Locomotion in viscoelastic fluids: pusher, puller & snowman



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Outline

- How does locomotion of micro-organisms change in visco-elastic fluids ?

- What can swim only in visco-elastic fluids? Can we use this?

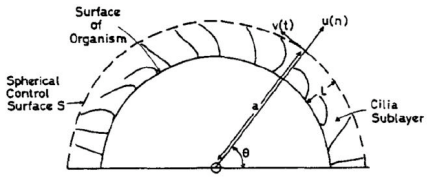
Introduction

- Study locomotion in biologically relevant non-Newtonian fluids



- Spheroid axisymmetric squirmer driven by tangential velocity (envelope method)

$$u_\theta = \sin(\theta) + \frac{\alpha}{2}\sin(2\theta)$$

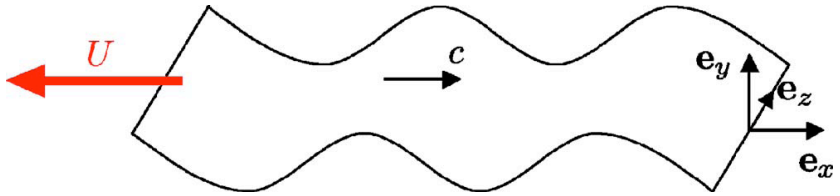


- Consider steady cilia beating with $\alpha \in [-5, 5]$

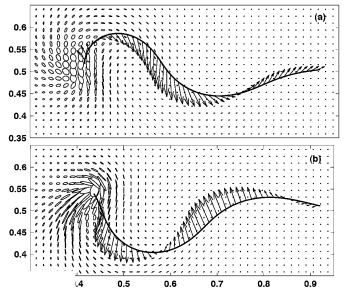
- Negative α: pusher. *Thrust comes from **rear** part of the body*
  Positive α: puller. *Thrust comes from **front** part of the body*

Introduction: locomotion in polymeric fluids

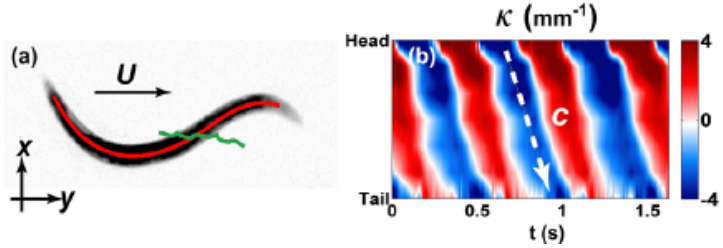
- Lauga (2007) and Fu et al. (2009) analytical work on waving sheet/filament



- Teran et al. (2010), numerical study of finite length filament



- Experiments with C. Elegans by Shen et al. (2011)



- Zhu et al. (2011), **Neutral swimmer by tangential surface deformation**

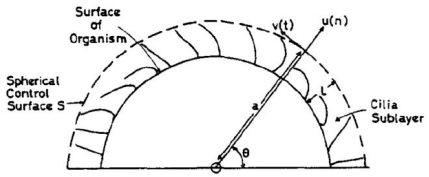
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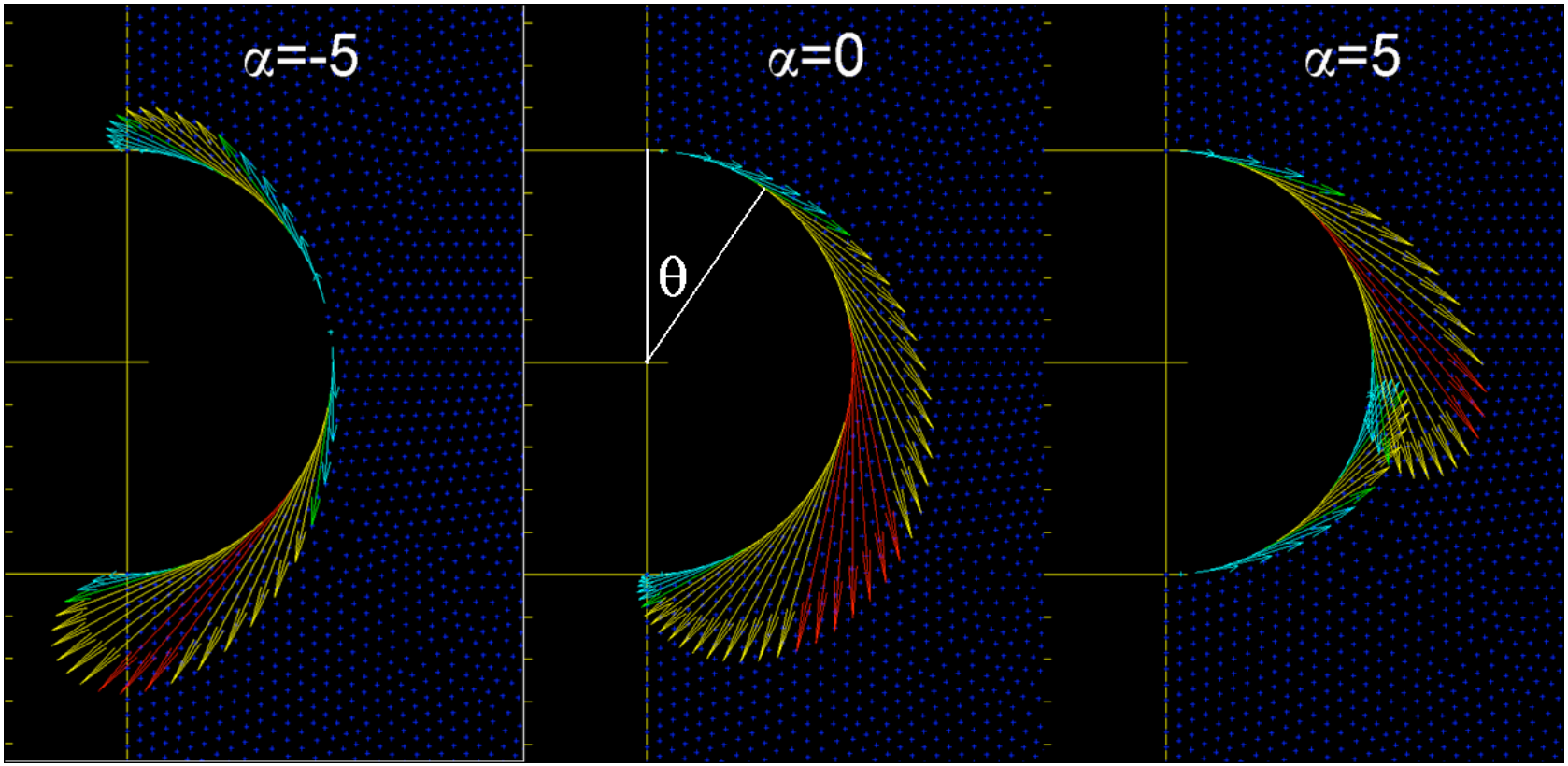


- Consider steady cilia beating with $\alpha \in [-5, 5]$

- Negative α: pusher. *Thrust comes from rear part of the body*
  Positive α: puller. *Thrust comes from front part of the body*

Tangential velocity $u_\theta = \sin(\theta) + \frac{\alpha}{2}\sin(2\theta)$

Pusher **Neutral** swimmer **Puller**



Swimming direction

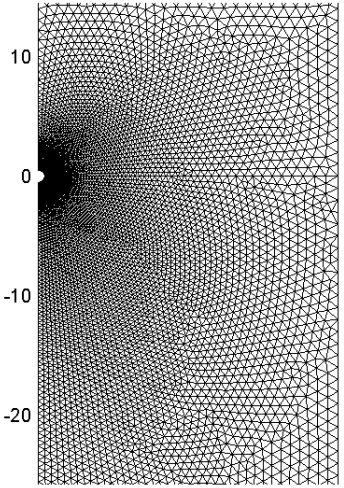
Negative α: pusher. *Thrust comes from rear part of the body*
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Model and assumption

- Steady locomotion
- Axisymmetric Stokes flow (size of the cell is small enough) $Re = 0$
- No Brownian effect (size of the cell is large enough)

Numerical method

- Finite Element Discretization DEVSS-G
- Streamline-upwind/Petrov- Galerkin (SUPG) method for the convective term in the constitutive equation



Polymeric fluid dynamics

Stokes flow and Giesekus model for the constitutive equation

Mobility factor

$$\alpha_m = 0.2$$

$$0 = -\nabla p + \beta \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau}_p$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\boldsymbol{\tau}^p + We\overset{\nabla}{\boldsymbol{\tau}}^p + \frac{We\alpha_m}{1-\beta}(\boldsymbol{\tau}^p \cdot \boldsymbol{\tau}^p) = (1-\beta)(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Viscosity ratio

$$\beta = \frac{\mu_s}{\mu} = 1 - \frac{\mu_p}{\mu}$$

Weissemberg number

$$We = \frac{\lambda U}{L}$$

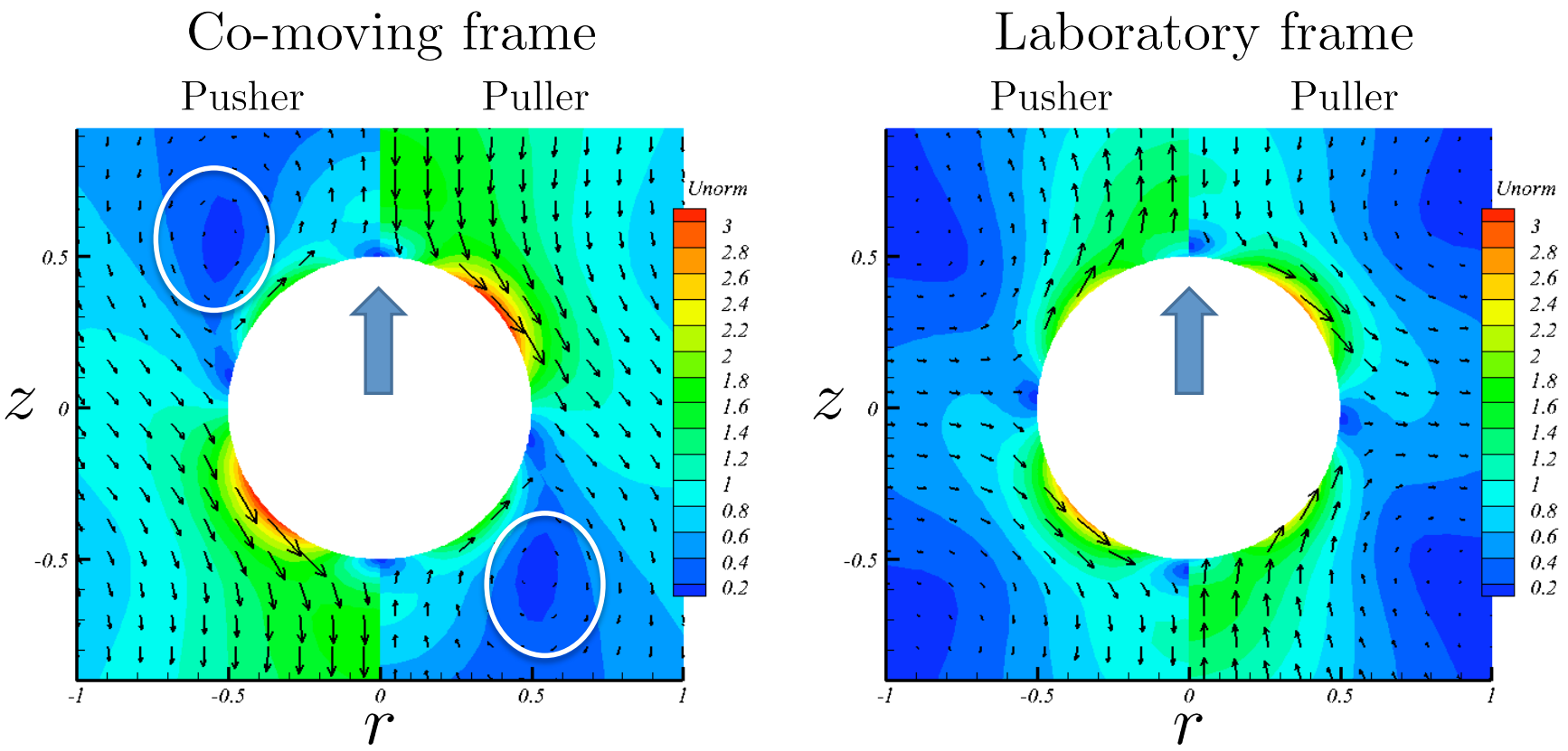
Results

- Integral quantities: swimming speed, power and efficiency

  Efficiency is defined as the ratio between the work rate necessary to pull a sphere/ ellipsoid at the swimmer speed in the same fluid and the swimming power P

