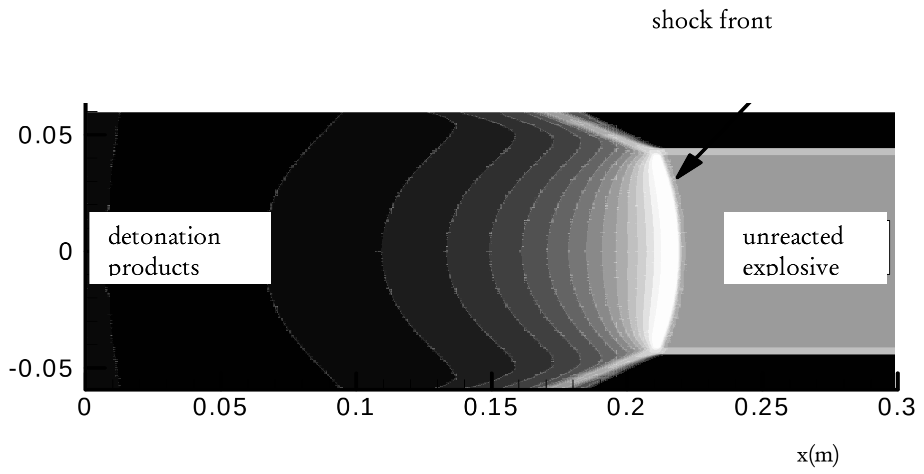

Figure 19. Detonation wave in a 84mm diameter cylindrical charge in a Plexiglas tube. Density contours

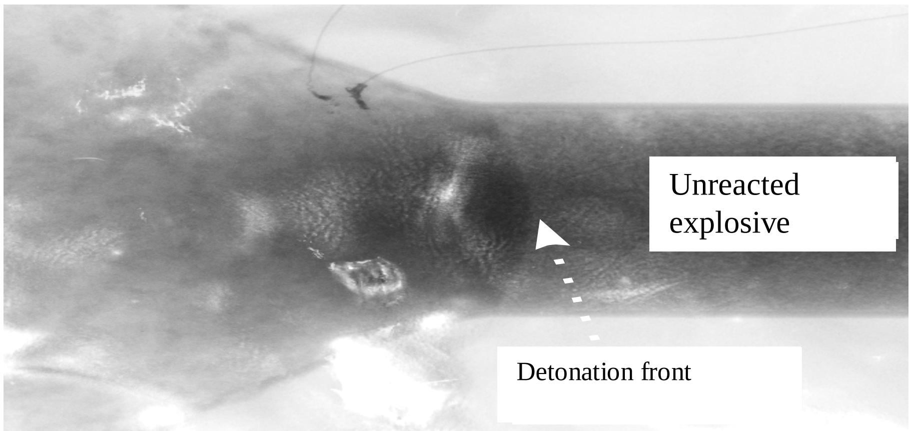

Figure 20. X ray measurement of the shock front and cylinder expansion from Saurel et al. [22]

