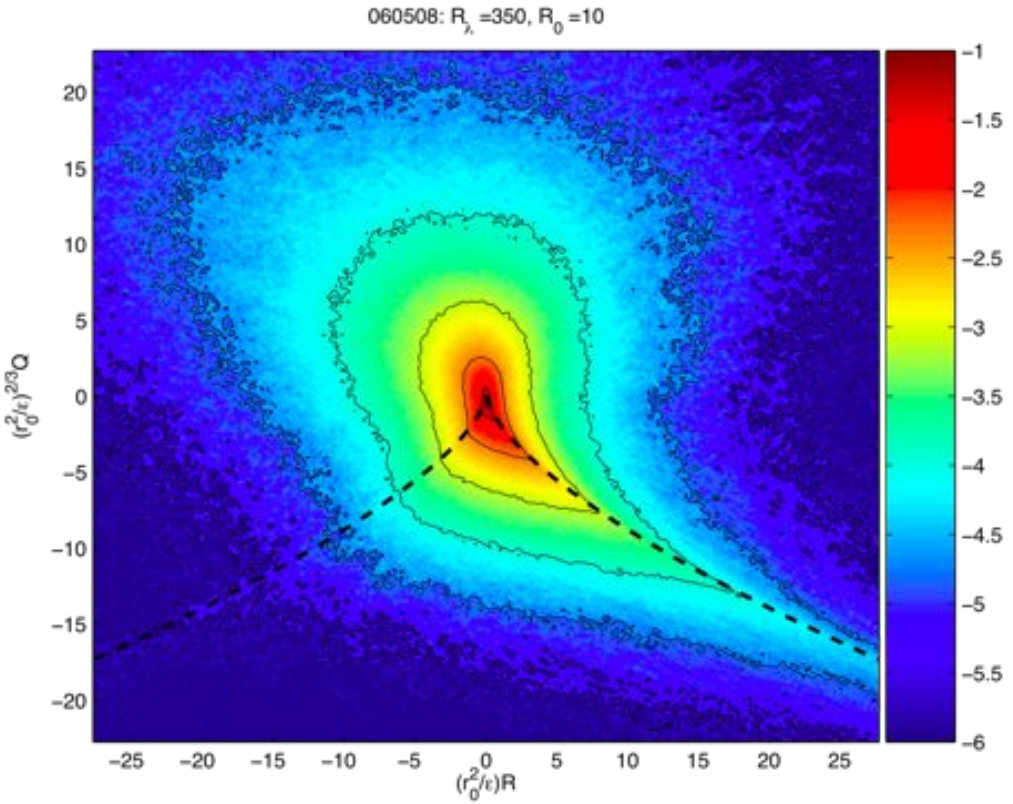
Water

PAM, ϕ =10ppm

Experimental Observation

$$R_\lambda = 350, \quad r_0/\eta \approx 119$$



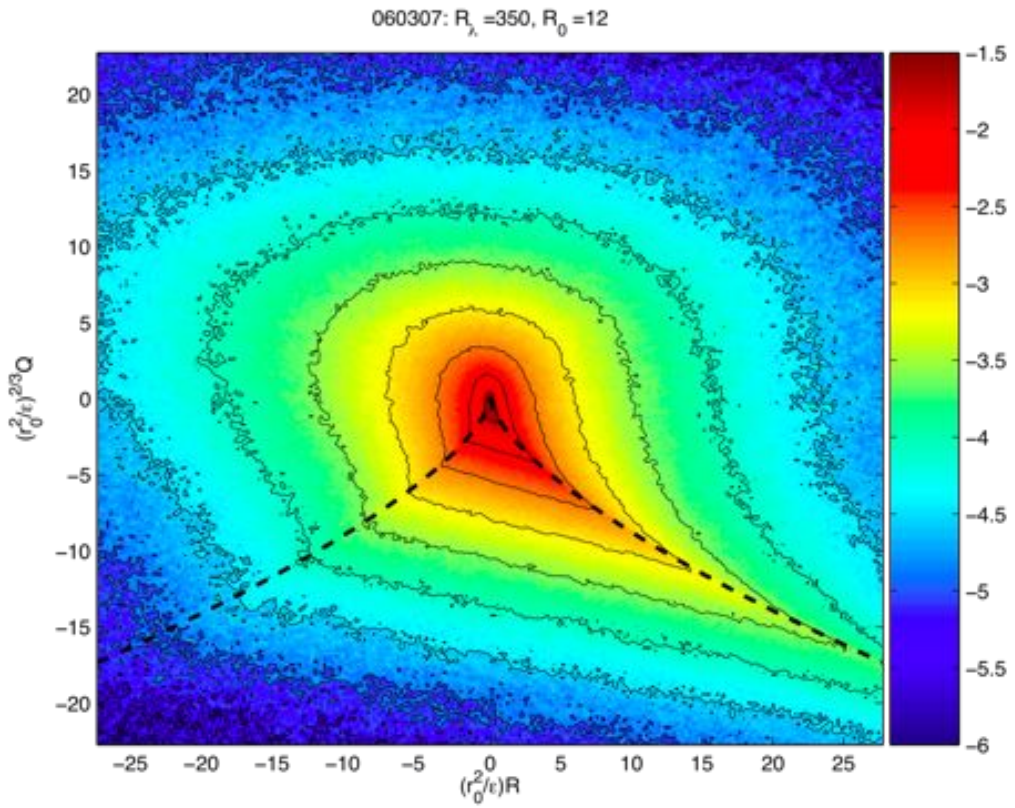


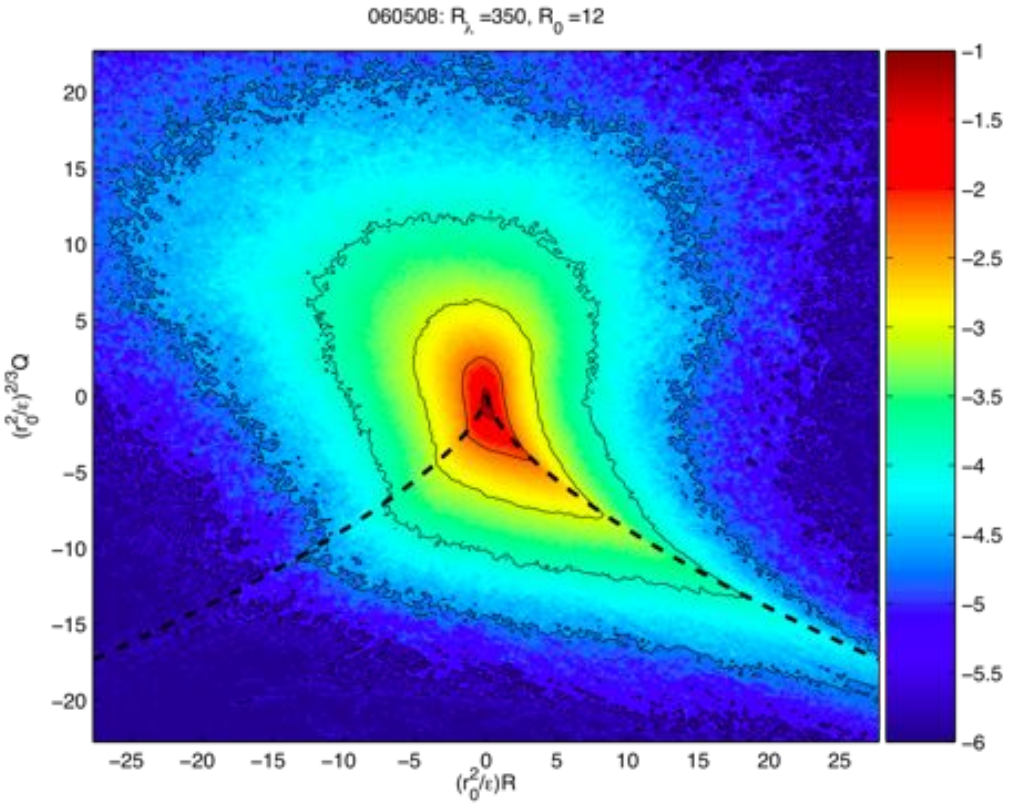
Water

PAM, ϕ =10ppm

Experimental Observation