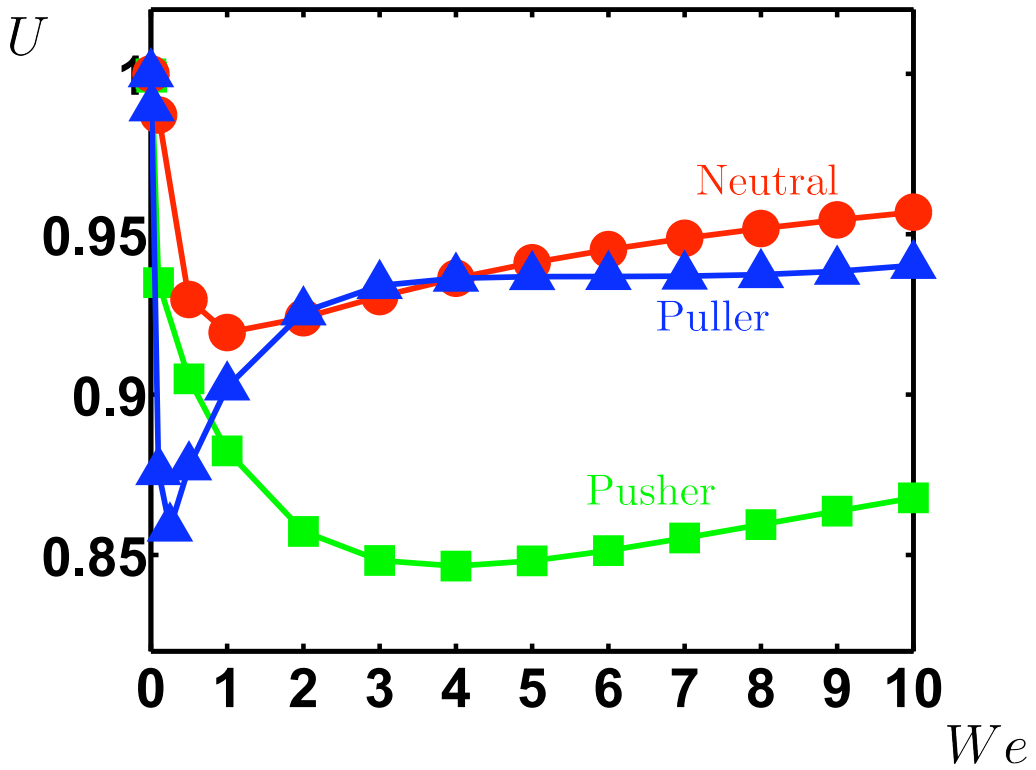
- Flow visualization: streamlines and polymer stretching

Pusher vs. puller



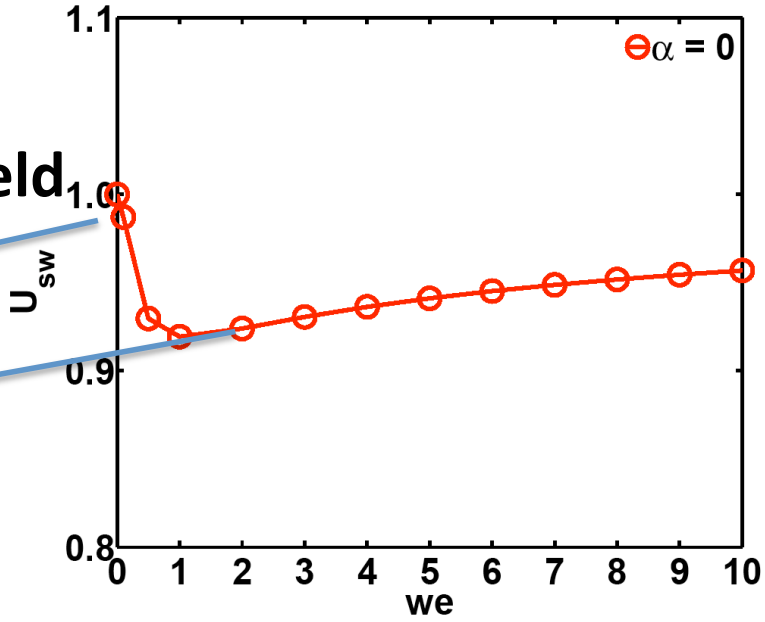
✓ Dissipating ring vortices

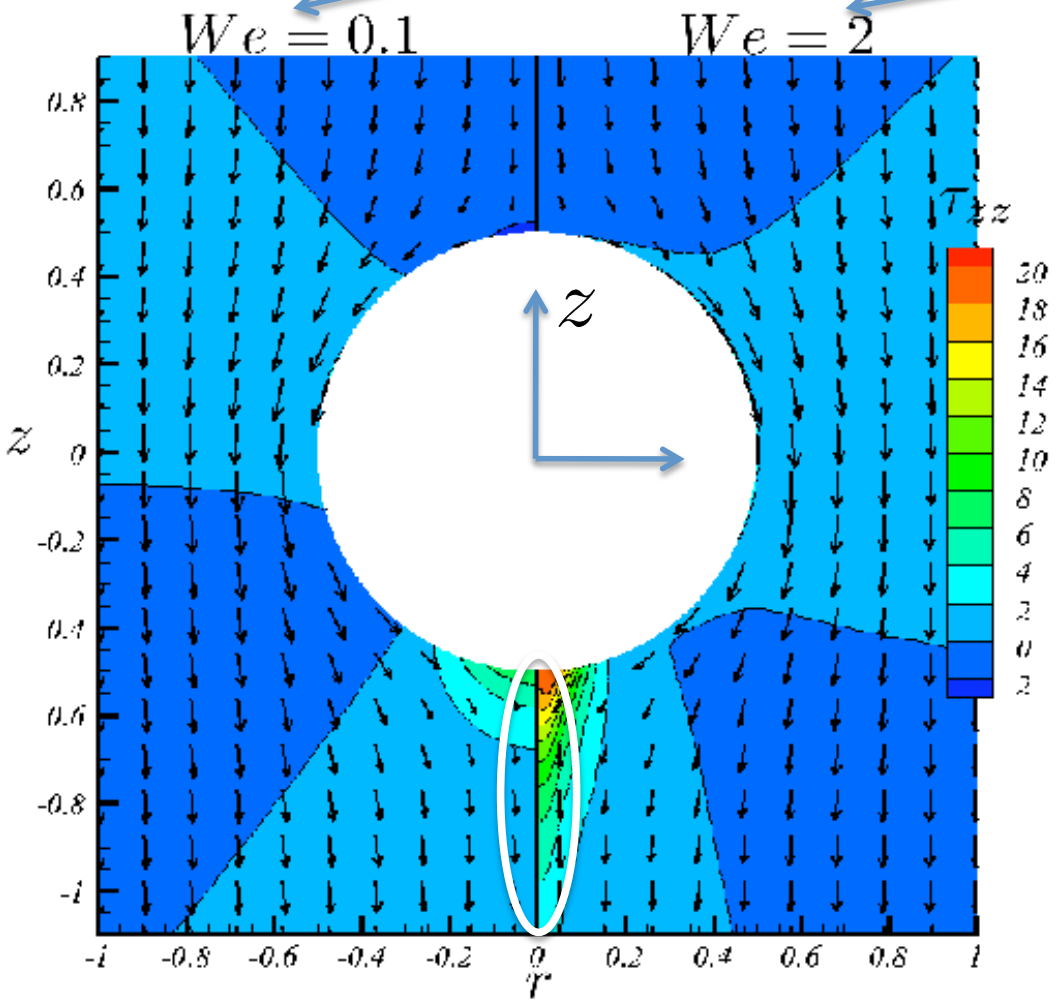
Swimming speed



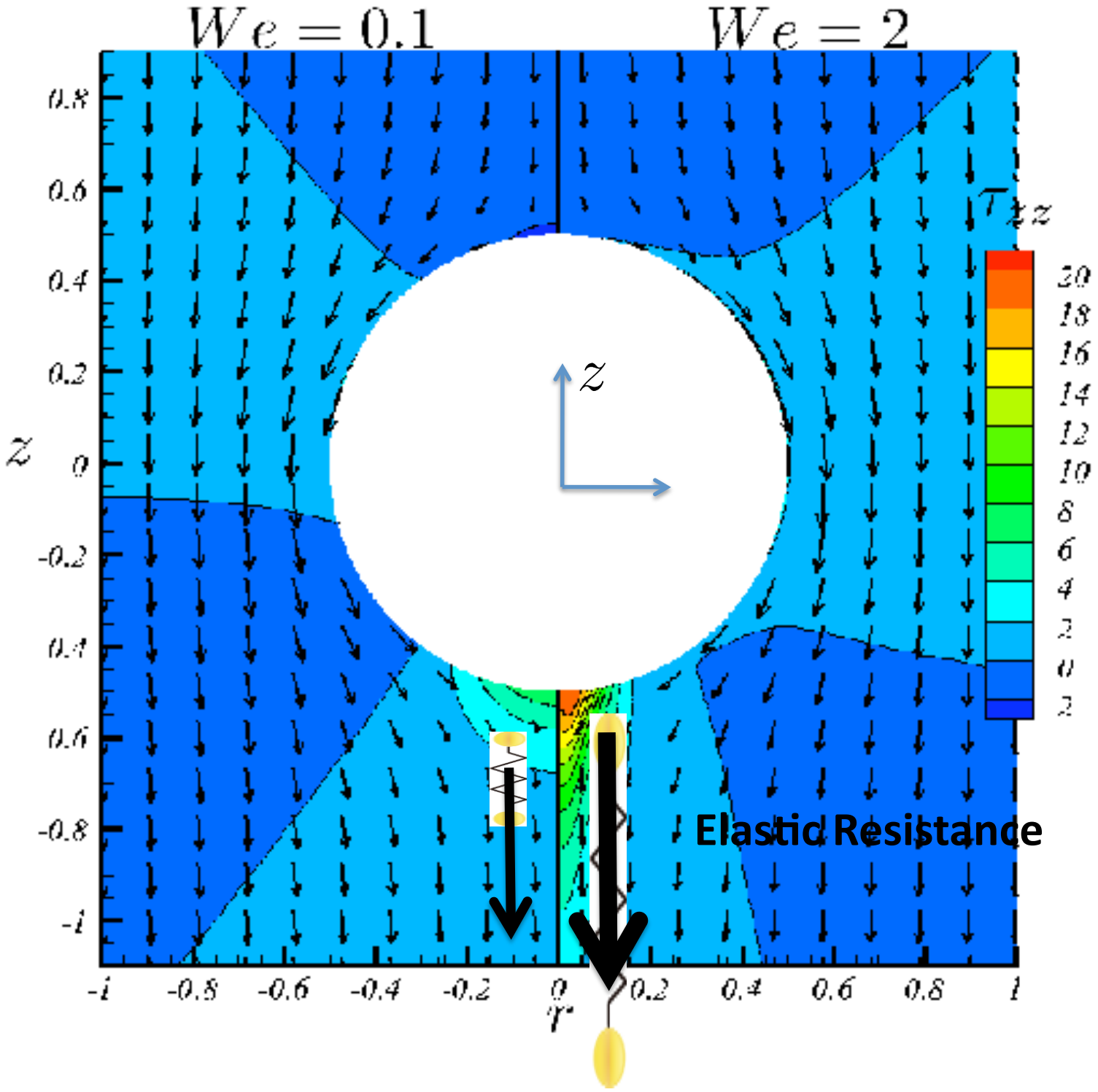
✓ Lower swimming speed in viscoelastic fluids