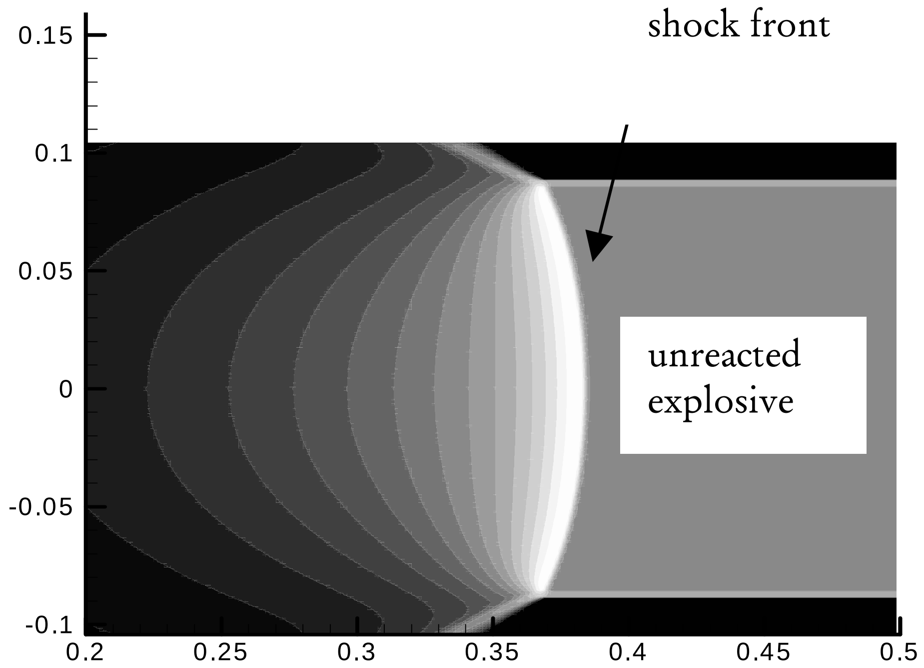

Figure 21. Detonation wave in a 174mm diameter cylindrical charge in a Plexiglas tube. Density contours.

Charge diameter	Detonation velocity Exp.	Comp.	Radius of curvature Exp.	Comp.	Expansion angle Exp.	Comp.
84 mm	3060 m/s	3160 m/s	96 mm	104 mm	32^o	31^o
104 mm	3467 m/s	3369 m/s	116 mm	120 mm	32^o	32^o
174 mm	4000 m/s	4006 m/s	290 mm	285 mm	33^o	32^o

Table 1
Experimental vs computational results for the propagation of detonation waves in explosive cylinders of various charge diameter. Experiments : left column. Computations: right column.

A. Derivation of the Roe average for the stiffness gas EOS

In the case of the stiffned gas, the equation of state is

$$\beta p + \theta = \rho\epsilon = E - \frac{1}{2}\rho u^2 \tag{40}$$

In the one dimensional case, the fluid is governed by

$$\partial_t W + \partial_x F = 0 \tag{41}$$

with initial and boundary conditions. The vector of conserved variables is $W = (\rho, \rho u, E, \rho\beta, \rho\theta) := (\rho, m, E, B, \Theta)$ and the flux is $F(W) = (\rho u, \rho u^2 + p, u(E+p), u\rho\beta, u\rho\theta)$. The Jacobian of F with respect to W is

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -u^2 + p_\rho & 2u + p_m & p_E & p_B & p_\Theta \\ u\,(-H+p) & H + up_m & u(1+p_E) & up_B & up_\Theta \\ -u\beta & \beta & 0 & u & 0 \\ -u\theta & \theta & 0 & 0 & u \end{pmatrix} \tag{42}$$

where

$$\begin{aligned} p_\rho &= \frac{\epsilon + \dfrac{u^2}{2}}{\beta} \quad & p_m &= -\frac{u}{\beta} \\ p_E &= \frac{1}{\beta} & p_B &= -\frac{p}{\rho\beta} \\ p_\Theta &= -\frac{1}{\rho\beta}. \end{aligned} \tag{43}$$

If we set $\Delta f = f_L - f_R$, the Roe average is defined by replacing in (42) the Roe average of the velocity and the total enthalpy H and by defining the average derivative of the pressure such that the following jump relation is satisfied,

$$\widetilde{\frac{\epsilon}{\beta}}\Delta\rho + \widetilde{\frac{1}{\beta}}\Delta(\rho\epsilon) - \widetilde{\frac{p}{\rho\beta}}\Delta(\rho\beta) - \widetilde{\frac{1}{\rho\beta}}\Delta(\rho\theta) = \Delta p. \tag{44}$$

Let $\lambda \in [0,1]$ and define for any function f, $\overline{f} = \lambda f_L + (1-\lambda) f_R$ and $\underline{f} = \lambda f_R + (1-\lambda) f_L$. For any functions f and g, we have

$$\Delta f g = \overline{f} \Delta g + \underline{g} \Delta f.$$

Last, if $\lambda = \frac{\sqrt{\rho_L}}{\sqrt{\rho_L}+\sqrt{\rho_R}}$, we have $\underline{\rho f} = \underline{\rho}\overline{f}$.

