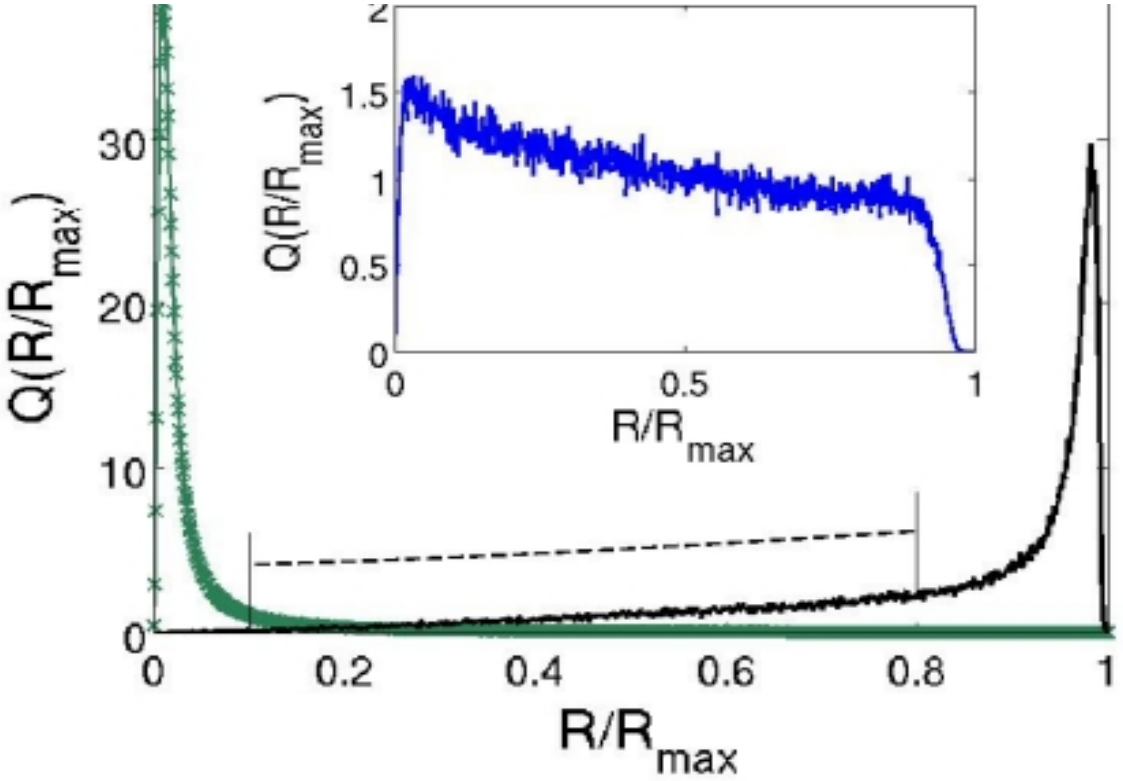
Statistics of polymer extensions in turbulent channel flow

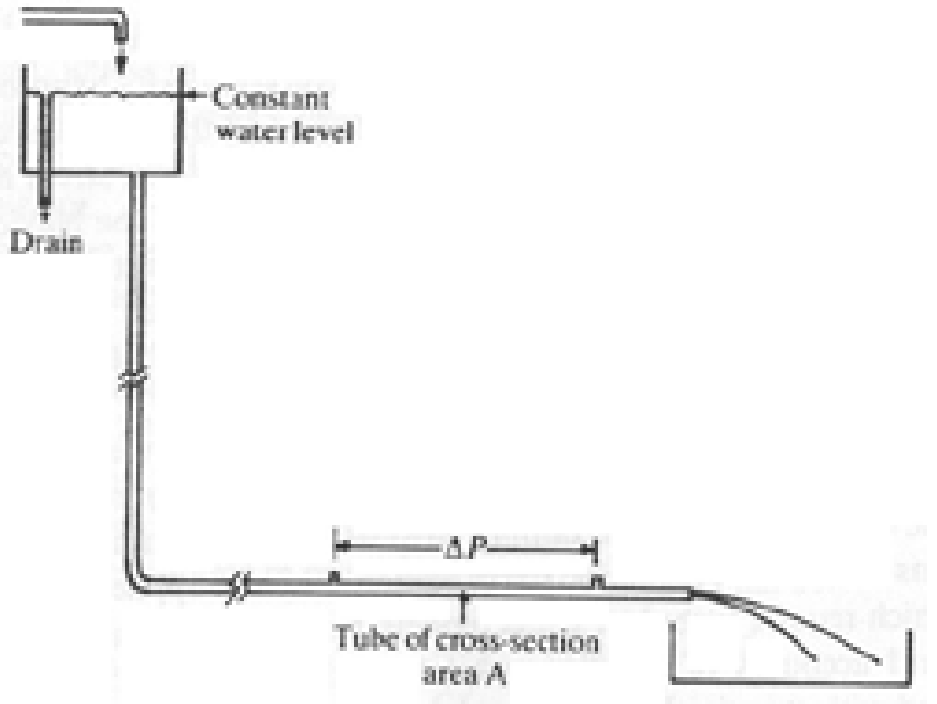
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Turbulent Drag Reduction (Toms, 1946)

Addition of small amount of polymer gives rise to massive drag reduction.



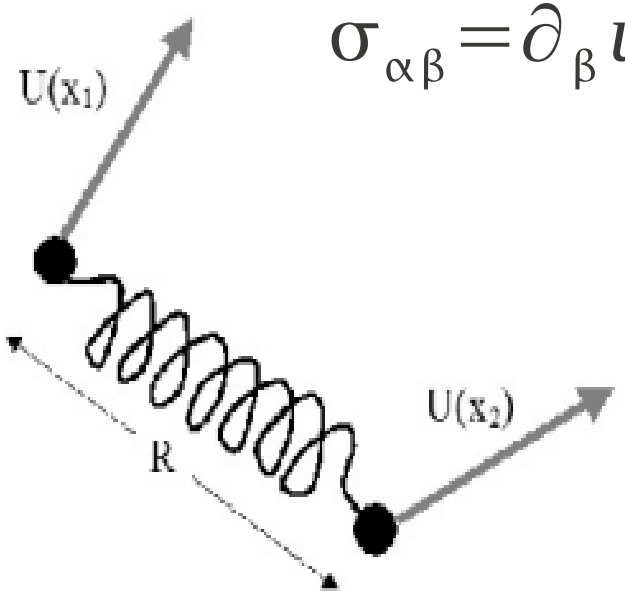
$$\% DR \equiv \frac{\Delta P^f - \Delta P^p}{\Delta P^f} \times 100$$