Correlate swimming speed with flow field



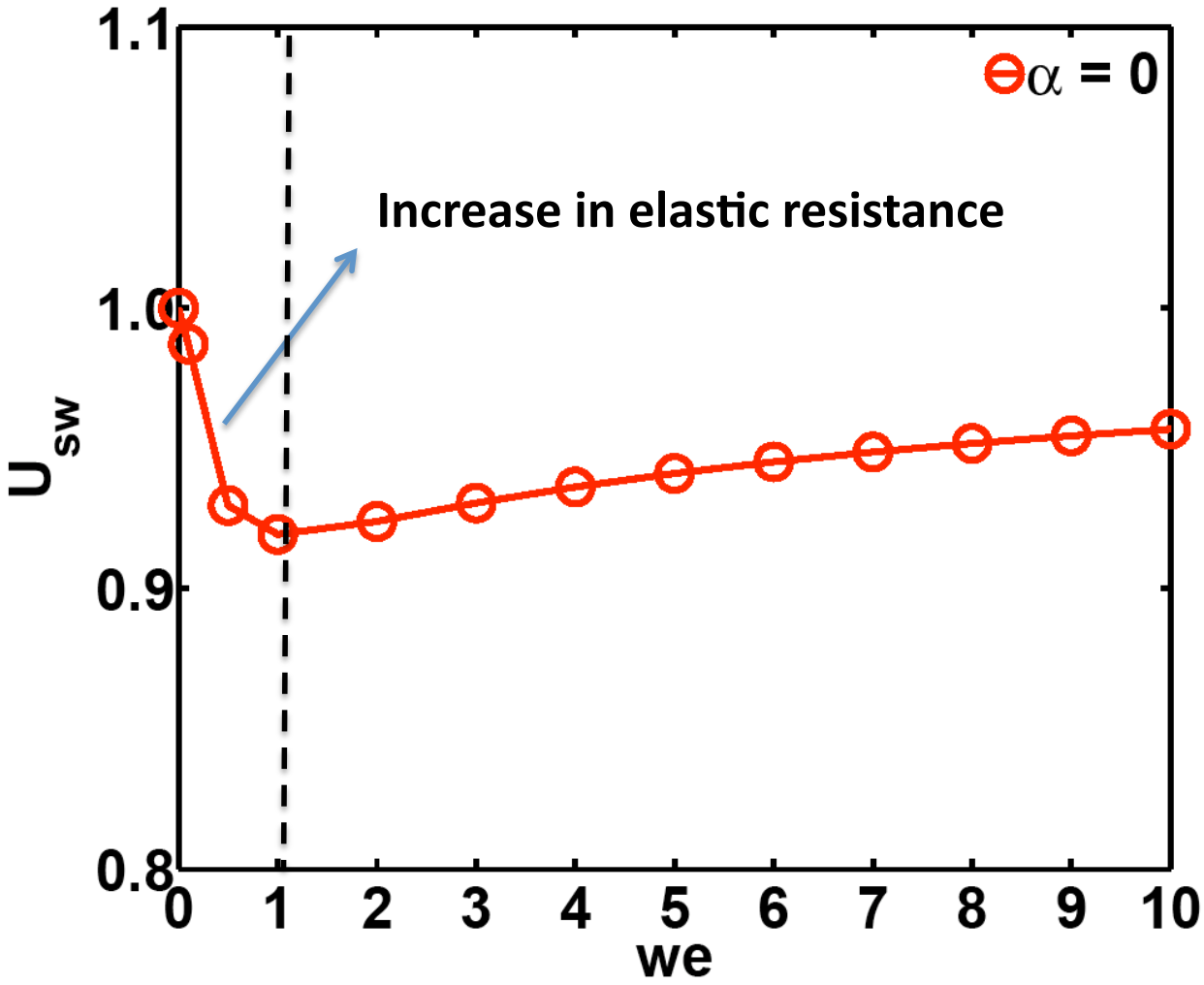


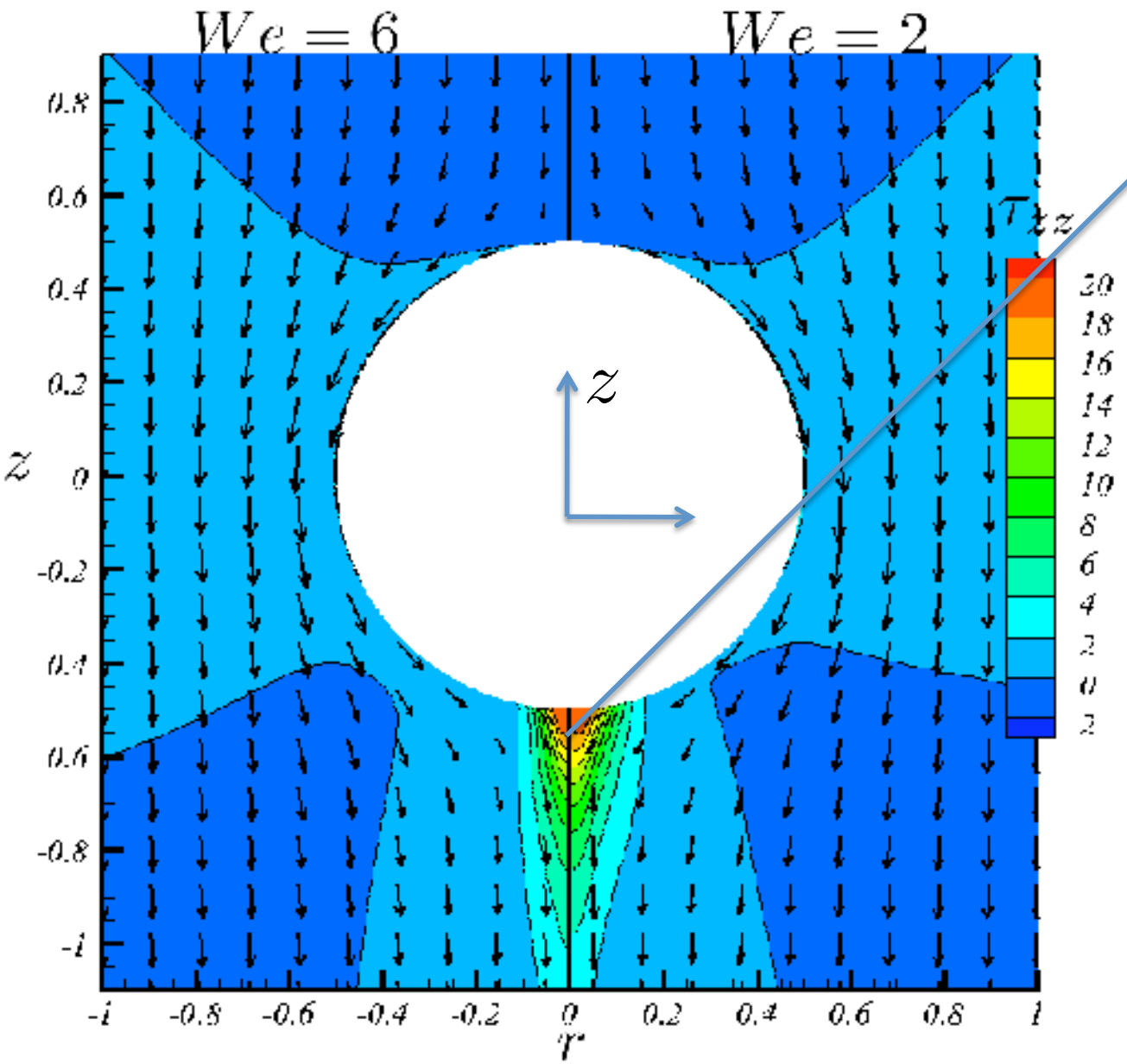
$\alpha = 0$ Swimmer


$We = 0.1$
$We = 2$
τ_{zz}
30
18
16
14
12
10
8
6
4
2
0
2
z
r
Elastic Resistance

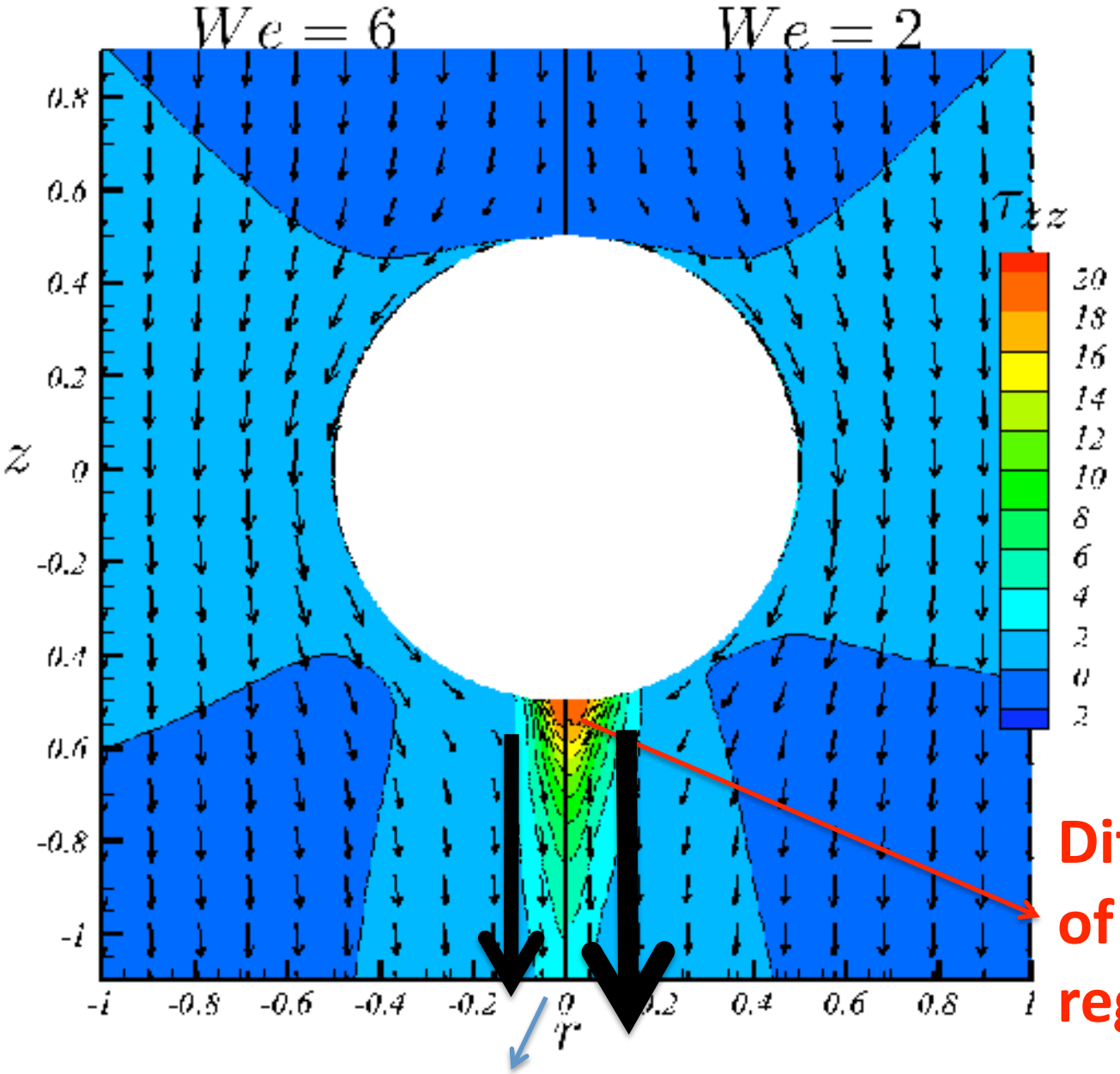
Correlate swimming speed with flow field