$$R_\lambda = 350, \quad r_0/\eta \approx 143$$



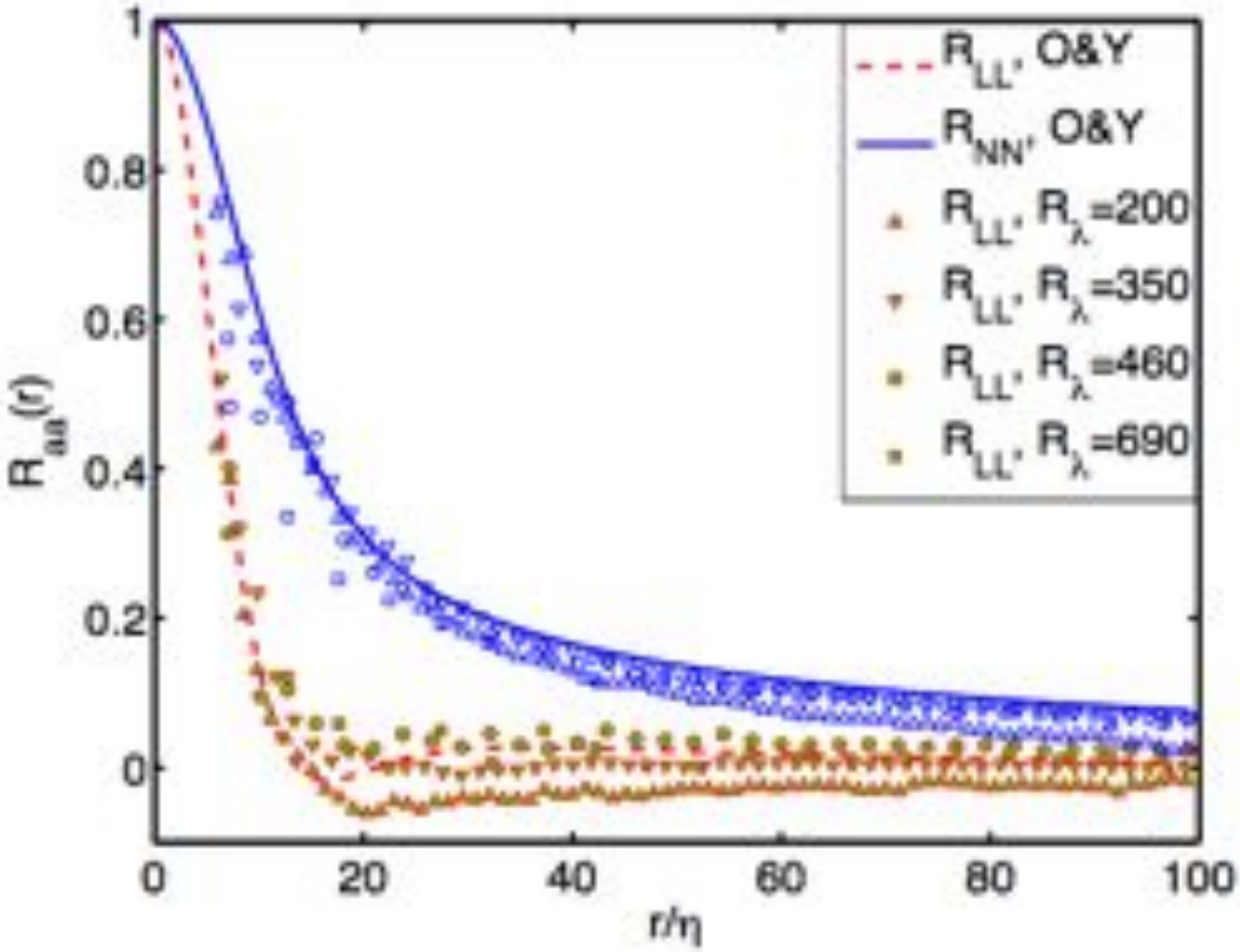


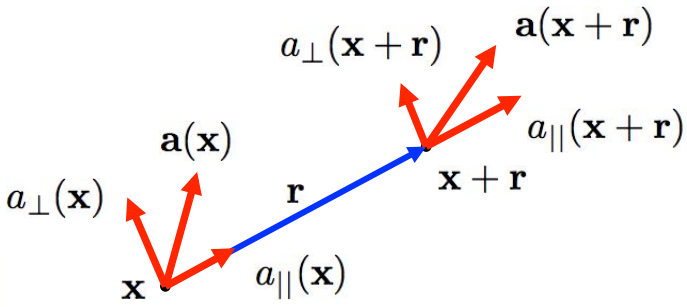
Water

PAM, ϕ =10ppm

Concentration Effect on Acceleration Correlations: Eulerian

Spatial Correlations of Acceleration