Simple model for single polymer

$$\partial_t R^j_\alpha(t) = \sigma^j_{\alpha\beta} R^j_\beta + f(R^j) + \sqrt{\frac{2R_0^2}{3\tau_{\text{poly}}}} B^j_\alpha .$$

$$\sigma_{\alpha\beta} = \partial_\beta u_\alpha$$

Gaussian, white Noise



Restoring force

$$f(R) = -R/\tau_{\text{poly}},$$

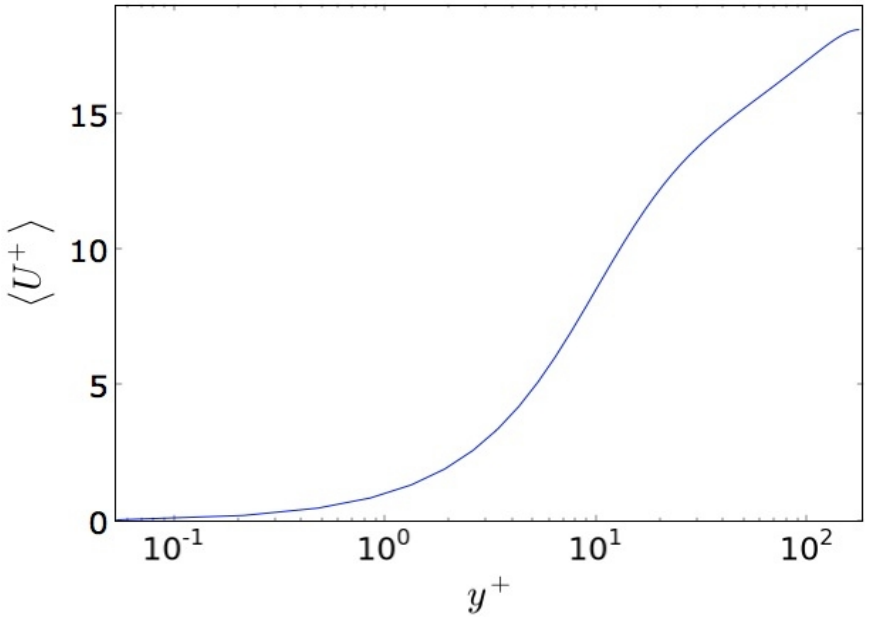
$$f(R) = -R/\tau_{\text{poly}}\{1 - (R/\bar{R}_{\max})^2\}.$$

Navier-Stokes equation

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = \nu \nabla^2 \boldsymbol{u} + \boldsymbol{\nabla} p$$

with the incompressibility constraint,

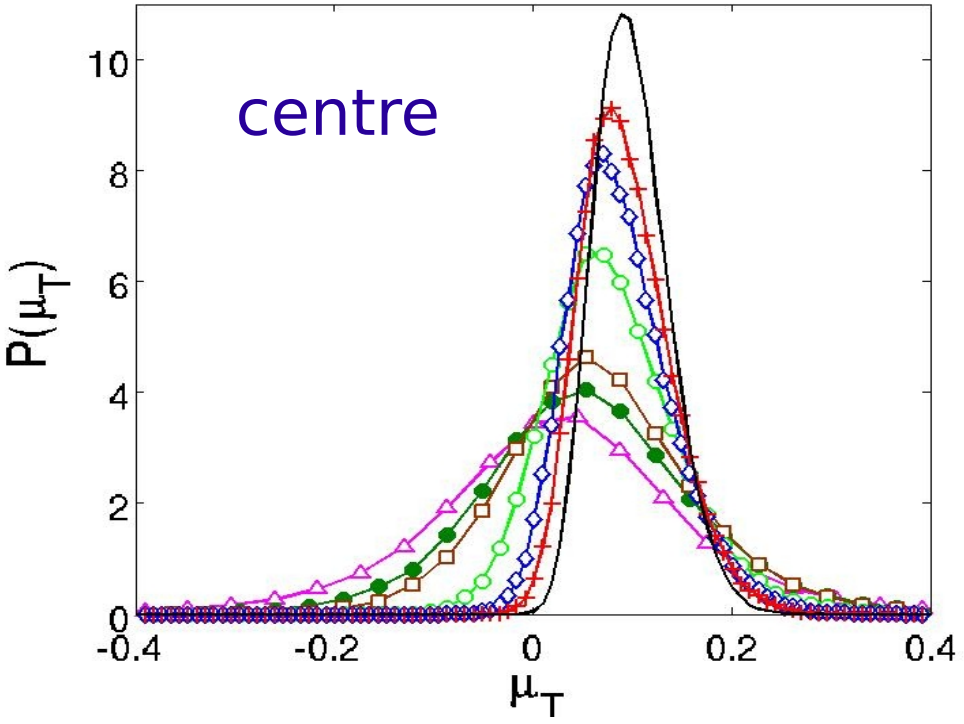
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0.$$