$\alpha = 0$ Swimmer





No obvious difference in the magnitude



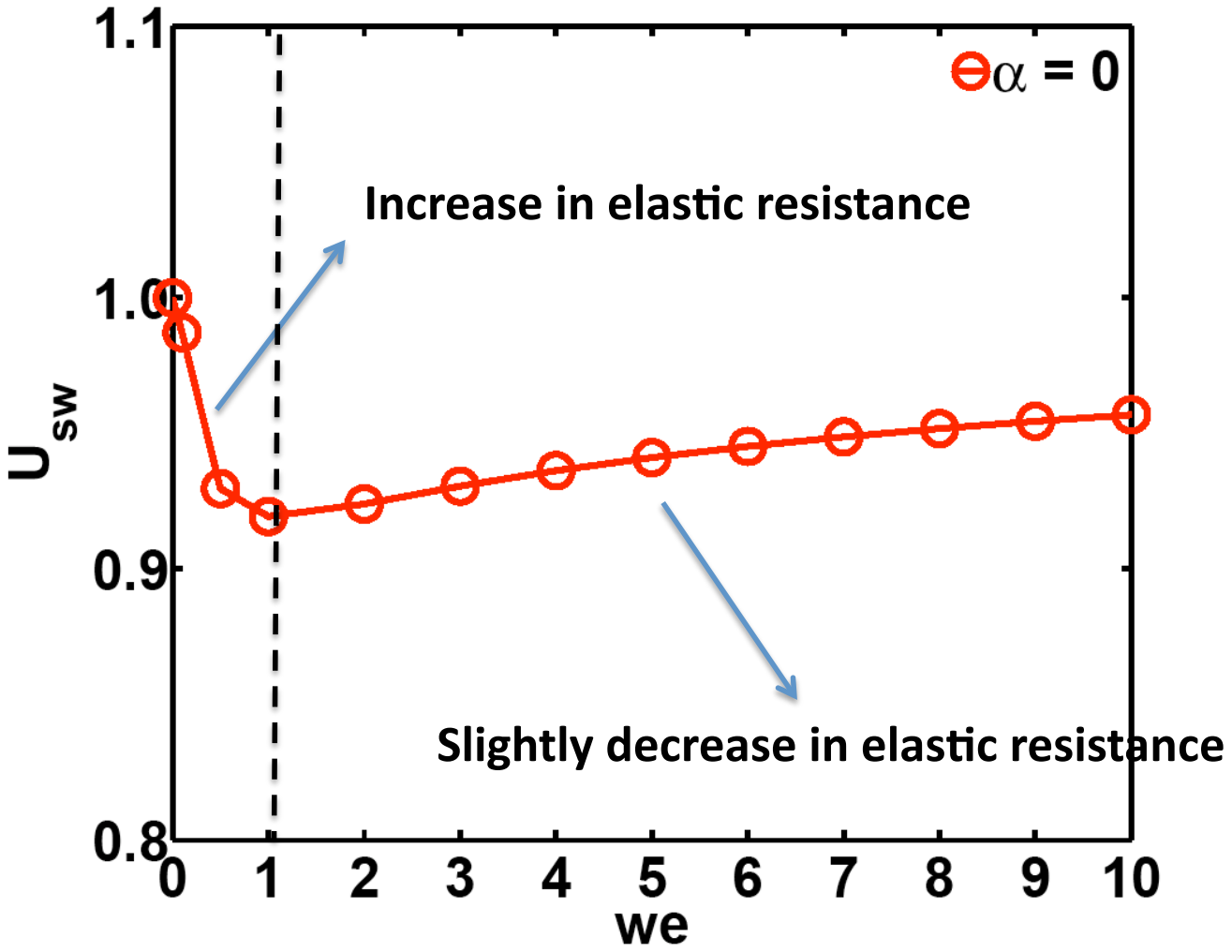
No obvious difference in the magnitude

Difference in size of highly concentrated region

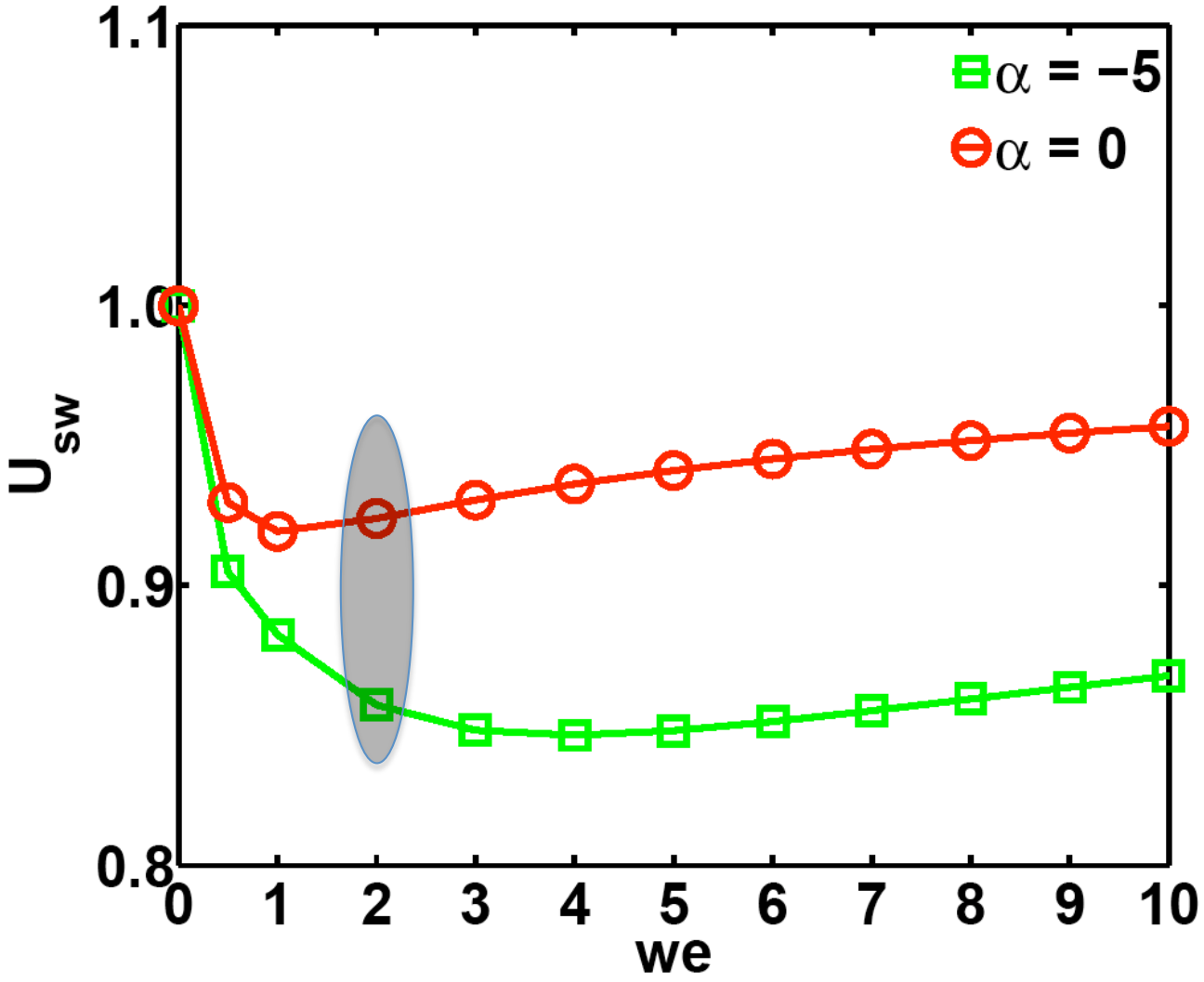
Slightly smaller elastic resistance with increase We

Correlate swimming speed with flow field

$\alpha = 0$ Swimmer

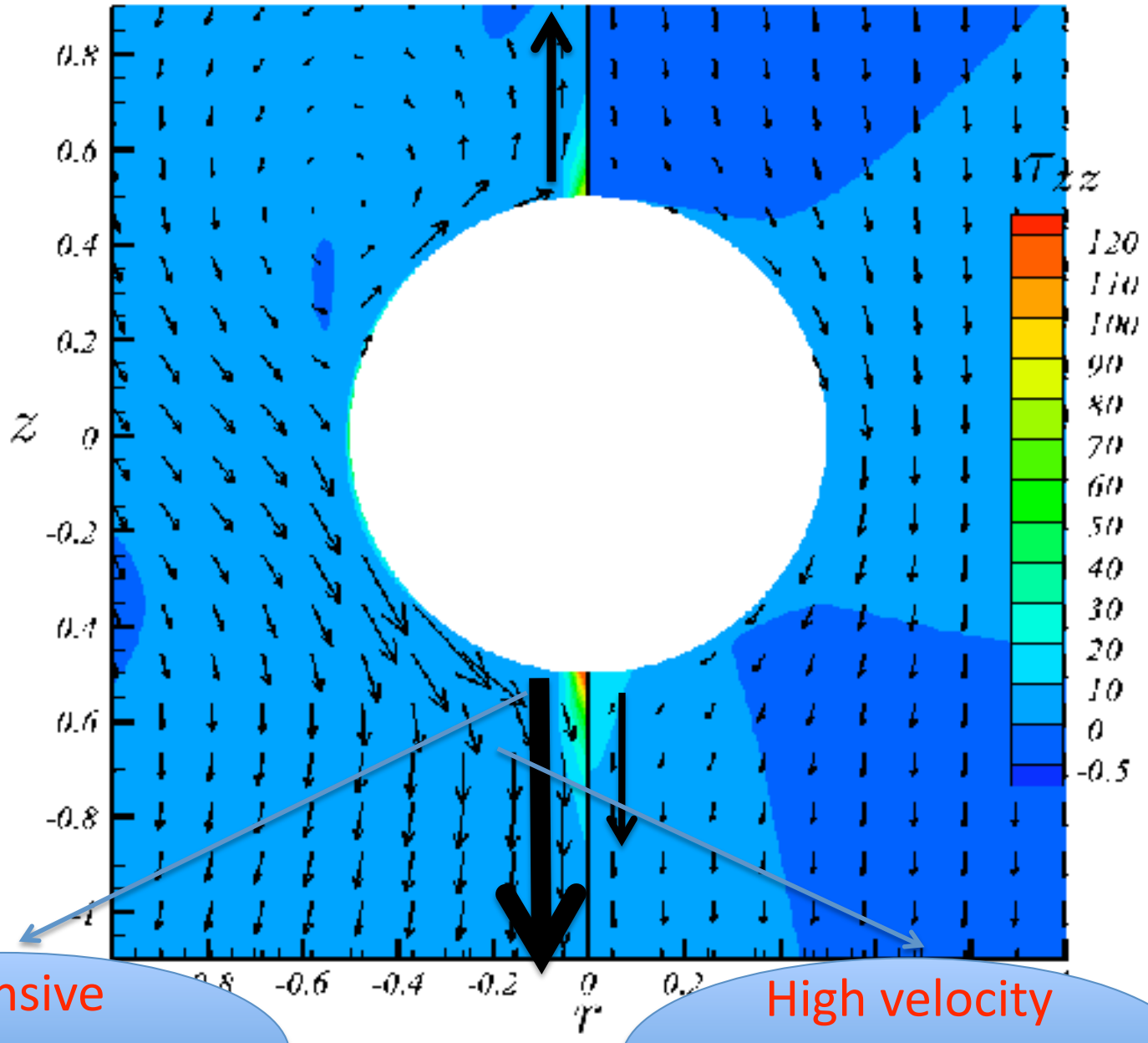


Why $\alpha = -5$ Pusher is slower?