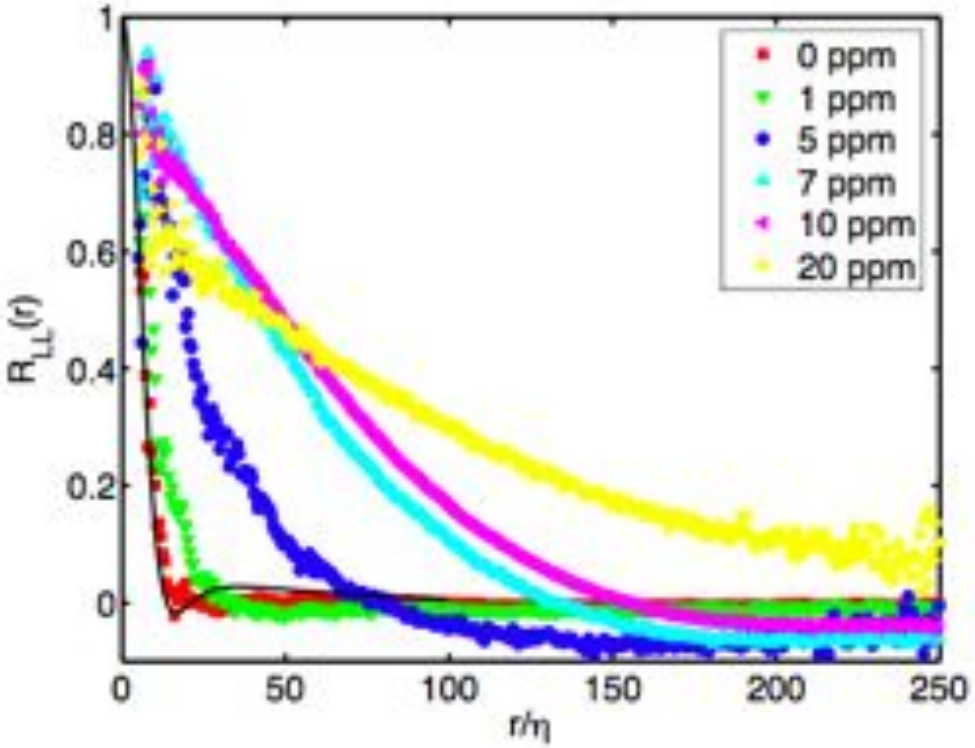
$$R_{LL}(r) = \langle a_{||}(\mathbf{x}+\mathbf{r}) a_{||}(\mathbf{x}) \rangle$$

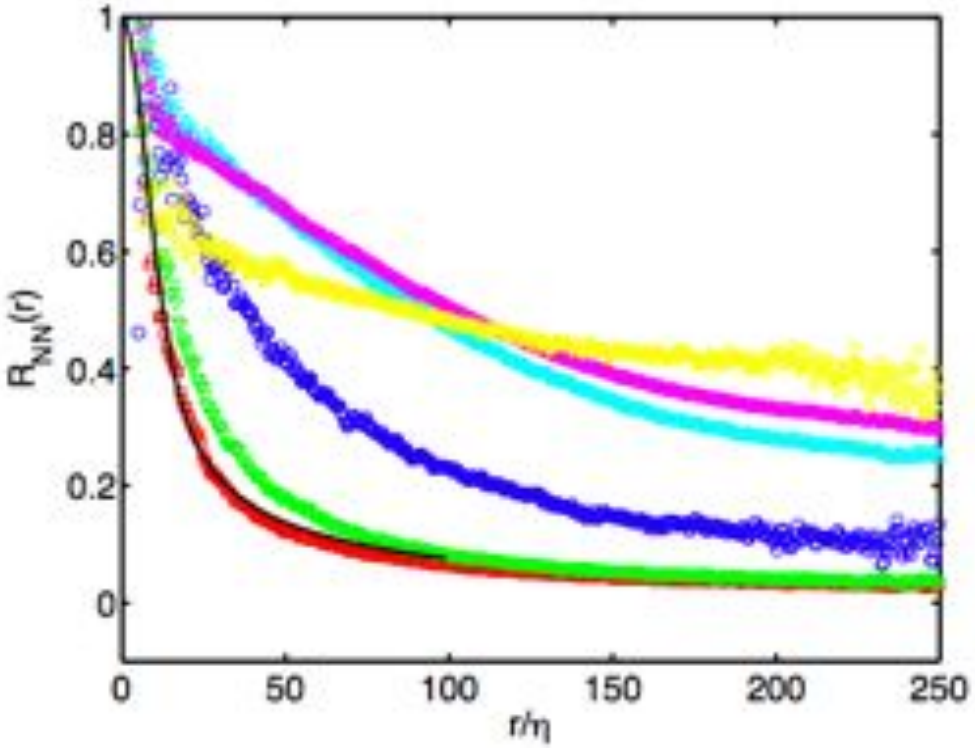
$$R_{NN}(r) = \langle a_{\perp}(\mathbf{x}+\mathbf{r}) a_{\perp}(\mathbf{x}) \rangle$$