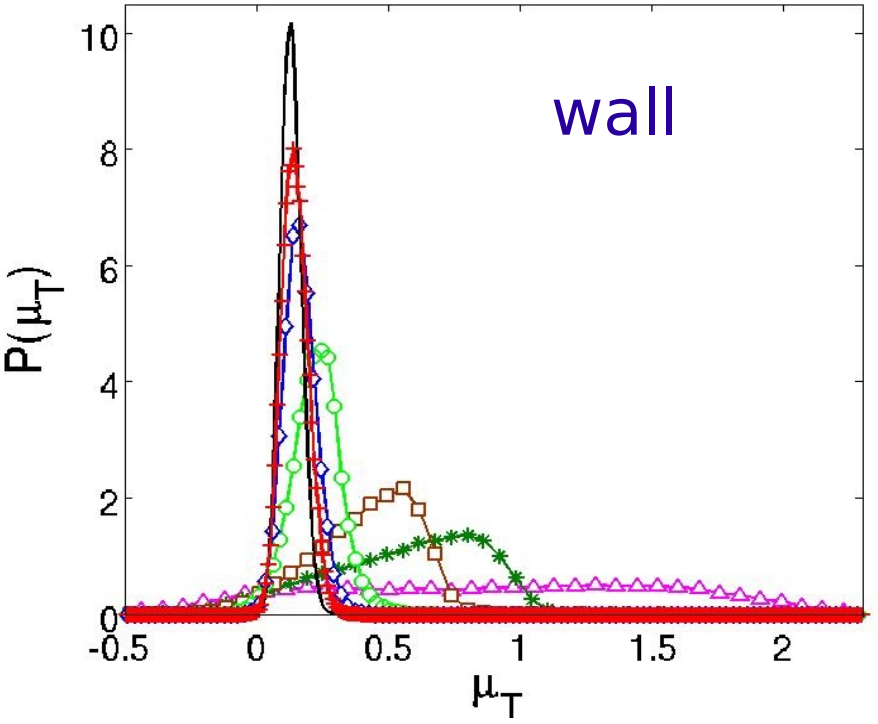


Lagrangian Particles

$$\partial_t \boldsymbol{r}^{\mathrm{j}}(t|t_0, \boldsymbol{r}_0^j) = \boldsymbol{v}^{\mathrm{j}}(t|t_0, \boldsymbol{r}_0^j).$$

Two particle separation in channel flow

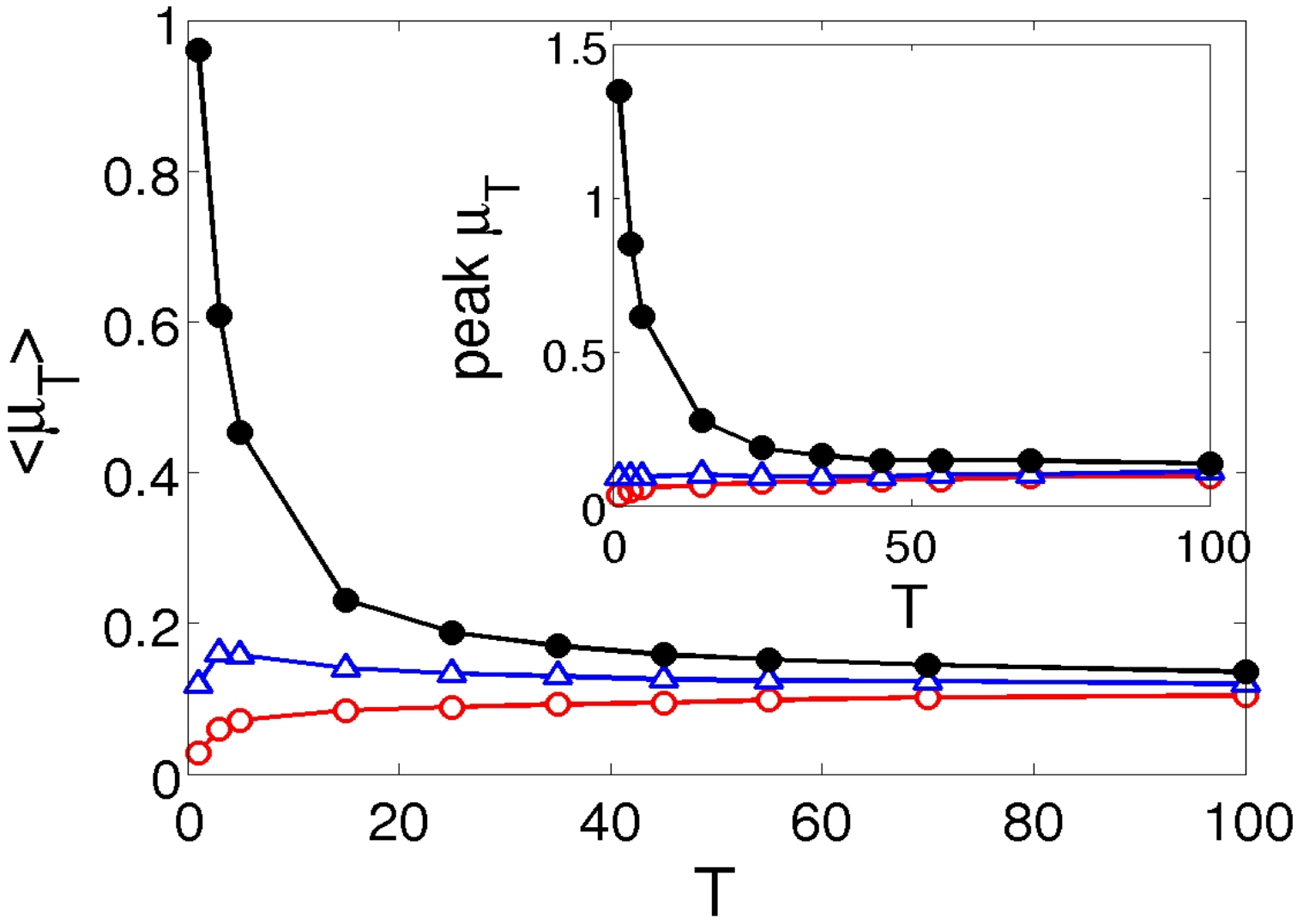




$$\partial_t \delta x^{\mathrm{j}}_{\alpha} = \sigma^{\mathrm{j}}_{\alpha,\beta} \delta x^{\mathrm{j}}_{\beta}.$$