$\alpha = -5$ Pusher $\alpha = 0$ Neutral swimmer

We =2

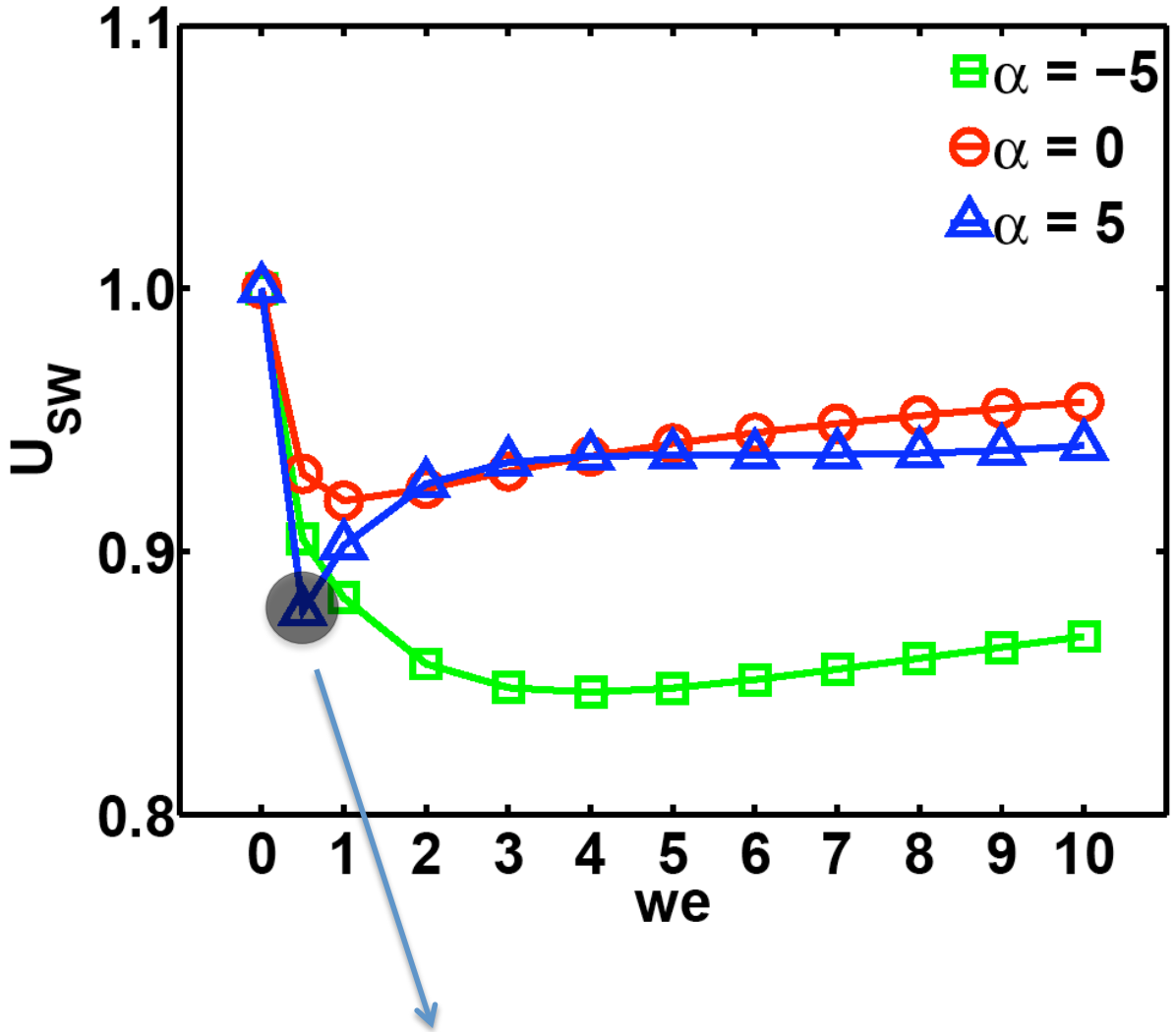


Intensive polymer accumulation

AND

High velocity gradient in z direction

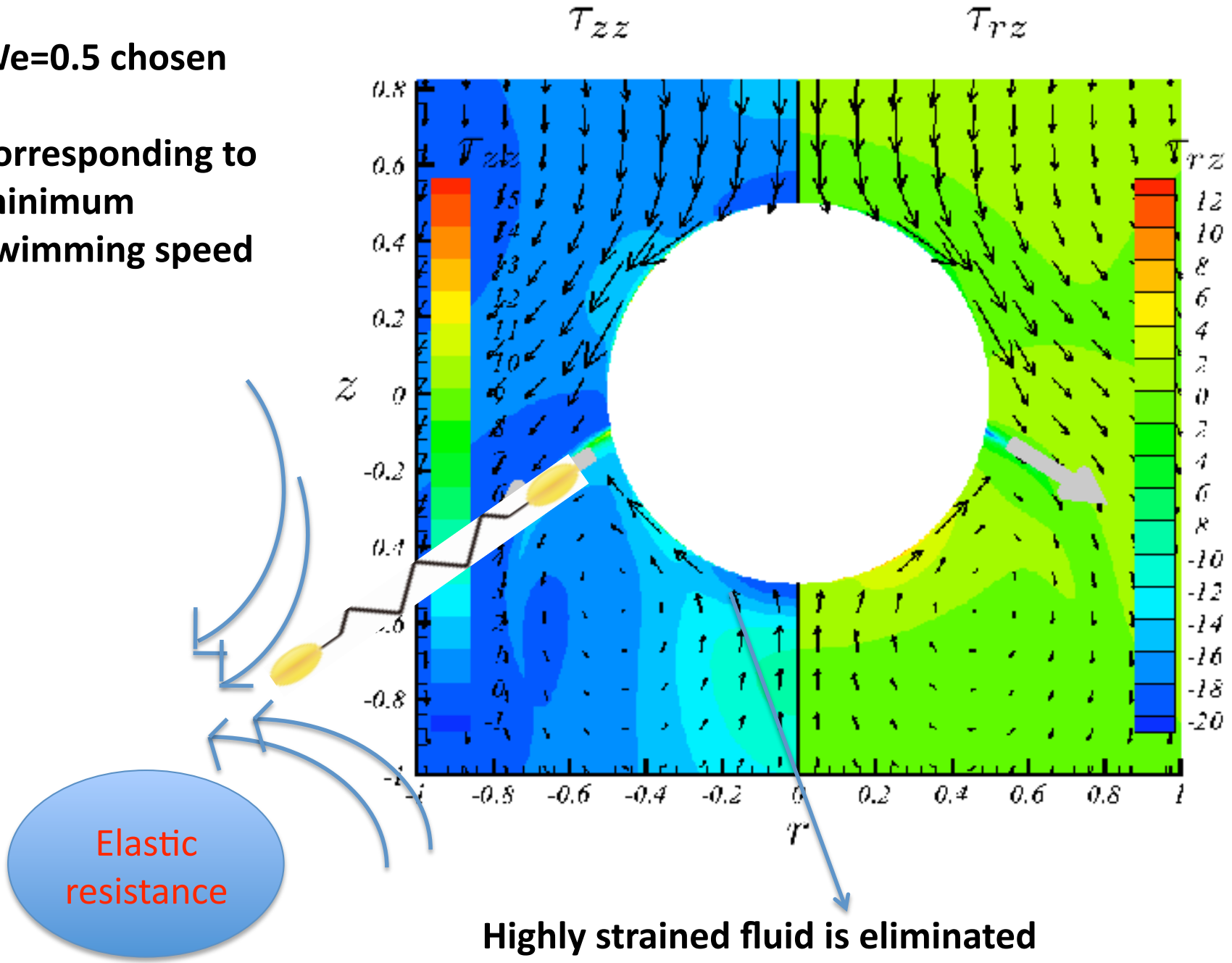
What about $\alpha = 5$ Puller ?



We = 0.5 is chosen for flow field based analysis

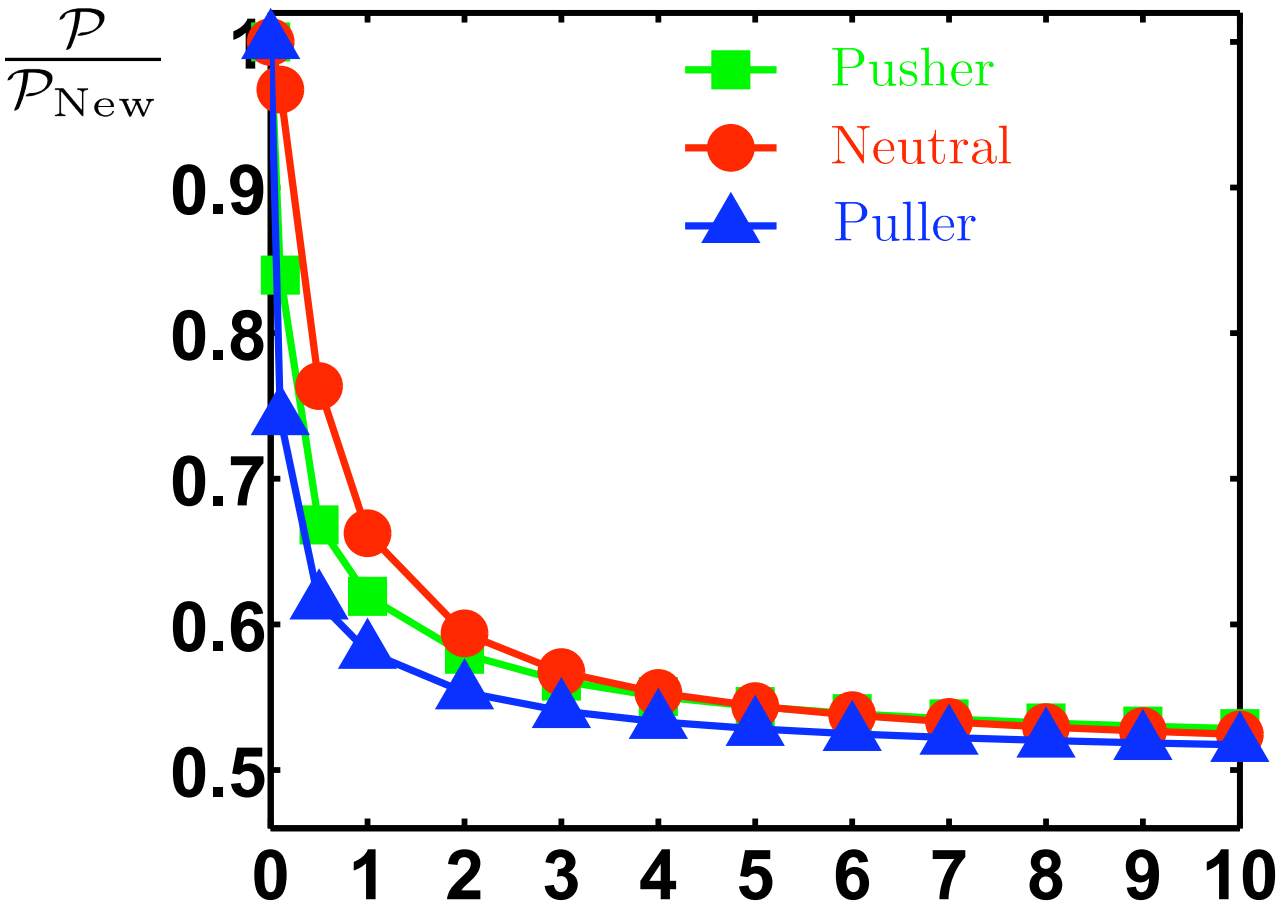
We=0.5 chosen

corresponding to minimum swimming speed

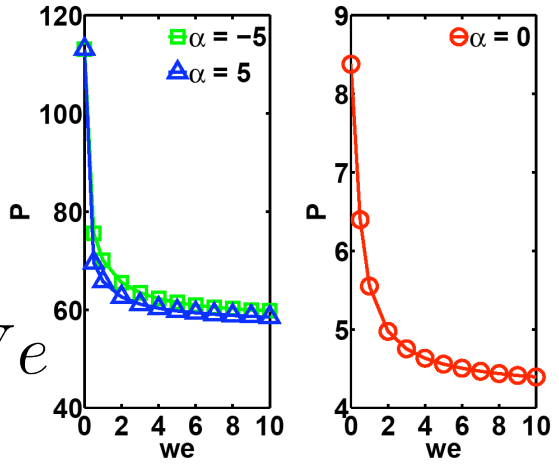


Highly strained fluid is eliminated but stretching in hyperbolic flow (Shen et al., PRL 2010)