Using the above calculation rules, we can rewrite Δp as

$$\Delta p = \overline{\frac{1}{\beta}}\underline{\rho}\Delta\epsilon + \overline{\frac{1}{\beta}}\overline{\epsilon}\Delta\rho - \frac{\underline{\rho\epsilon} - \underline{\theta}}{\beta_*^2}\Delta\beta - \overline{\frac{1}{\beta}}\Delta\theta$$

where we have set $\beta_*^2 = \beta_L \beta_R$. last, by proceeding by identification, we get

$$\begin{aligned} &\widetilde{\frac{\epsilon}{\beta}} := \overline{\epsilon}\frac{1}{\underline{\beta}} && \widetilde{\frac{1}{\beta}} := \overline{\frac{1}{\beta}} = \frac{\underline{\beta}}{\beta_*^2} \\ &\widetilde{\frac{p}{\rho\beta}} := \overline{\frac{1}{\underline{\rho}}}\left(\frac{\underline{\rho\epsilon} - \underline{\theta}}{\beta_*^2}\right) && \\ &\widetilde{\frac{1}{\rho\beta}} := \overline{\frac{1}{\rho\beta}} && \end{aligned} \tag{45}$$

The matrix of eigenvectors is easily computed,

$$R = \begin{pmatrix} r_1 & r_2 & r_3 & r_4 & r_5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ \overline{u} & \overline{u} & \overline{u} & \overline{u} - \overline{c} & \overline{u} + \overline{c} \\ \dfrac{\overline{u}^2}{2} - \overline{\epsilon} & \dfrac{\overline{u}^2}{2} & \dfrac{\overline{u}^2}{2} & \overline{H} - \overline{uc} & \overline{H} + \overline{uc} \\ 0 & \overline{\beta} & 0 & \overline{\beta} & \overline{\beta} \\ 0 & \underline{\theta} & \underline{\rho\epsilon} & \underline{\theta} & \underline{\theta} \end{pmatrix} \tag{46}$$

associated to the eigenvalues $\overline{u}$, $\overline{u}$, $\overline{u}$, $\overline{u} - \overline{c}$, $\overline{u} + \overline{c}$ where $\overline{c}^2 = \overline{\beta}\left(\overline{H} - \dfrac{\overline{u}^2}{2}\right)$.

Similarly, any vector $W = (A, B, C, D, E)$ in $\mathbb{R}^5$ can be decomposed as

$$W = \sum_{i=1}^{5} l_i(W) r_i.$$

We get, for $W = (\Delta\rho, \Delta m, \Delta E, \Delta B, \Delta\Theta)$,

$$\begin{aligned} l_1(W) &= \frac{1}{\overline{\epsilon}}\left(\overline{\beta}\Delta\theta - \overline{\theta}\Delta\beta\right) \\ l_2(W) &= \Delta\rho - \frac{\Delta p}{\overline{c}^2} - l_1 - l_3 \\ l_3(W) &= \frac{\underline{\theta} - \underline{\rho\epsilon}}{\overline{\beta}\underline{\epsilon}}\Delta\beta - \frac{\Delta\theta}{\underline{\epsilon}} \\ l_4(W) &= \frac{1}{2}\left(\frac{\Delta p}{\overline{c}^2} - \frac{\underline{\rho}}{\overline{c}}\Delta u\right) \\ l_5(W) &= \frac{1}{2}\left(\frac{\Delta p}{\overline{c}^2} + \frac{\underline{\rho}}{\overline{c}}\Delta u\right) \end{aligned} \tag{47}$$