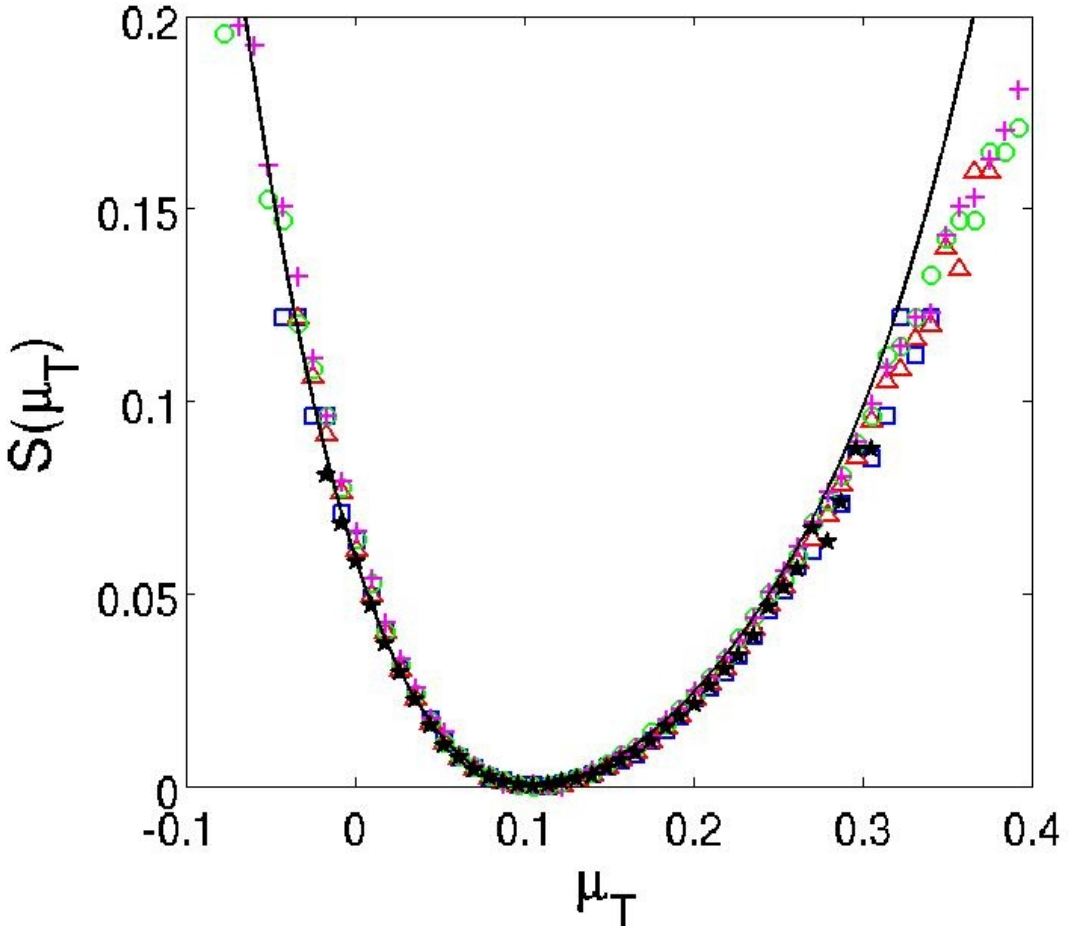
$$\mu^{\mathrm{j}}_{\mathrm{T}} = \frac{1}{T} \ln \left[\frac{|\delta \boldsymbol{x}^{\mathrm{j}}(t)|}{|\delta \boldsymbol{x}^{\mathrm{j}}(t-T)|} \right].$$

Change with time:



Cramer's function:

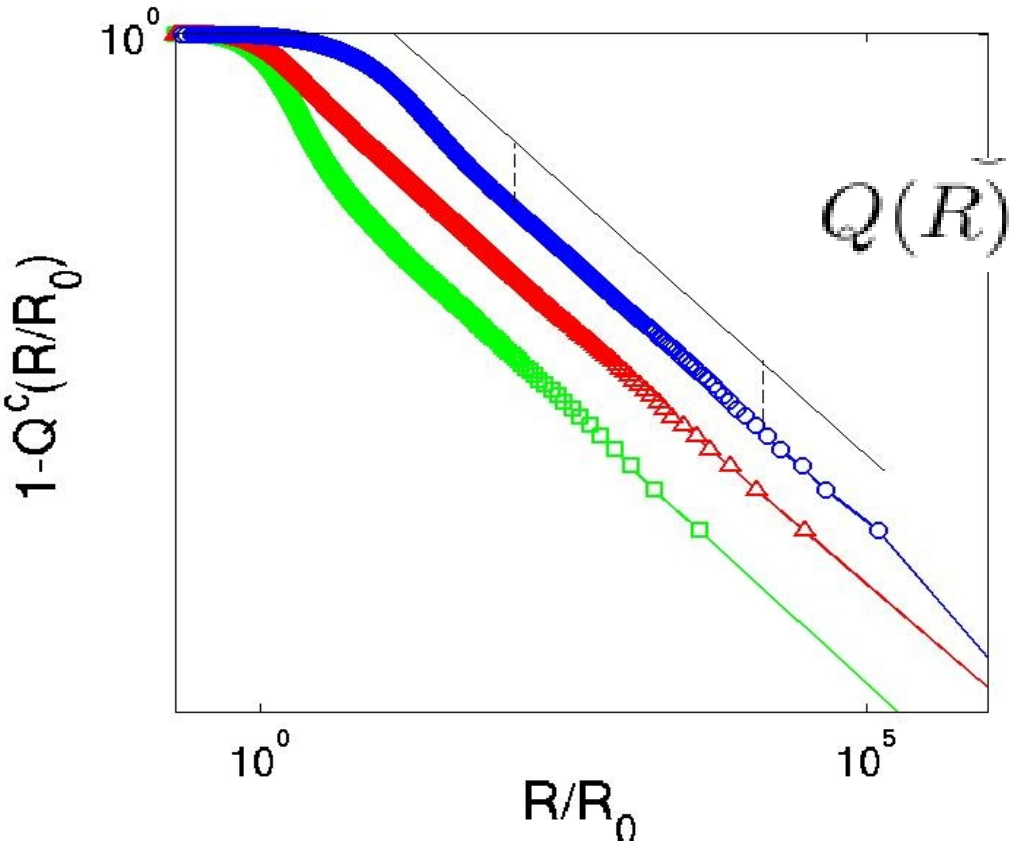
$$P(\mu_T) \sim \exp[-TS(\mu_T)]$$



$$\mathrm{Wi} \equiv \bar{\mu}\tau_{\mathrm{poly}}$$

$$S(\mu) = a_2(\mu - \bar{\mu})^2 + a_3(\mu - \bar{\mu})^3 + a_4(\mu - \bar{\mu})^4$$

Cramer's function and PDF of polymer extensions:



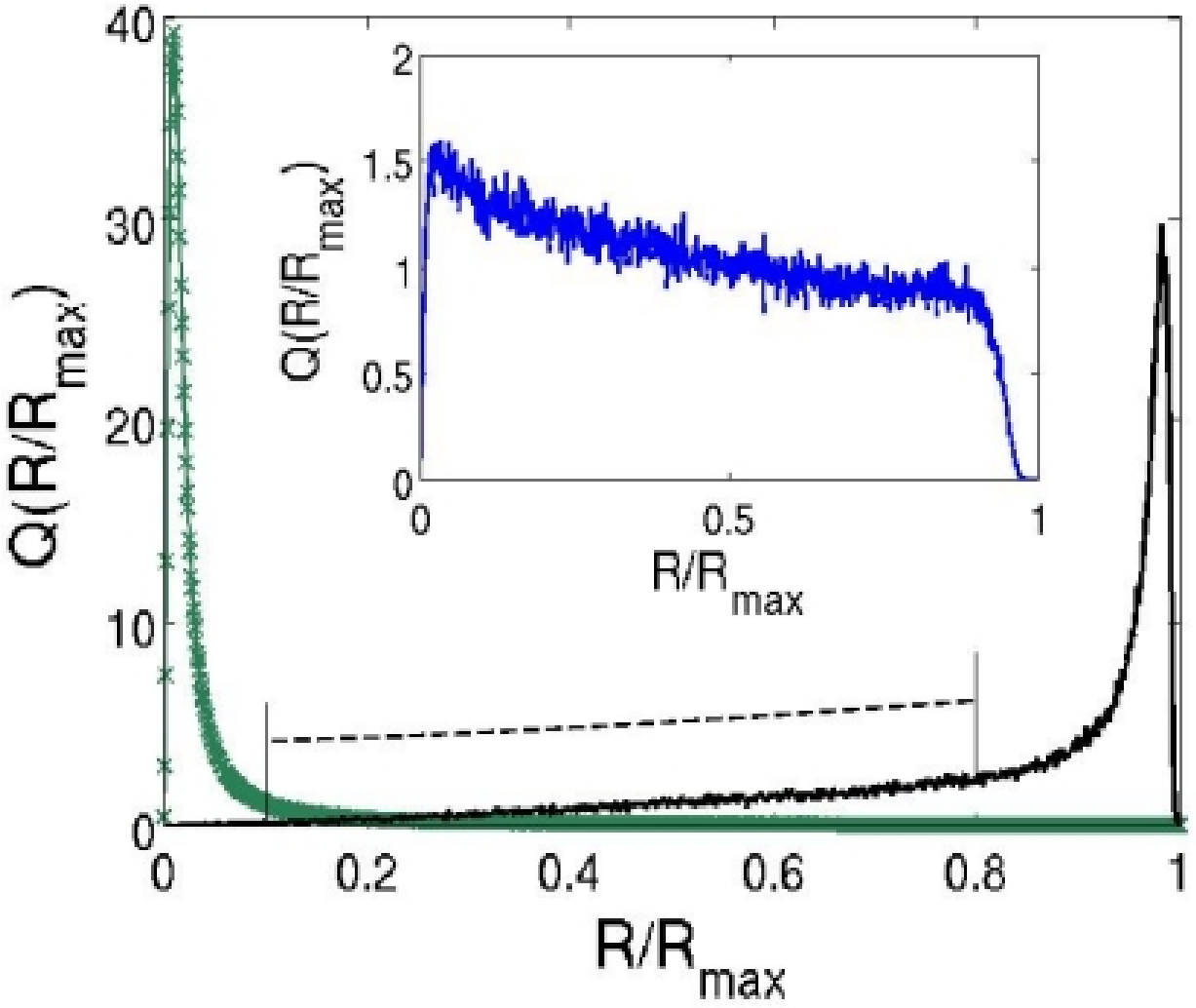
$$\mathcal{P}(R) \sim R^{-\alpha-1}$$

$$Q(R) \sim R^{-\alpha}$$

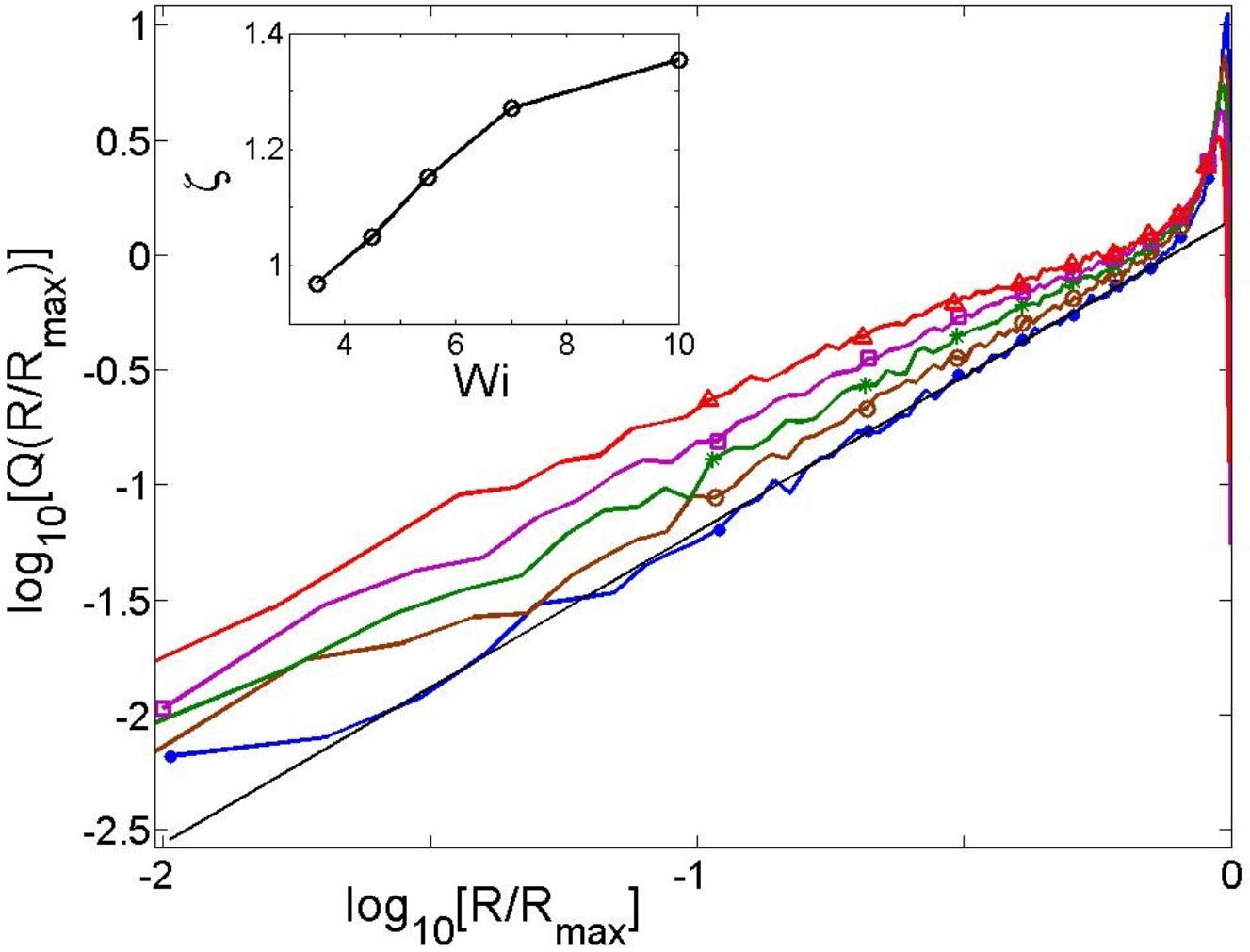
$$\alpha = S'(\beta + \frac{1}{\tau_{\text{poly}}} - \bar{\mu})$$