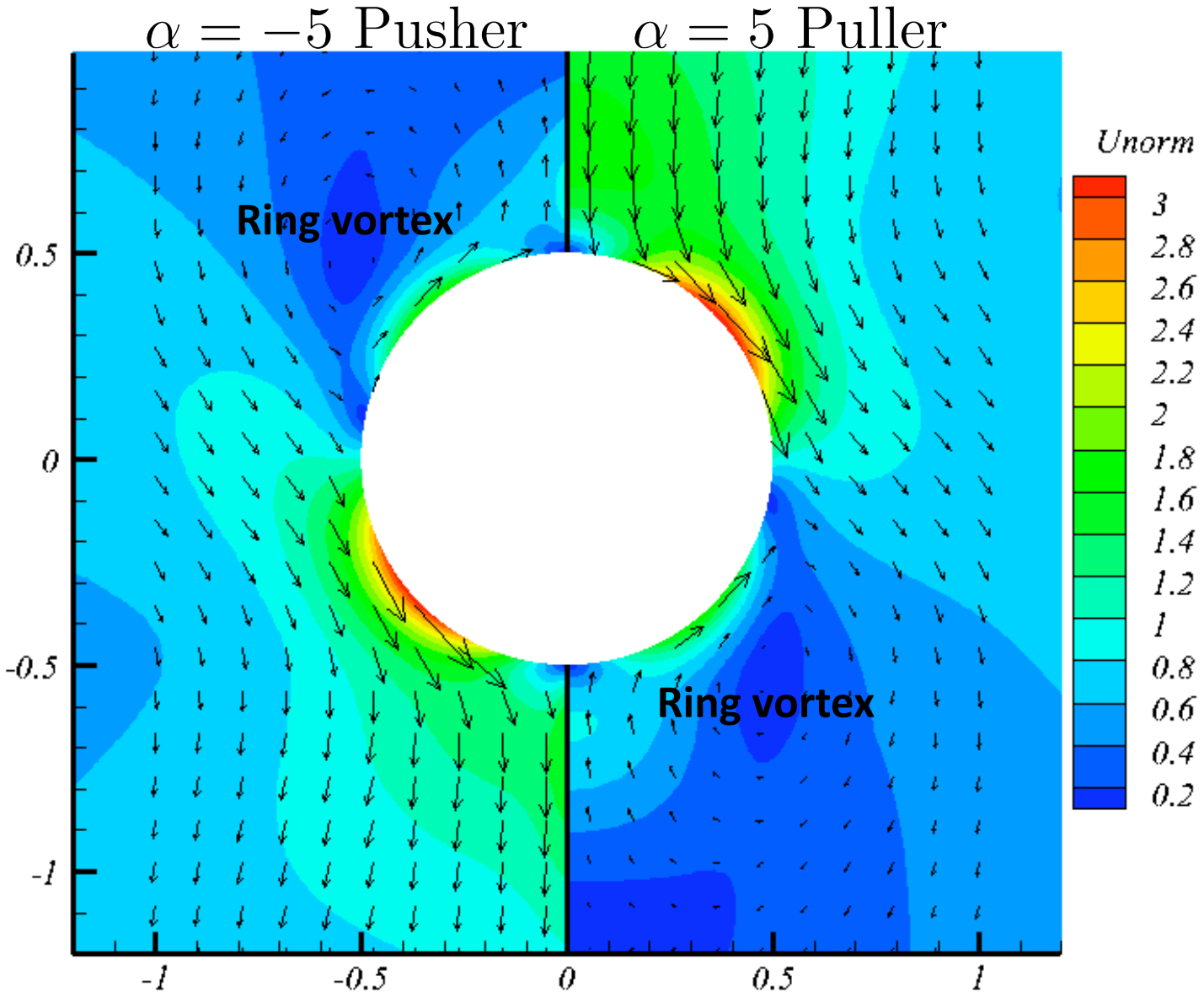
Power consumption



Pusher and puller:
most unefficient



✓ Lower consumption in viscoelastic fluids



Ring vortex responsible for much higher power consumption

Power Decomposition Analysis

$$P = \int_V 2\mu \mathbf{E} : \mathbf{E} dV =$$

$$\int_V \mu\omega^2 dV \qquad + \qquad \int_V 2\mu \nabla \mathbf{u} : \nabla \mathbf{u} dV$$

Stone & Samuel, PRL, 1996

Power Decomposition Analysis

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$$P =$$

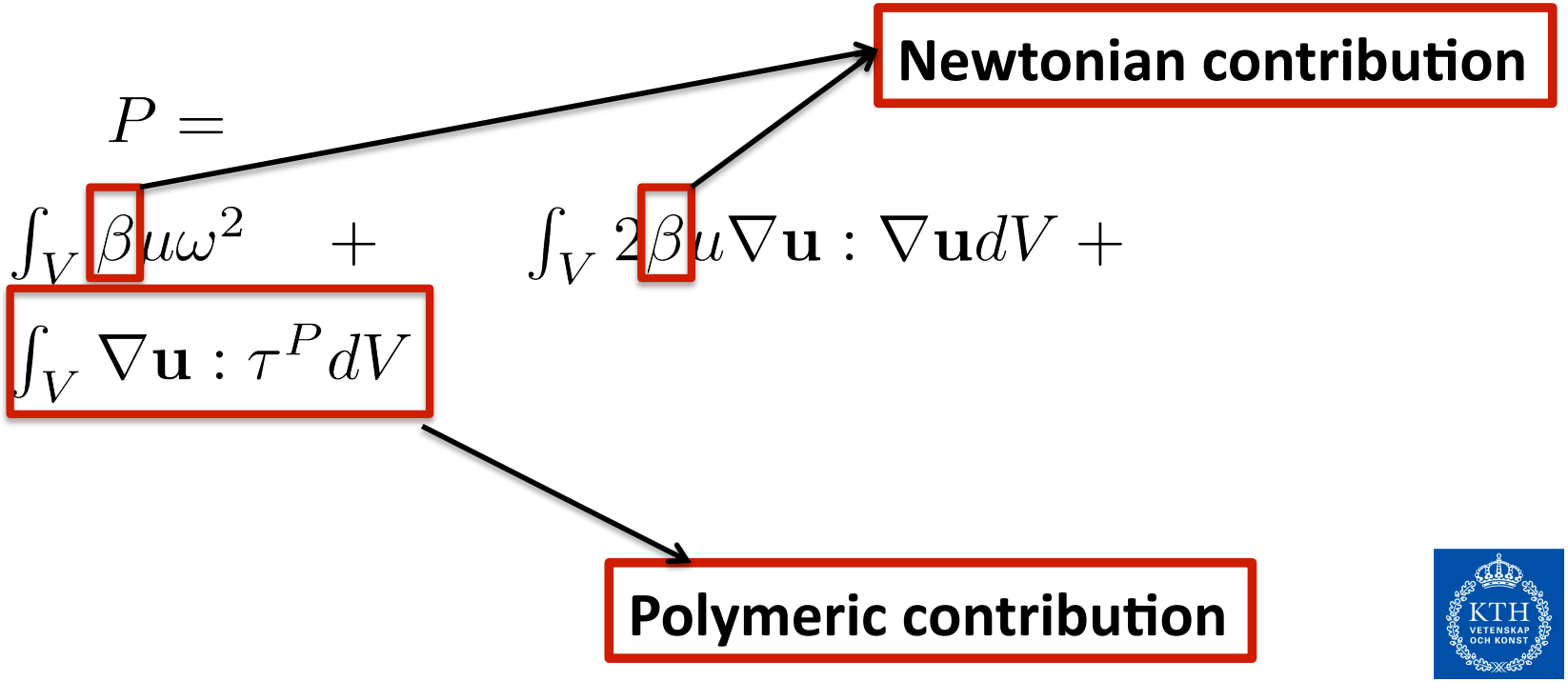
$$\int_V \quad \mu\omega^2 \quad + \qquad \int_V 2 \quad \mu \nabla \mathbf{u} : \nabla \mathbf{u} dV$$

Power Decomposition Analysis

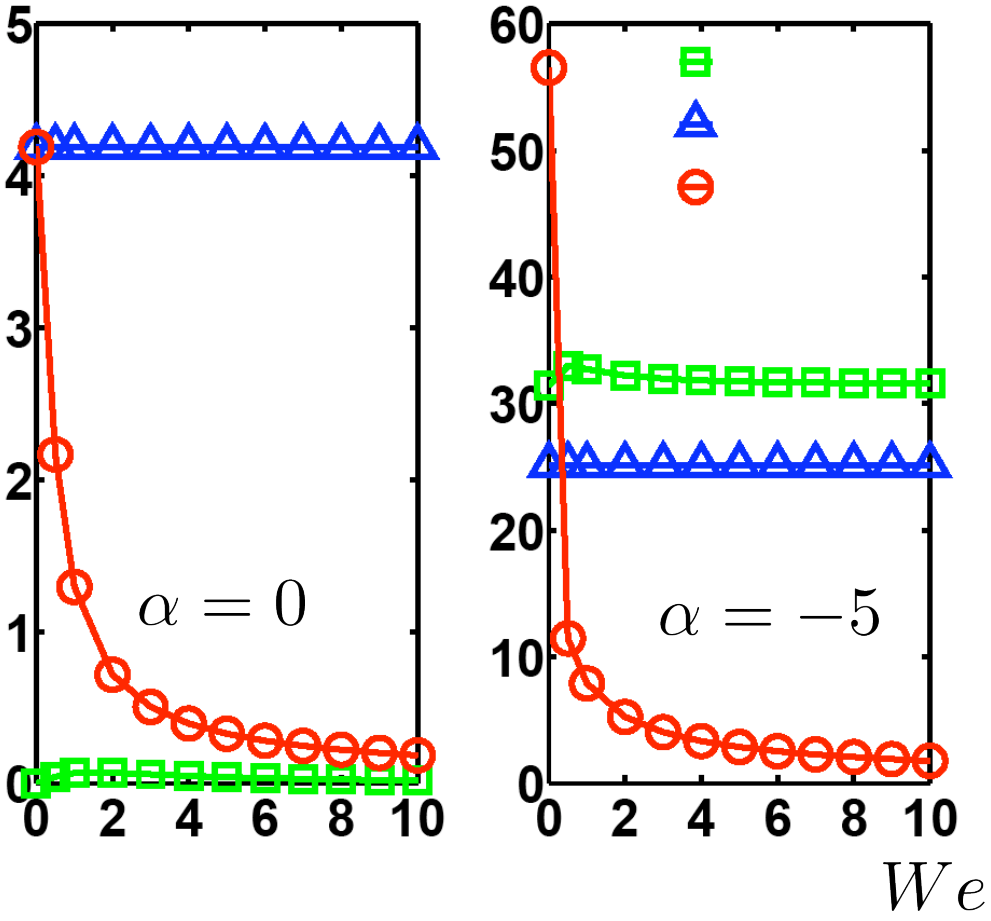
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Power Decomposition Analysis



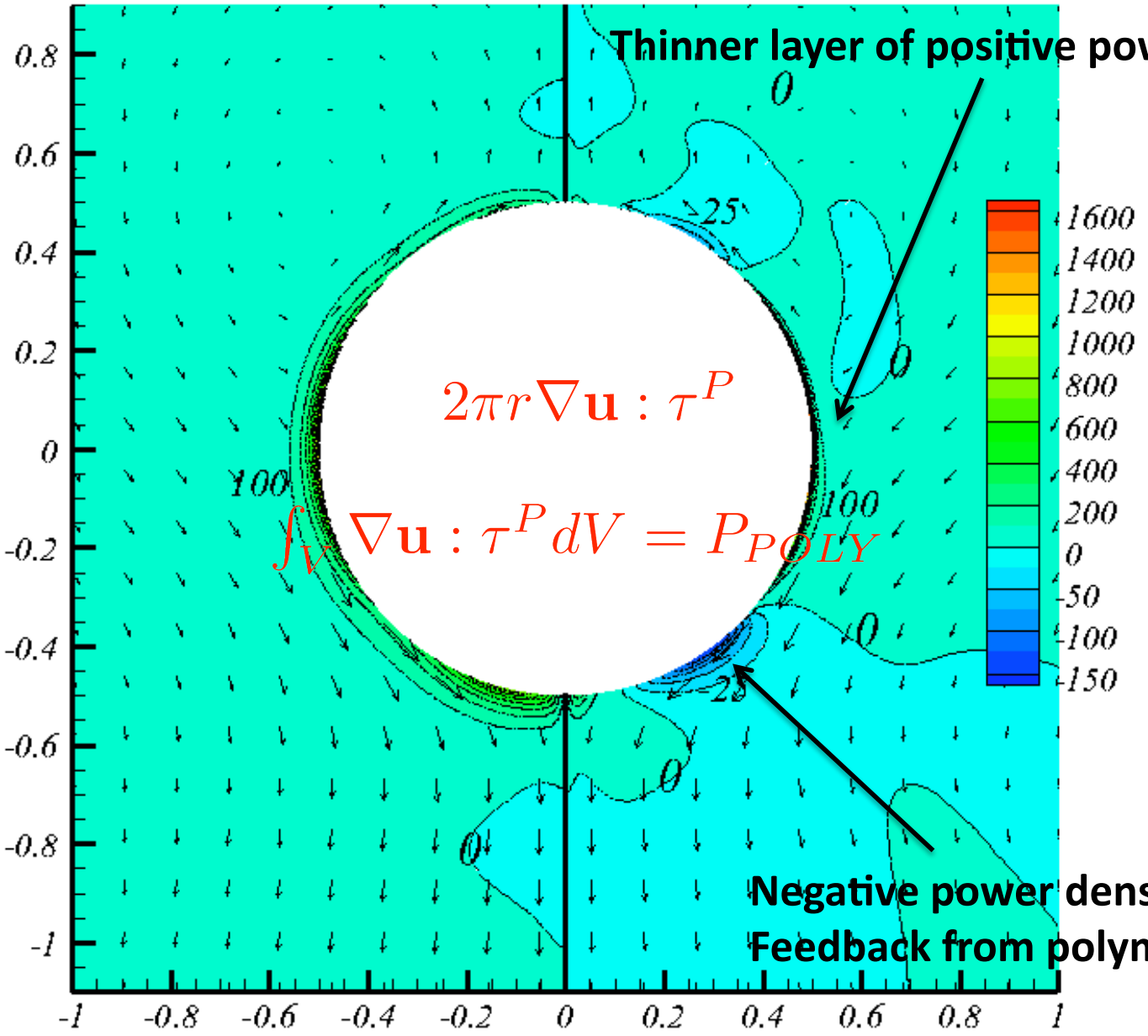
$$\int_V \beta\mu\omega^2 dV = P_\Omega$$

$$\int_V 2\beta\mu\nabla\mathbf{u} : \nabla\mathbf{u} dV = P_{DV}$$

$$\int_V \nabla\mathbf{u} : \tau^P dV = P_{POLY}$$

$$P = P_\Omega + P_{DV} + P_{POLY}$$

$\alpha = -5$ Pusher $We = 0.1$ $We = 2$



$$\boldsymbol{\tau}^p + \boxed{We\overset{\nabla}{\boldsymbol{\tau}}^p + \frac{We\alpha_m}{1-\beta}(\boldsymbol{\tau}^p \cdot \boldsymbol{\tau}^p)} = (1-\beta)(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$$

