

Two phase Darcy flows in fractured porous media

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Hennicker

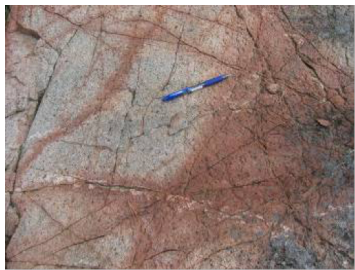
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Numérique
Meknès, 16-18 décembre 2019*

Outline

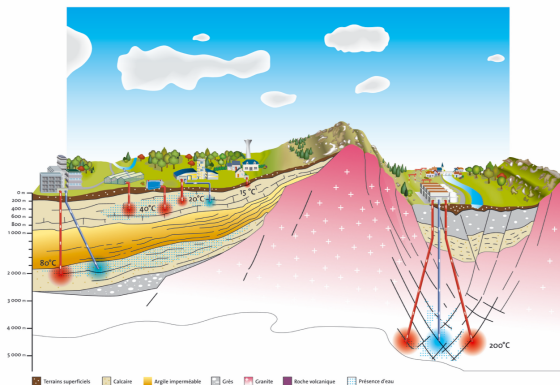
- Modelling Two-Phase Darcy flows in Discrete Fracture Networks
- Finite Volume Discretizations
- Numerical examples

Fractured/faulted porous media: multiple scales (figures from J. R. de Dreuzy, Geosciences Rennes and Inria)



Fractured/faulted porous media: applications

- Oil and gas
- Hydrogeology
- Geothermal energy
- Geological storages
- Soil remediation
- ...

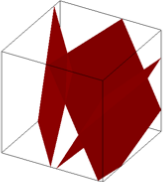
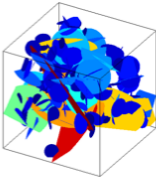
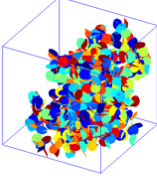
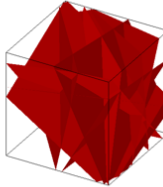
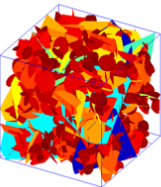
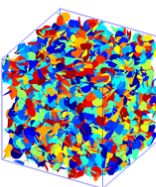


Flow in Fractured porous media: two main approaches

- Single or double Continuum Media: homogeneization for dense fracture networks
- **Discrete Fracture Matrix (DFM) models**: 2D fracture model coupled to 3D matrix medium
- Possibility to couple both approaches
 - Double Continuum media for small fractures coupled with DFM for large fractures
 - Numerical Homogeneization: parameters of the Double Continuum media computed by a DFM model

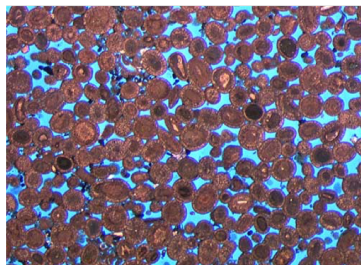
Discrete fracture networks (DFN) (figures from J. R. de Dreuzy, Geosciences Rennes and Inria)

Stochastic models of fractured media

Crossing fractures "LONG"	Power-law length "DIST"	Small fractures "SHORT"	
			threshold
			"3*threshold"

Modelling Two-Phase Darcy flows in Discrete Fracture Networks

Porous medium: Darcy scale and porosity



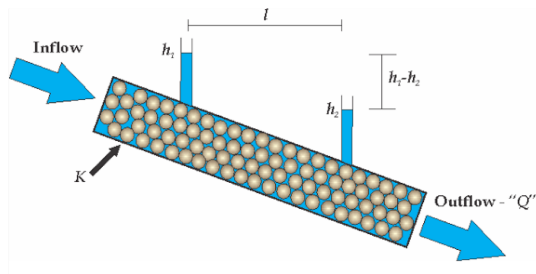
Sandstone: porous and permeable rock

$\phi(\mathbf{x})$: porosity = pore volume / total volume, $0.01 \leq \phi \leq 0.4$

Pore scale = micron meters

Darcy scale = mm

Darcy flow



Henry Darcy (1803-1858)

$$\mathbf{q} = -\frac{\Lambda(\mathbf{x})}{\mu}(\nabla u - \rho\mathbf{g}) \quad (\text{m}\cdot\text{s}^{-1})$$

u : fluid pressure (in Pa)

$\Lambda(\mathbf{x})$: rock permeability tensor in m^2 (1 Darcy = 10^{-12} m^2).