Xu, Ouellette, Vincenzi, Bodenschatz (**PRL**, 2007)

Spatial Correlations of Acceleration

$$R_\lambda = 350$$



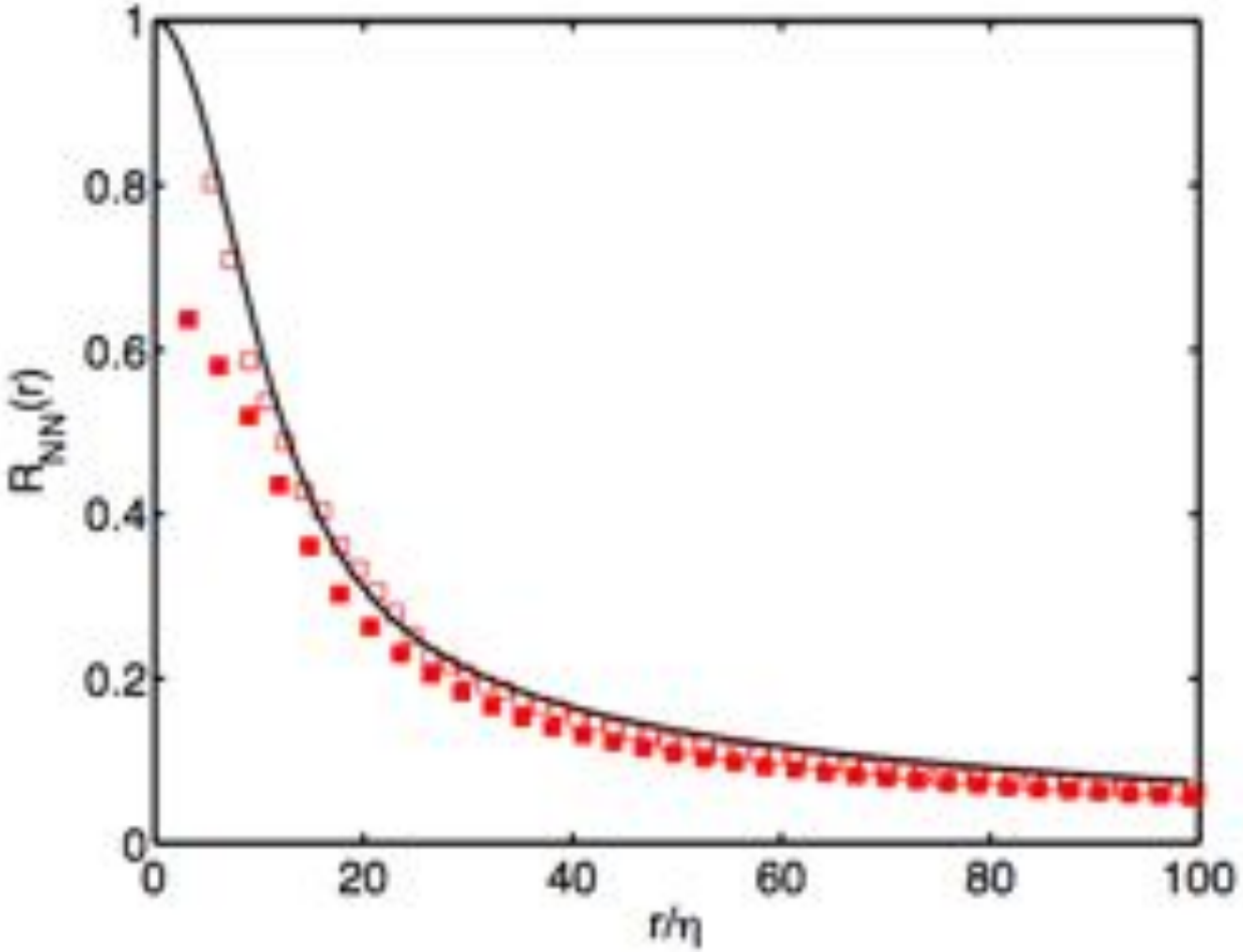


Isotropic Relation

$$\mathbf{a} = -\nabla p + \nabla \cdot \mathbf{T}^p$$

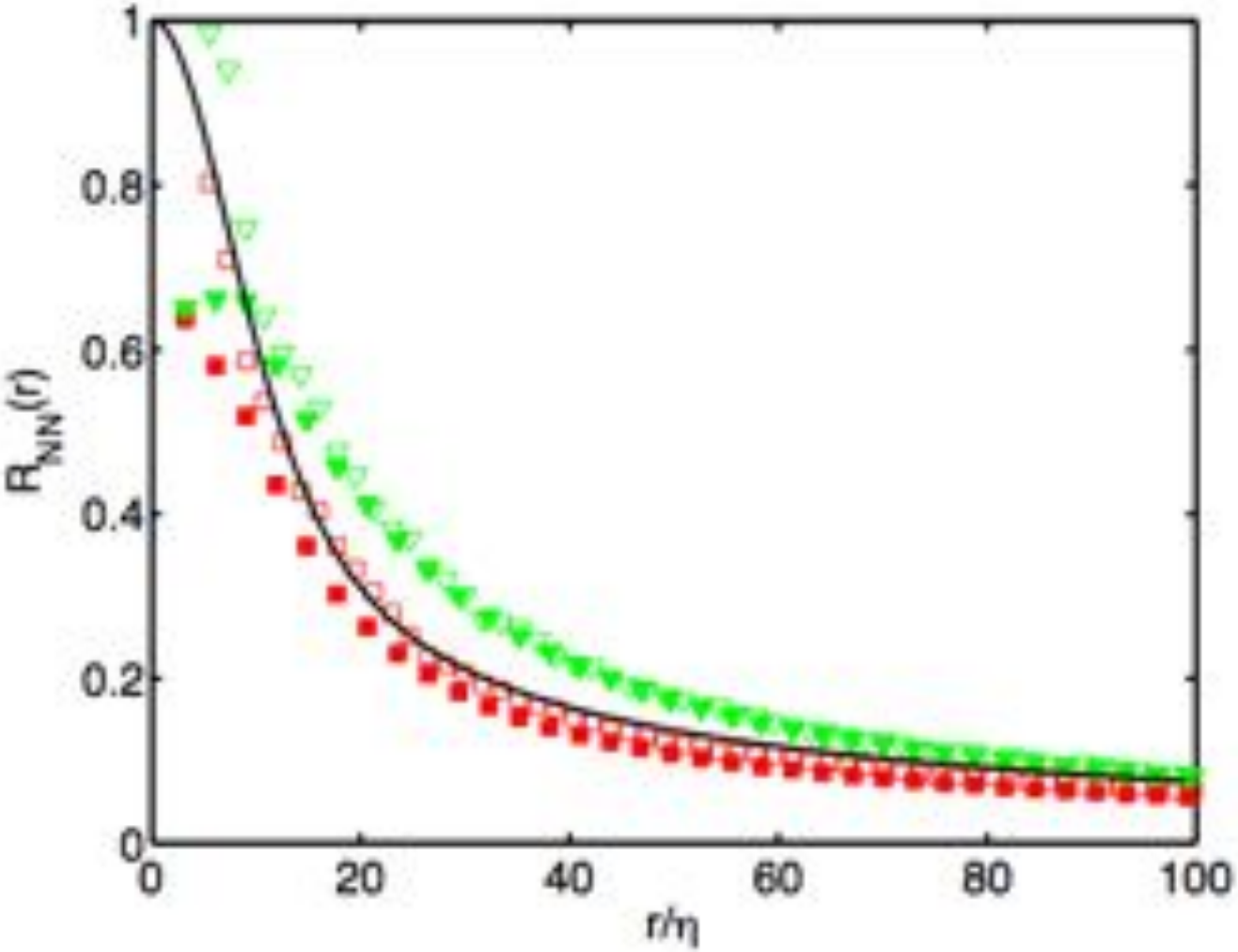
$$\Rightarrow \quad R_{NN}(r) \stackrel{?}{=} \frac{1}{r}\int_0^r R_{LL}(r)dr$$