B. 2D Roe type scheme

This section is devoted to the derivation of equation (25) in section §5.2.
We first introduce some notations. We set

$$Z = \begin{pmatrix} \mathcal{W} \\ \rho\mathcal{V} \\ \rho\beta \\ \rho\theta \end{pmatrix}, \quad \mathcal{G} = \begin{pmatrix} \mathcal{F} \\ \rho\mathcal{V}\mathcal{U} \\ \rho\beta\mathcal{U} \\ \rho\theta\mathcal{U} \end{pmatrix}, \quad \mathcal{M}_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \boldsymbol{\eta}_x & \boldsymbol{\eta}_y & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \boldsymbol{\eta}_x^{\perp} & \boldsymbol{\eta}_y^{\perp} & 0 \end{pmatrix},$$

$$\mathcal{Z} = \mathcal{M}_{ij} W, \quad W = \mathcal{M}_{ij}^{-1} \mathcal{Z}$$

and

$$\mathcal{J}\left(\boldsymbol{u}, H, \beta, \theta, \mathcal{S}_0, \mathcal{S}_1\right) = \partial_Z \mathcal{G} = \mathcal{U}\ Id + \begin{pmatrix} \mathcal{S}_0 \\ \mathcal{U}\mathcal{S}_0 + \mathcal{S}_1 \\ H\mathcal{S}_0 + \mathcal{U}\mathcal{S}_1 \\ \mathcal{V}\mathcal{S}_0 \\ \beta\mathcal{S}_0 \\ \theta\mathcal{S}_0 \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{S}_0 &= \begin{pmatrix} -\mathcal{U} & 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \rho\nabla_Z \cdot \mathcal{U} \\ \mathcal{S}_1 &= \begin{pmatrix} \partial_\rho p & \partial_{\rho\mathcal{U}} p & \partial_E p & \partial_{\rho\mathcal{V}} p & \partial_{\rho\beta} p & \partial_{\rho\theta} p \end{pmatrix} = \nabla_Z \cdot p. \end{aligned}$$

We denote by Z_i and Z_j the values of the conservative variables in the cells i and j. For any function f, we set $\overline{f} = \lambda f_i + (1-\lambda) f_j$ and $\underline{f} = \lambda f_j + (1-\lambda) f_i$. These two averages have the property that

$$\Delta(fg) = \overline{f}\Delta g + \underline{g}\Delta f.$$

Moreover, if

$$\lambda = \frac{\sqrt{\rho_i}}{\sqrt{\rho_i} + \sqrt{\rho_j}},$$

we have $\underline{\rho g} = \underline{\rho}\overline{g}$. These remarks simplify the following calculations.

We define the matrix $\tilde{\mathcal{J}}\left(\overline{\boldsymbol{u}}, \overline{H}, \overline{\beta}, \overline{\theta}, {\mathcal{S}_0}^*, {\mathcal{S}_1}^*\right)$ with $\underline{\rho\tilde{\epsilon}} = \overline{\beta}\underline{p} + \overline{\theta}$,

$$\begin{aligned} {\mathcal{S}_0}^* &= \begin{pmatrix} -\overline{\mathcal{U}} & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \\ {\mathcal{S}_1}^* &= \frac{1}{\overline{\beta}}\begin{pmatrix} \tilde{\epsilon} + \tilde{\mathcal{Q}} & -\overline{\mathcal{U}} & 1 & -\overline{\mathcal{V}} & -\frac{\underline{p}}{\underline{\rho}} & -\frac{1}{\underline{\rho}} \end{pmatrix} \end{aligned}$$

where $\tilde{\mathcal{Q}} = \frac{\overline{\mathcal{U}}^2 + \overline{\mathcal{V}}^2}{2}$. The matrix $\tilde{\mathcal{J}}$ satisfy the jump condition $\Delta\mathcal{G} = \tilde{\mathcal{J}} \cdot \Delta Z$ ($\Delta Z = \Delta_{ij} Z = Z_i - Z_j$). Moreover, we have the relations:

$${\mathcal{S}_0}^* \Delta Z = \underline{\rho}\Delta\mathcal{U}, \qquad {\mathcal{S}_1}^* \Delta Z = \Delta p$$