Integral of polymeric part: $\nabla\mathbf{u} : \boldsymbol{\tau}^p \equiv \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T) : \boldsymbol{\tau}^p$

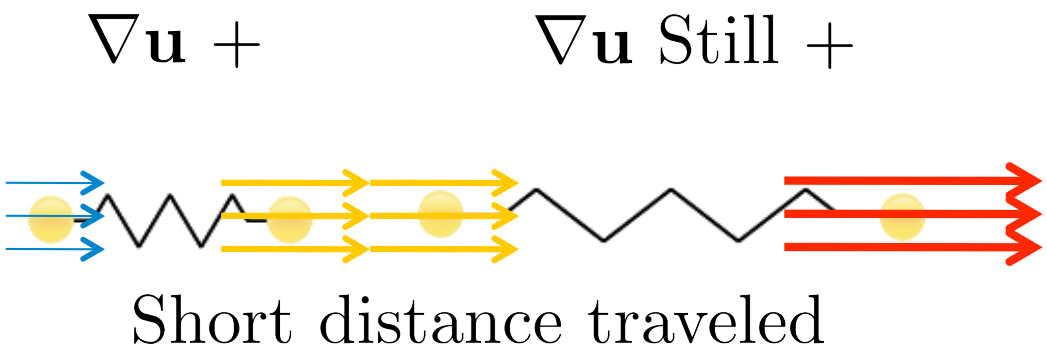
$$\text{As } We \to 0 \quad \boxed{We\overset{\nabla}{\boldsymbol{\tau}}^p + \frac{We\alpha_m}{1-\beta}(\boldsymbol{\tau}^p \cdot \boldsymbol{\tau}^p)} \to 0$$

$$\boldsymbol{\tau}^p \to (1-\beta)(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$$

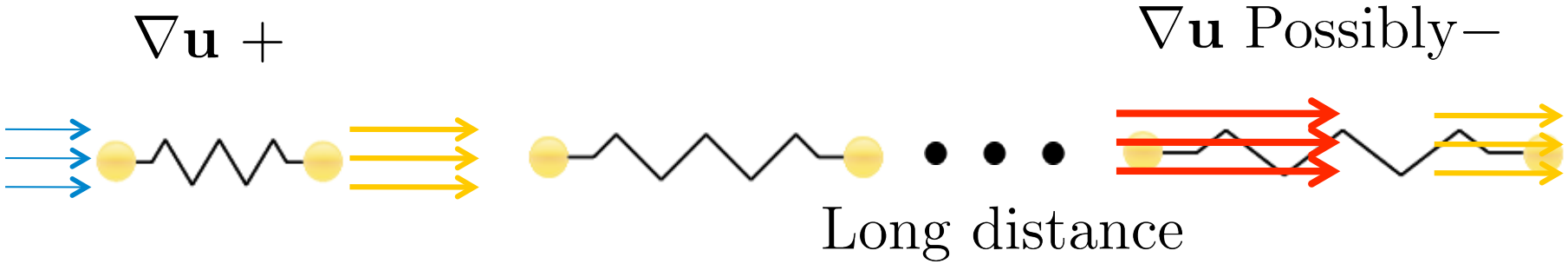
High spatial correlation between $(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ And $\boldsymbol{\tau}^p$

Optimal to maximum value of inner product →

Gives high value of $\frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T) : \boldsymbol{\tau}^p$

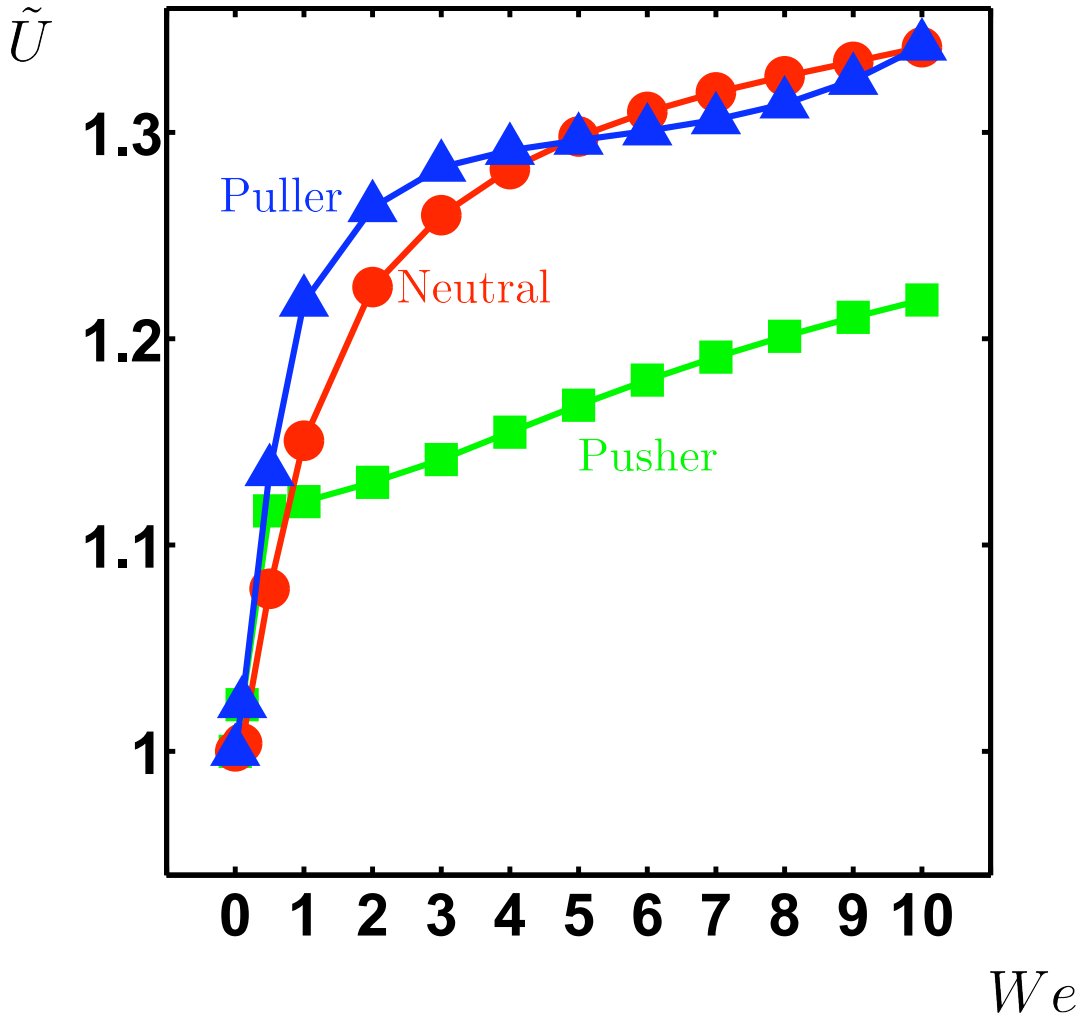


Small relaxation time Quick response to activation



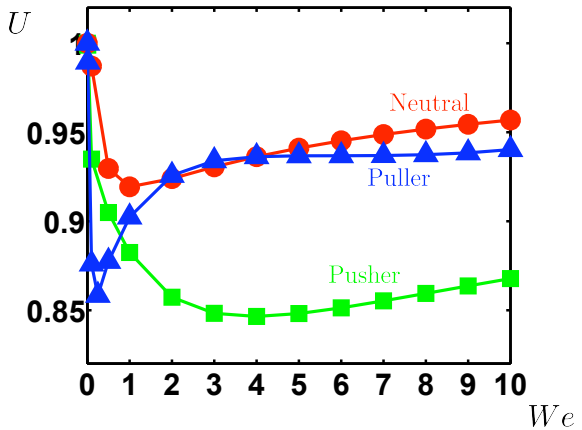
Large relaxation time Slow response to activation

Swimming speed with Constant power consumption

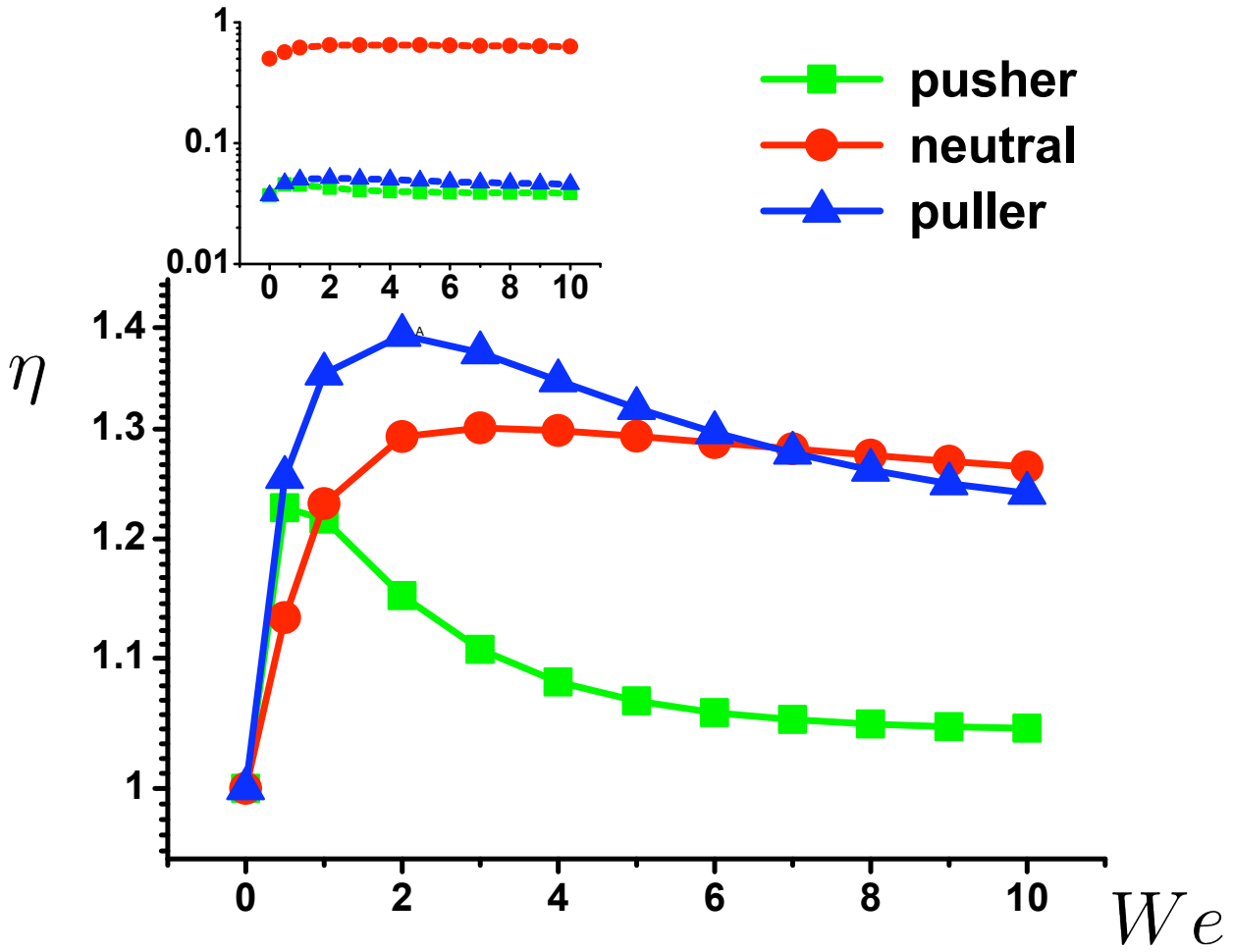


✓ Swimming speed increases !