$$S(\beta + \frac{1}{\tau_{\text{poly}}} - \bar{\mu}) - \beta S'(\beta + \frac{1}{\tau_{\text{poly}}} - \bar{\mu}) = 0$$

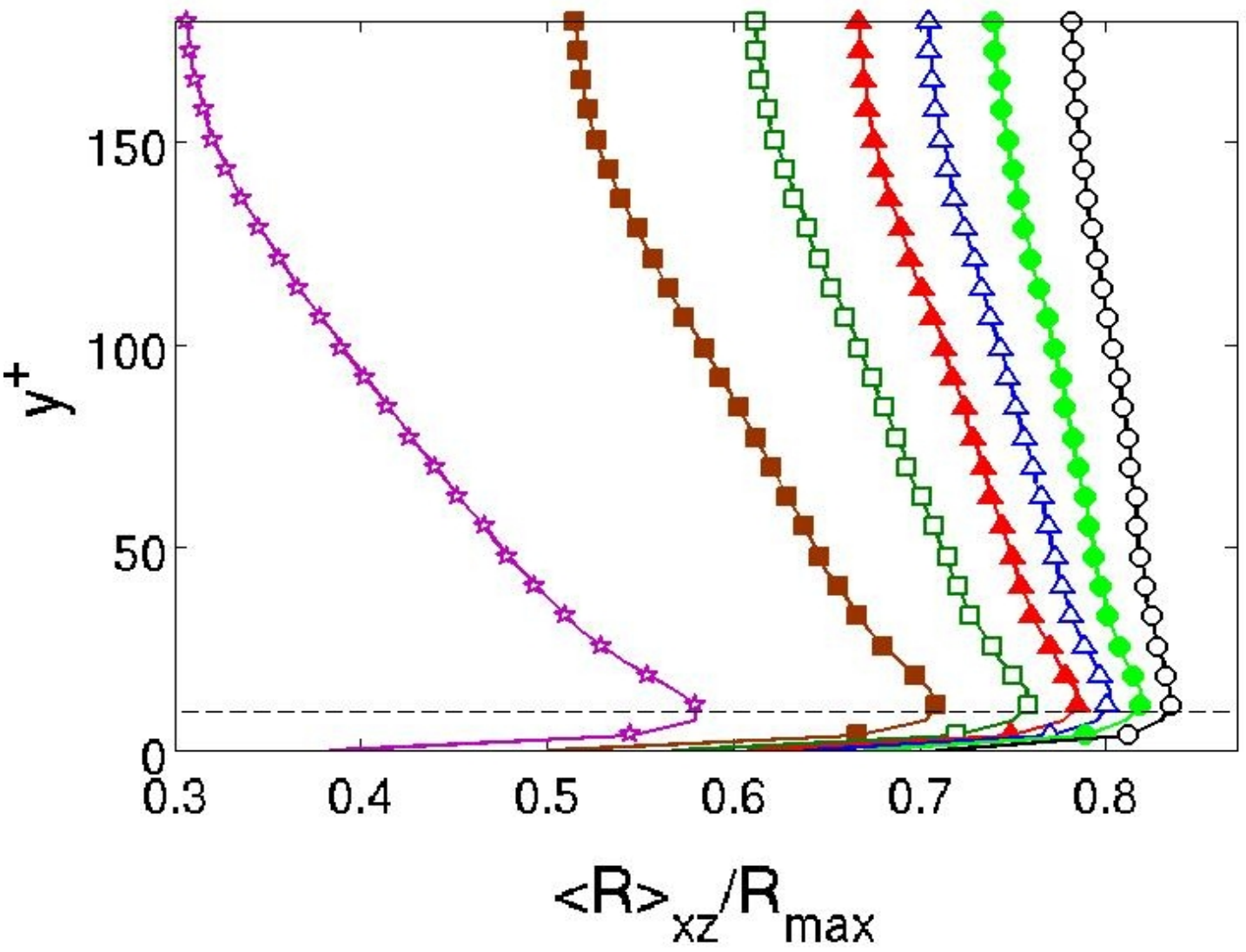
Coil-stretch Transition



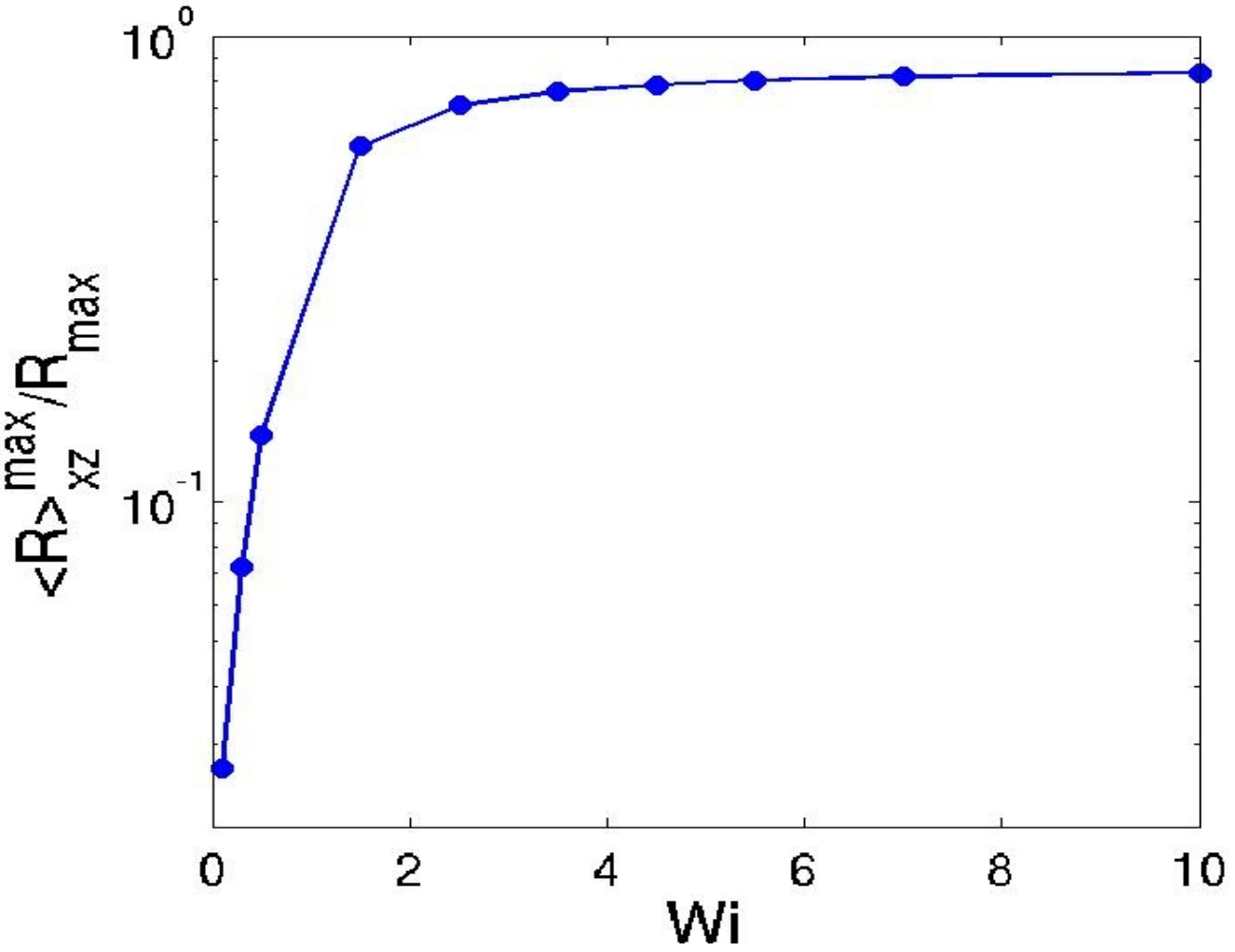
Left tail of the PDF



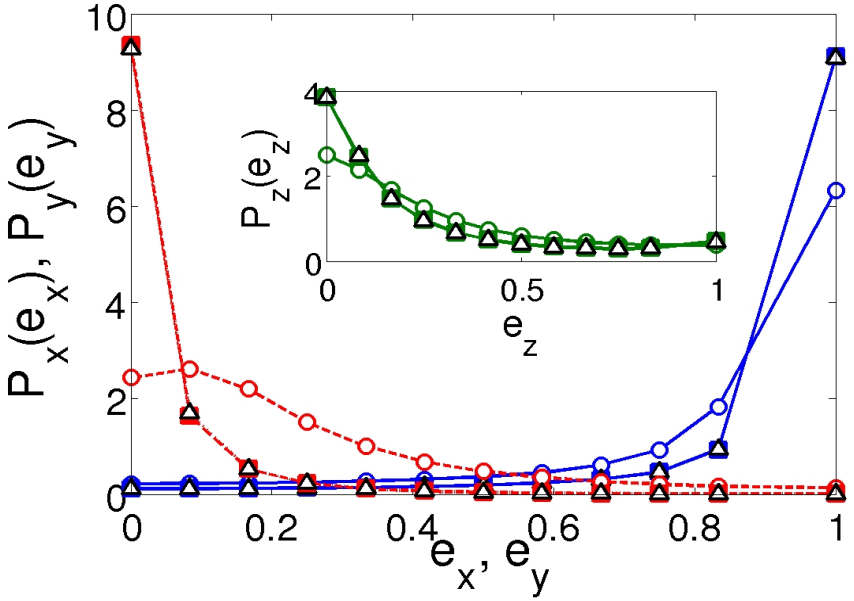
PDF of polymer extension