Isotropic Relation



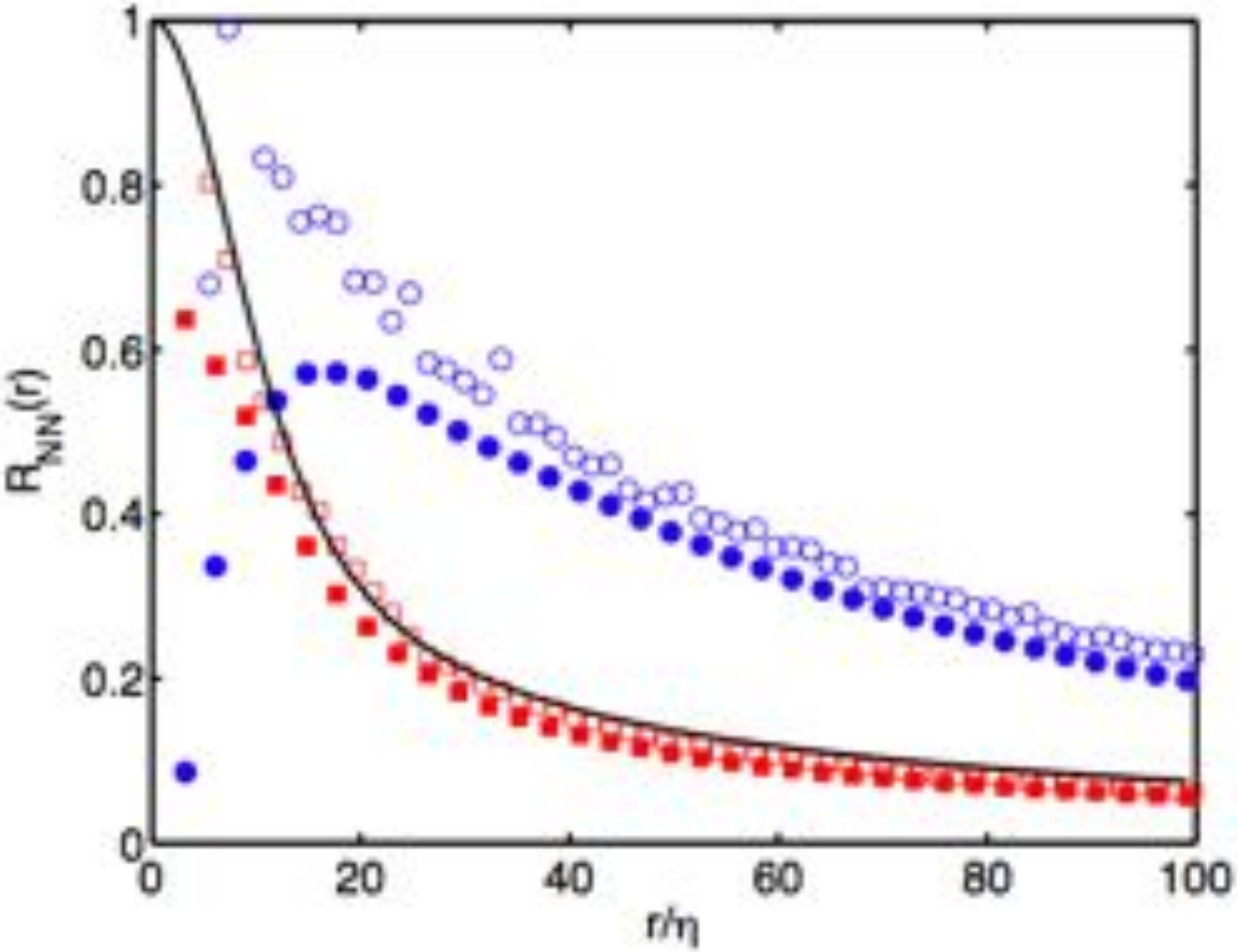
water

Isotropic Relation



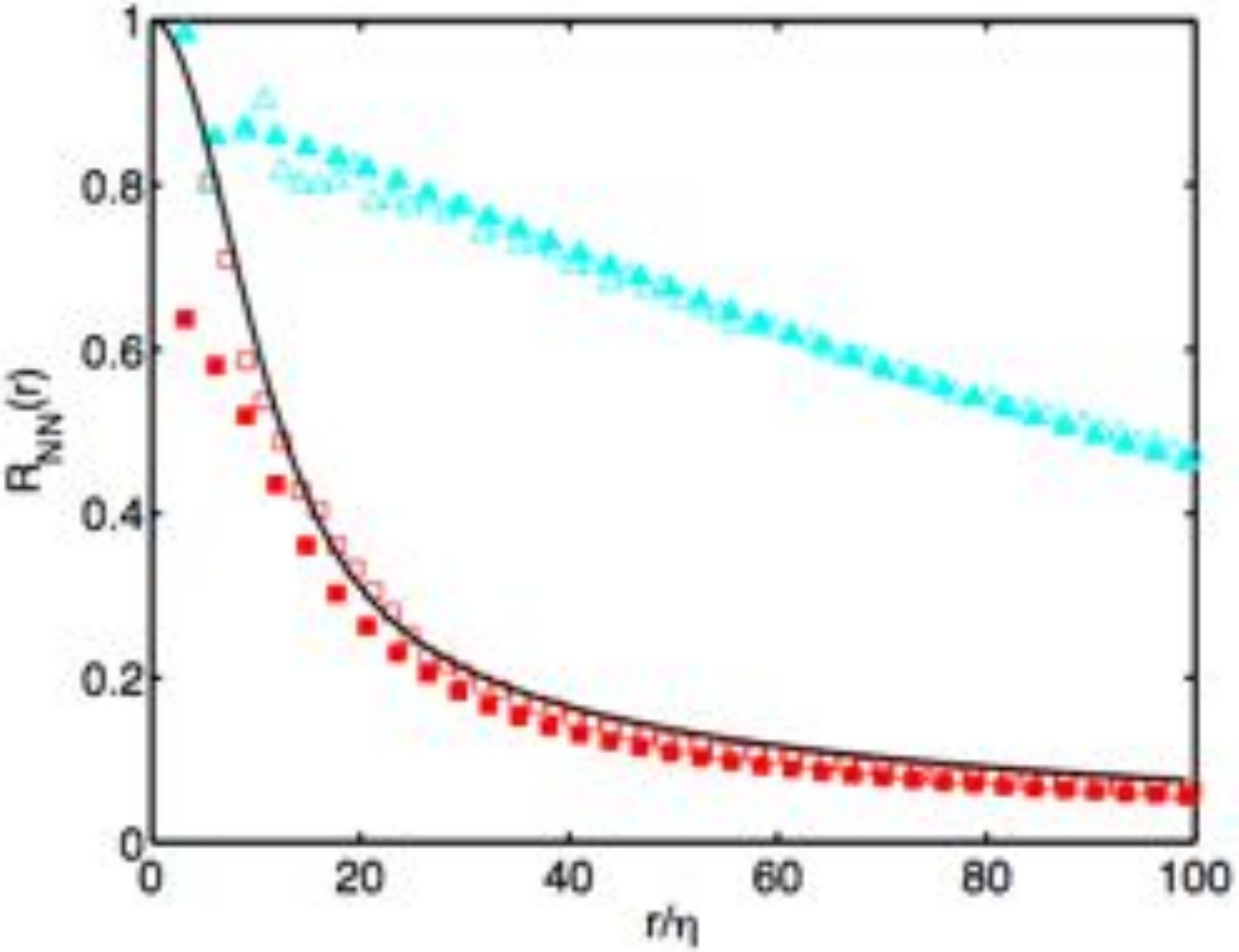
Water +
1ppm

Isotropic Relation