Speed at constant gait

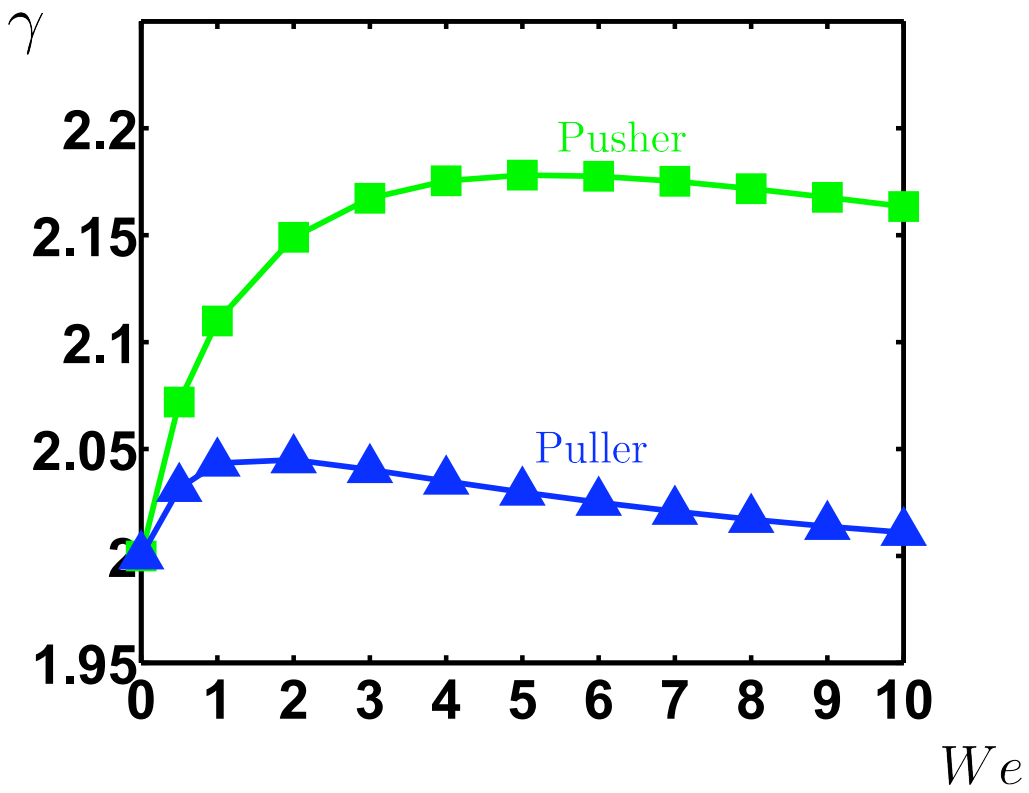


Efficiency



✓ Larger efficiency in viscoelastic fluids

Active suspensions: Velocity decay



Faster decay of velocity perturbation induced by a pusher

Stresslet

- Batchelor (1970) …Ishikawa & Pedley (2006) considered rheology of suspensions in dilute regime

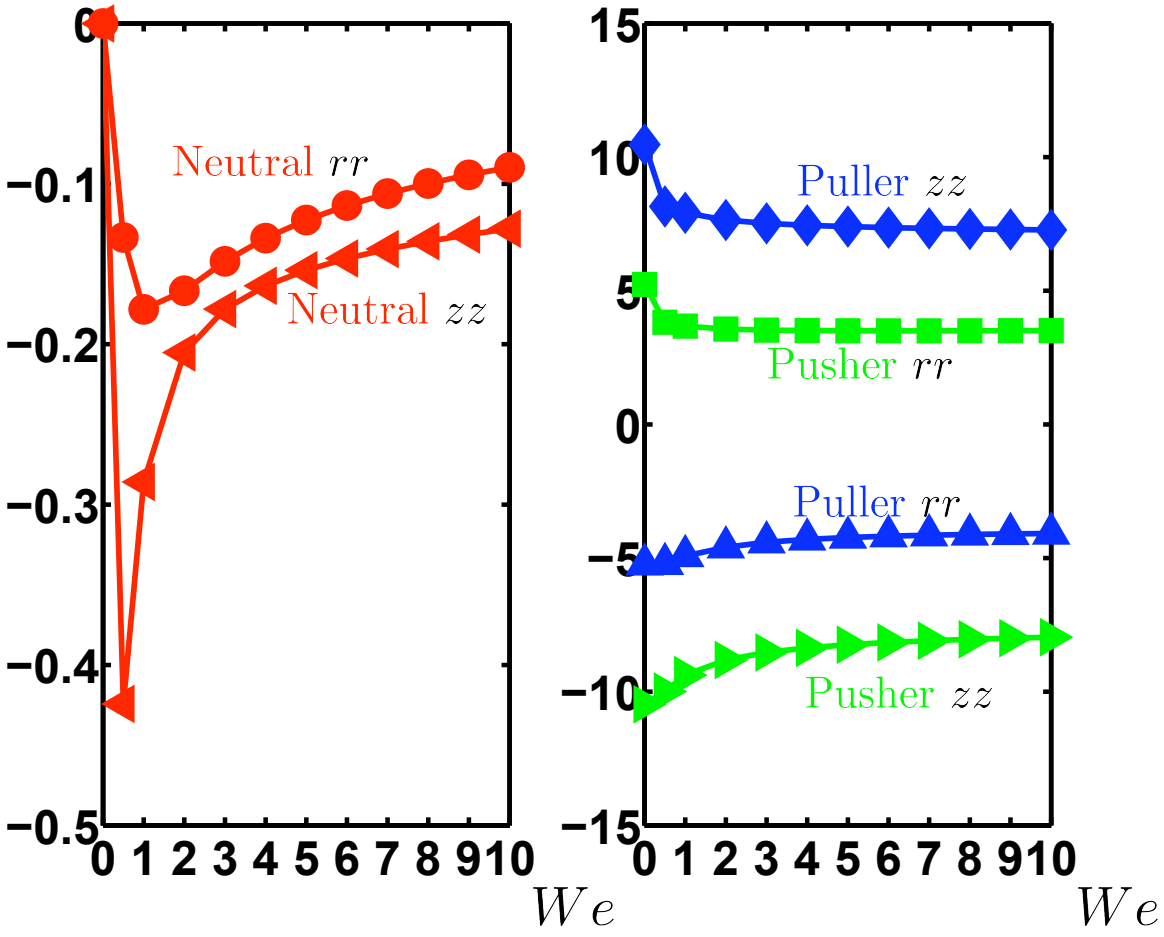
Bulk stress $$\hat{\boldsymbol{\sigma}} = \mathbf{I}.\mathbf{T}. + 2\mu\mathbf{E} + \bar{\boldsymbol{\tau}}_{\boldsymbol{p}} + \boldsymbol{\sigma}'$$

Particle bulk stress $$\boldsymbol{\sigma}' = \frac{1}{V}\sum \mathbf{S}$$

- Stresslet in polymeric flows require integration of solution inside the squirmer

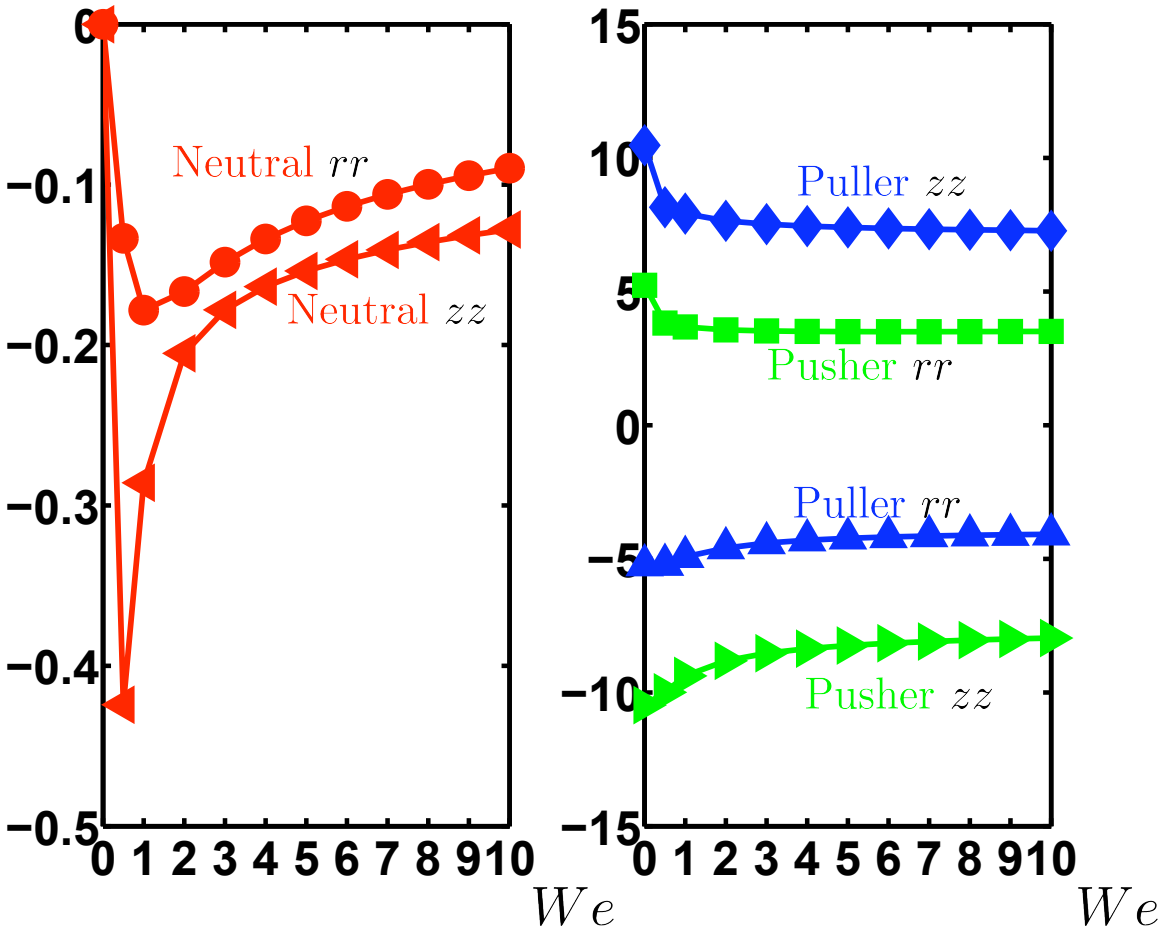
$$\mathbf{S} = \int_{A_0} \left[\frac{1}{2}\left\{ (\boldsymbol{\sigma}\cdot\mathbf{n})\,\mathbf{x} + \mathbf{x}\,(\boldsymbol{\sigma}\cdot\mathbf{n}) \right\} - \frac{1}{3}\mathbf{x}\cdot\boldsymbol{\sigma}\cdot\mathbf{n}\mathbf{I} - \mu\left(\mathbf{un} + \mathbf{nu}\right) \right] dA - \int_{V_0} \boldsymbol{\tau}^p \, dV$$

Stresslet



Decrease by 20% of stresslet amplitude

Stresslet



Velocity field induces separation between swimming pairs

Conclusions I

- Numerical simulations locomotion by tangential deformation in viscoelastic fluid
- Squirmer: Decrease in swimming speed and consumed power. Increase in efficiency!
- Spherical pullers are faster and more efficient in polymeric fluids (agreement with observation in nature)
- Decreasing effect of swimmers in polymeric suspensions

Outline

- How does locomotion of micro-organisms change in visco-elastic fluids ?

- What can swim only in visco-elastic fluids? Can we use this?

Thank you!