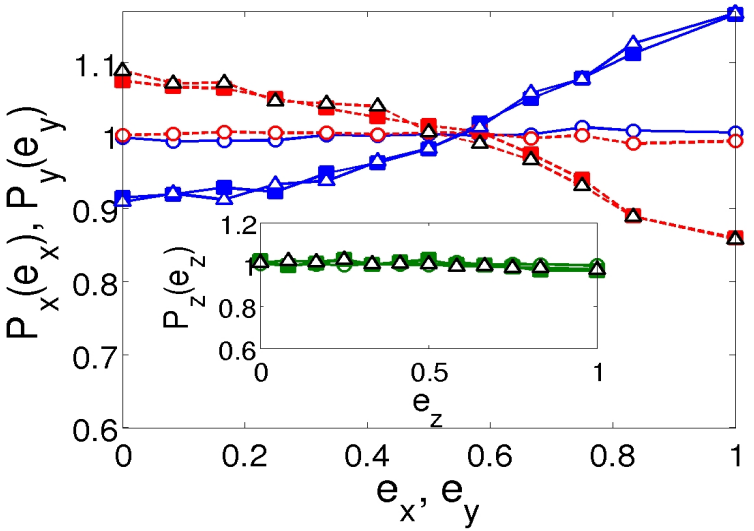
Maximum of polymer extensions



Orientation w.r.t. Channel geometry

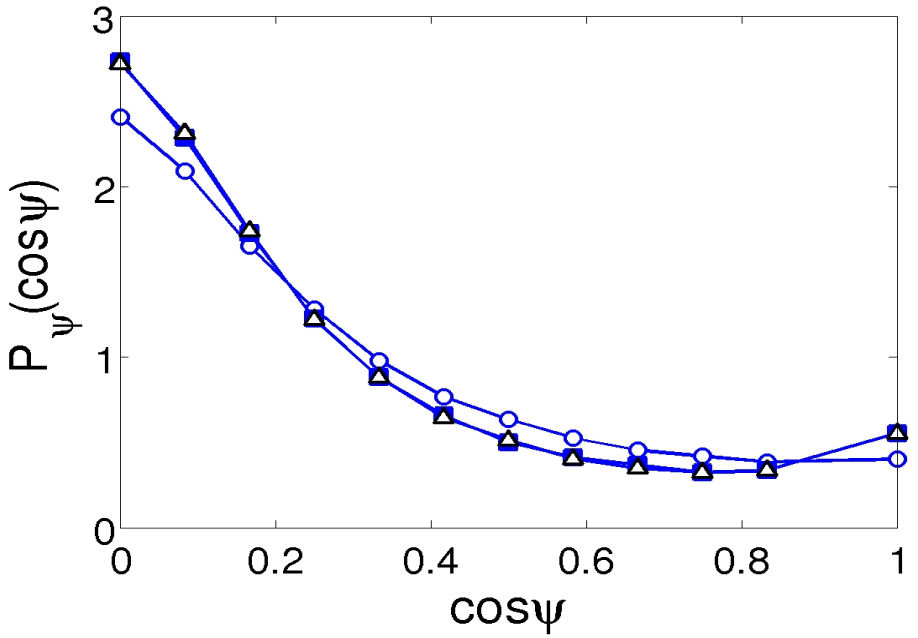


wall

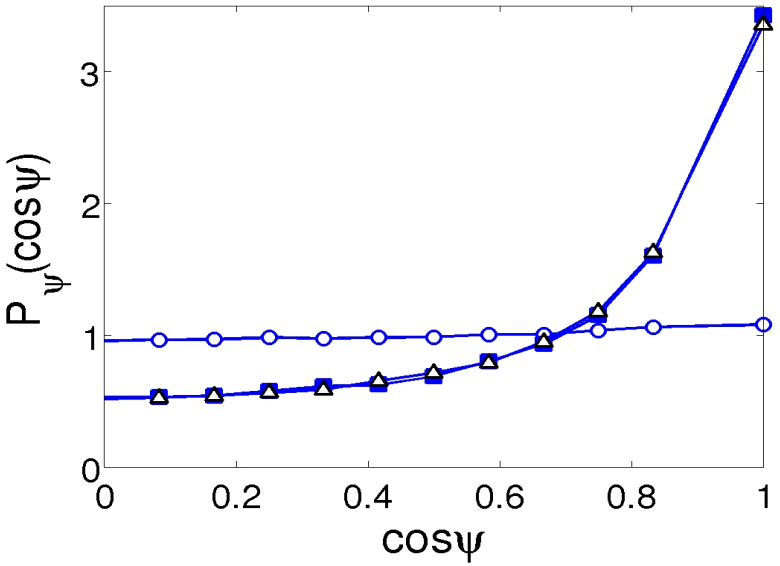


centre

Orientation w.r.t. Local vorticity

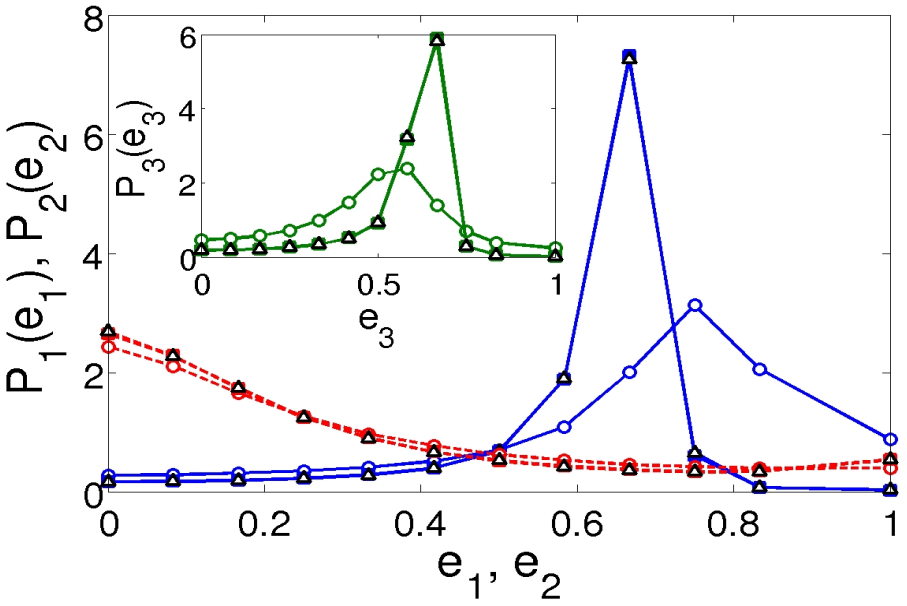


wall

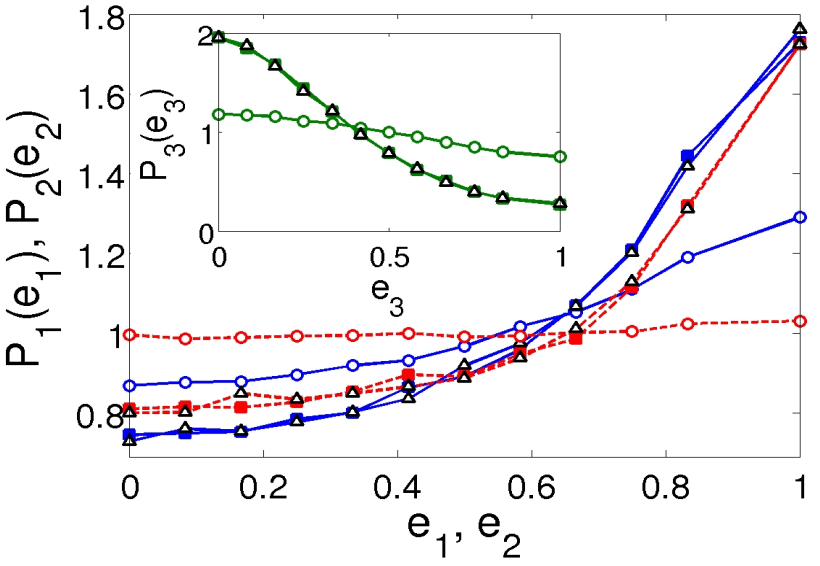


centre

Orientation w.r.t. Local strain



wall



centre

Conclusion

First calculation of Cramer's function in channel flows

Verification of analytical theory of polymer stretching

Coil-stretch transition

Polymers are more stretched near the wall.

The orientation of the polymers is determined by the mean shear.