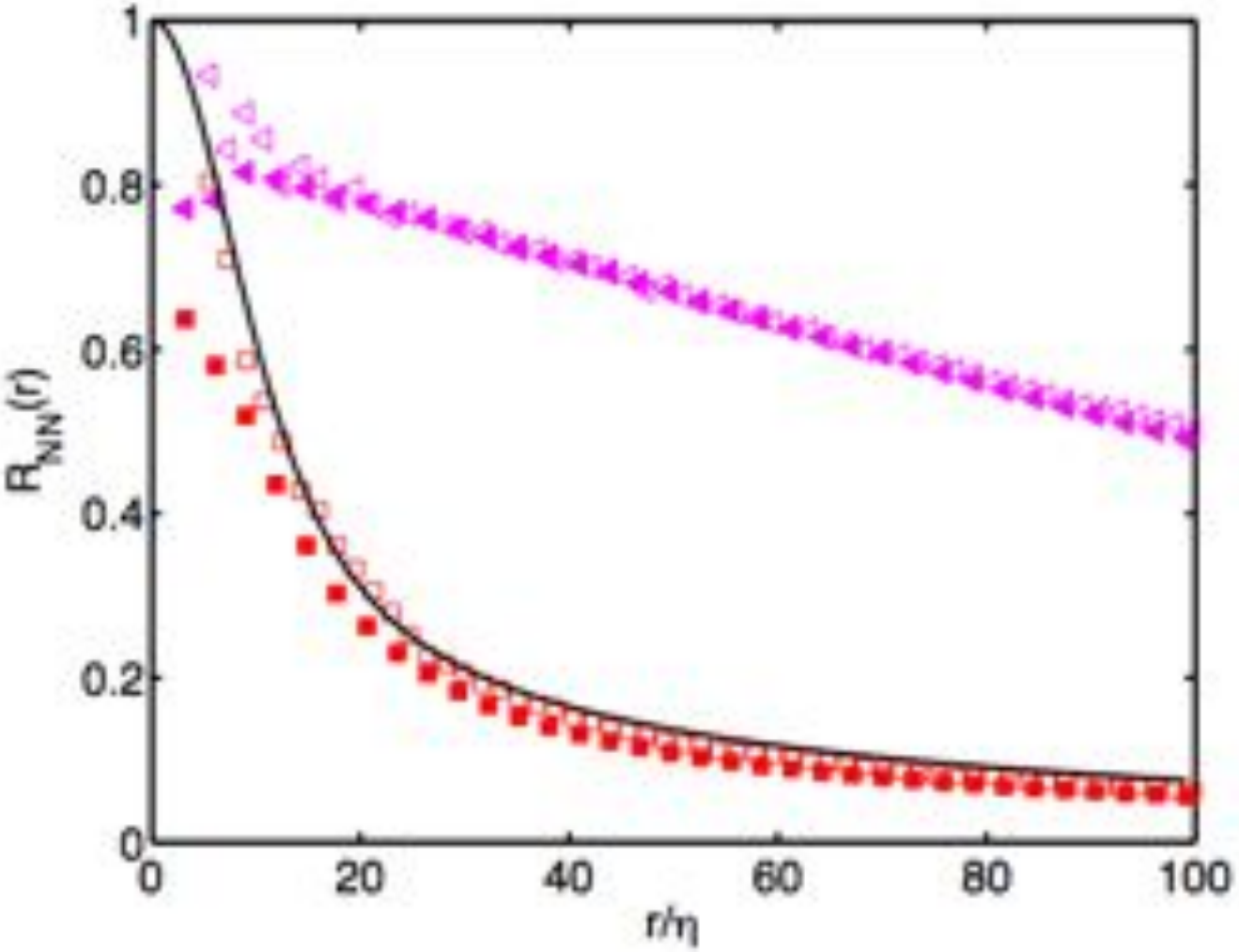
Water + 5ppm

Isotropic Relation



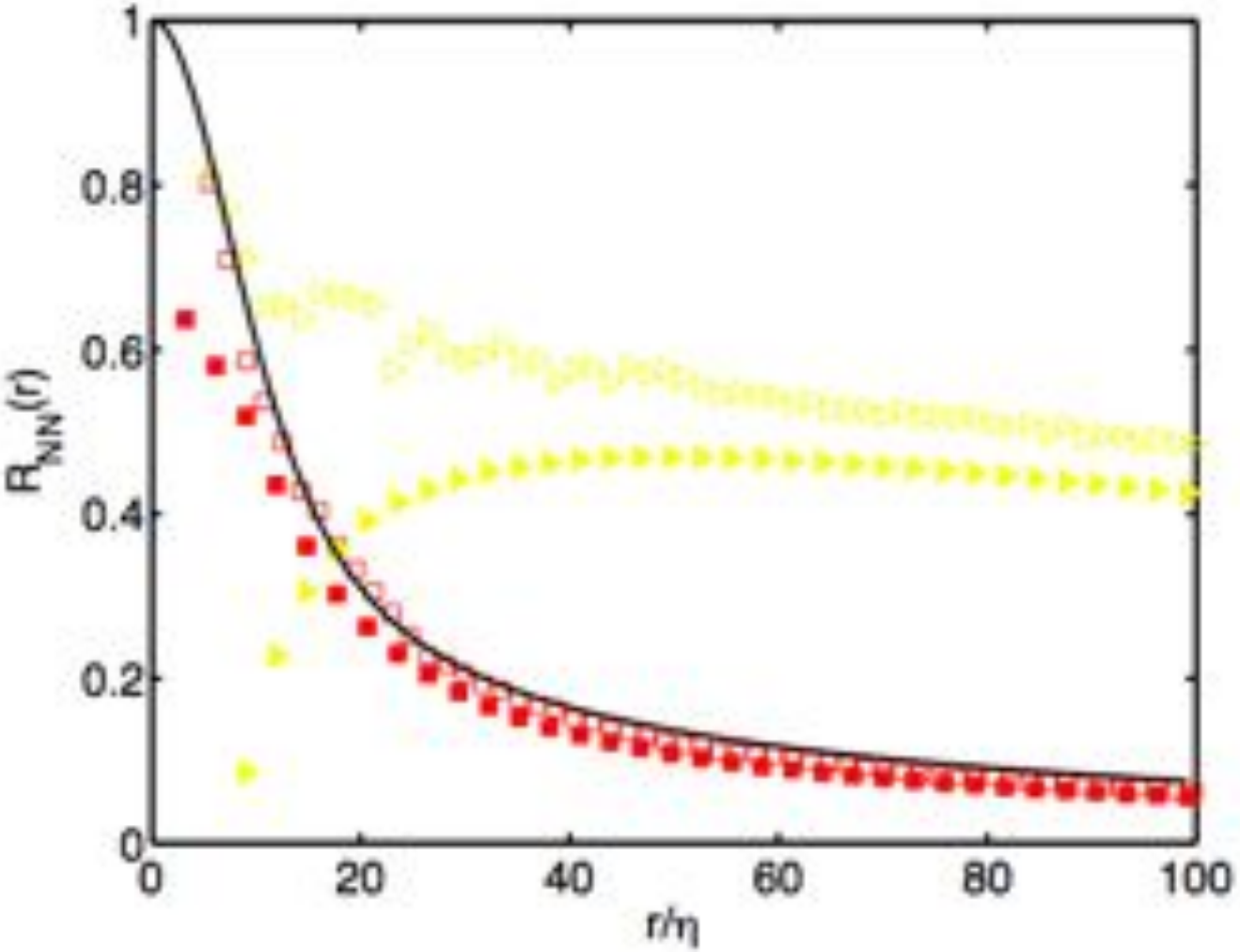
Water +
7ppm

Isotropic Relation