The matrix $\tilde{\mathcal{J}}$ is diagonalisable and the eigenvectors are:

$$\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ \overline{\mathcal{U}} \\ \tilde{\mathcal{Q}} \\ \overline{\mathcal{V}} \\ \overline{\beta} \\ \overline{\theta} \end{pmatrix}, \quad \boldsymbol{r}_2 = \begin{pmatrix} 1 \\ \overline{\mathcal{U}} - \tilde{c} \\ \overline{H} - \tilde{c}\overline{\mathcal{U}} \\ \overline{\mathcal{V}} \\ \overline{\beta} \\ \overline{\theta} \end{pmatrix}, \quad \boldsymbol{r}_3 = \begin{pmatrix} 1 \\ \overline{\mathcal{U}} + \tilde{c} \\ \overline{H} + \tilde{c}\overline{\mathcal{U}} \\ \overline{\mathcal{V}} \\ \overline{\beta} \\ \overline{\theta} \end{pmatrix},$$

$$\boldsymbol{r}_4 = \begin{pmatrix} 0 \\ 0 \\ \overline{\mathcal{V}} \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{r}_5 = \begin{pmatrix} 0 \\ 0 \\ \tilde{\epsilon} \\ 0 \\ \overline{\beta} \\ \overline{\theta} \end{pmatrix}, \quad \boldsymbol{r}_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \underline{\rho} \end{pmatrix},$$

where $\overline{\beta}\tilde{c}^2 = \overline{H} - \tilde{\mathcal{Q}}$. The associated eigenvalues are:

$$\lambda_1 = \overline{\mathcal{U}},\ \ \lambda_2 = \overline{\mathcal{U}} - \tilde{c},\ \ \lambda_3 = \overline{\mathcal{U}} + \tilde{c},\ \ \lambda_4 = \overline{\mathcal{U}},\ \ \lambda_5 = \overline{\mathcal{U}},\ \ \lambda_6 = \overline{\mathcal{U}},$$

In the eigenvectors basis, the jump of conservative variables is given by:

$$\begin{aligned} \Delta Z = \ & \Big(\Delta(\rho) - \xi\Big)\,\boldsymbol{r}_1 + \tfrac{\xi-\Theta}{2}\boldsymbol{r}_2 + \tfrac{\xi+\Theta}{2}\boldsymbol{r}_3 \\ & + \Big(\Delta(\rho\mathcal{V}) - \overline{\mathcal{V}}\Delta(\rho)\Big)\,\boldsymbol{r}_4 + \tfrac{\Delta(\rho\beta)-\overline{\beta}\Delta\rho}{\overline{\beta}}\boldsymbol{r}_5 + \tfrac{\overline{\beta}\Delta(\rho\theta)-\overline{\theta}\Delta(\rho\beta)}{\underline{\rho}\overline{\beta}}\boldsymbol{r}_6 \end{aligned} \tag{48}$$

where

$$\xi = \frac{1}{\overline{\beta}\tilde{c}^2}\Big(-\overline{\mathcal{V}}\Delta(\rho\mathcal{V}) - \overline{\mathcal{U}}\Delta(\rho\mathcal{U}) + \overline{\mathcal{Q}}\Delta(\rho) + \Delta(E) - \tilde{\epsilon}\tfrac{\Delta(\rho\beta)-\overline{\beta}\Delta\rho}{\overline{\beta}} - \tfrac{\overline{\beta}\Delta(\rho\theta)-\overline{\theta}\Delta(\rho\beta)}{\underline{\rho}\overline{\beta}}\Big) = \frac{\Delta p}{\tilde{c}^2}$$

and $\Theta = \dfrac{\Delta(\rho\mathcal{U}) - \overline{\mathcal{U}}\Delta(\rho)}{\tilde{c}}$