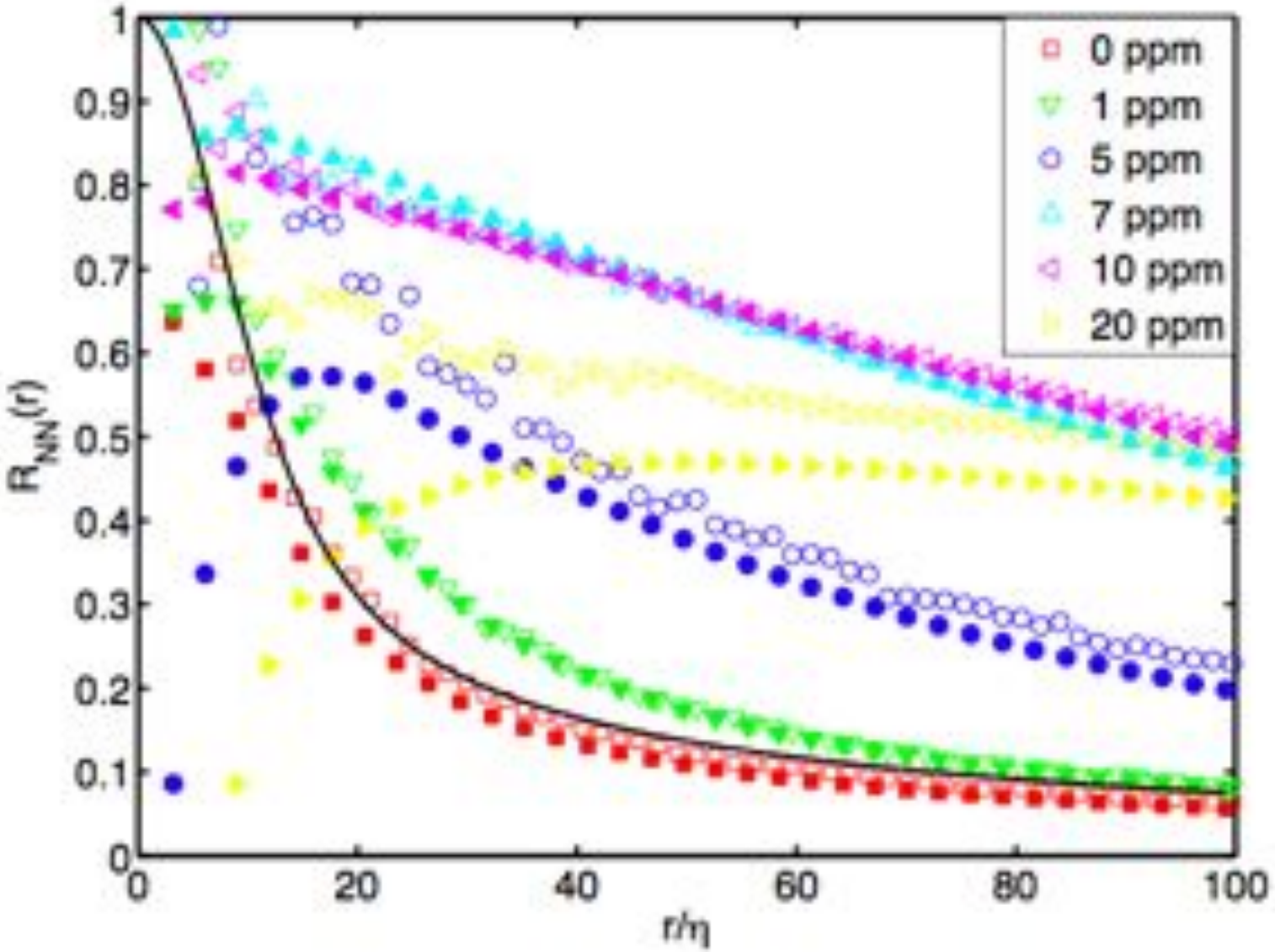
Water +
10ppm

Isotropic Relation



Water +
20ppm

Isotropic Relation



Acknowledgment

Alice Crawford (NIST, USA),

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Dario Vincenzi (Nice, France)

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Alain Pumir (Lyon, France)