μ : fluid dynamic viscosity (Pa.s)

ρ : fluid mass density ($\text{Kg}\cdot\text{m}^{-3}$)

Two-phase Darcy flow

$\alpha = 1$: wetting phase

$\alpha = 2$: non wetting phase

u^α : phase pressure

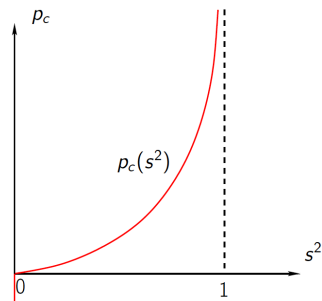
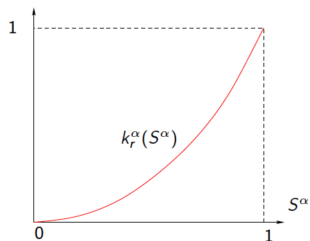
s^α : phase saturation

$p_c(s^2) = u^2 - u^1$: capillary pressure

$k_r^\alpha(s^\alpha)$: phase relative permeability

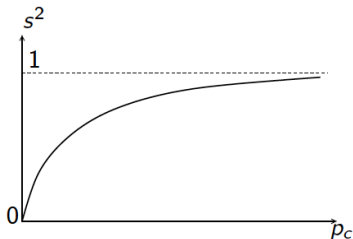
$$\mathbf{q}^\alpha = - \underbrace{\frac{k_r^\alpha(s^\alpha)}{\mu^\alpha}}_{k^\alpha(s^\alpha)} \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g})$$

$k^\alpha(s^\alpha)$: phase mobility



Two-phase Darcy flow

$$\begin{aligned} s^2 &= S^2(p_c), \\ s^1 &= S^1(p_c) = 1 - S^2(p_c) \end{aligned}$$



Fluids are assumed incompressible

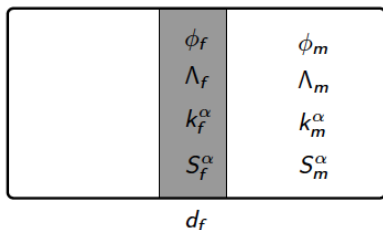
Phase pressure formulation of two-phase Darcy flows:

$$\begin{cases} \phi(\mathbf{x})\partial_t S^1(p_c) + \operatorname{div}(\mathbf{q}^1) = 0, \\ \phi(\mathbf{x})\partial_t S^2(p_c) + \operatorname{div}(\mathbf{q}^2) = 0, \\ p_c = u^2 - u^1 \end{cases}$$

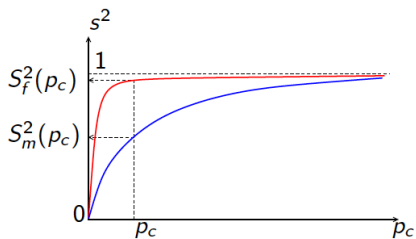
with

$$\mathbf{q}^\alpha = -k^\alpha(S^\alpha(p_c)) \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g}), \quad \alpha = 1, 2,$$

Matrix and fracture domains



- Fracture width: $d_f \ll$ matrix size L
- Fracture rocktype (f): $\Lambda_f, \phi_f, k_f^\alpha, S_f^\alpha$
- Matrix rocktype (m): $\Lambda_m, \phi_m, k_m^\alpha, S_m^\alpha$
- The fracture can act as a drain or a barrier for the flow

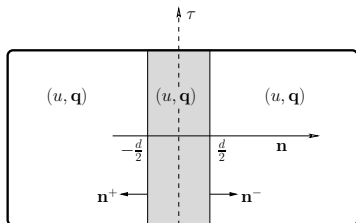


Dimensional reduction (codimension 1 in the fracture)

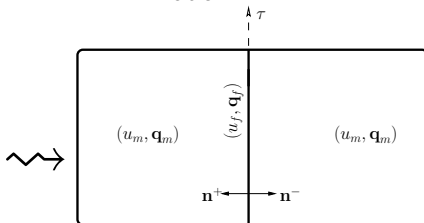
[Granet et al 2001], [Jaffré et al. 2002], [Bogdanov et al 2003], [Faille et al 2003], [Karimi Fard 2004], [Jaffré et al. 2005], [Angot et al. 2009]

- **Dimensional reduction:** averaging the model equations over the fracture width
- **Objectives:** facilitate the mesh generation and lower the number of degrees of freedom

equi-dimensional model:



DFM model:



Dimensional reduction

$$\Lambda_f = \begin{pmatrix} \Lambda_{f,\tau} & 0 \\ 0 & \Lambda_{f,n} \end{pmatrix} \text{ in } (\tau, n) \text{ coordinates}$$

$$\mathbf{q}^\alpha = -k_f^\alpha(S_f^\alpha(p_c)) \Lambda_f (\nabla u^\alpha - \rho^\alpha \mathbf{g})$$

$$= \underbrace{-k_f^\alpha(S_f^\alpha(p_c)) \Lambda_{f,\tau} (\nabla_\tau u^\alpha - \rho^\alpha \mathbf{g}_\tau)}_{\text{tangential flux } \mathbf{q}_\tau^\alpha} \quad \underbrace{-k_f^\alpha(S_f^\alpha(p_c)) \Lambda_{f,n} (\partial_n u^\alpha - \rho^\alpha \mathbf{g} \cdot \mathbf{n})}_{\text{normal flux } (\mathbf{q}^\alpha \cdot \mathbf{n})}$$

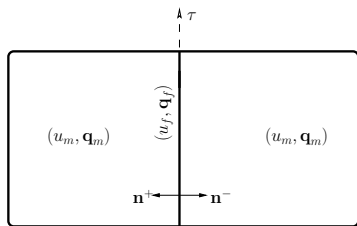
$$\text{div}(\mathbf{q}^\alpha) = \text{div}_\tau(\mathbf{q}_\tau^\alpha) + \partial_n(\mathbf{q}^\alpha \cdot \mathbf{n}).$$

Dimensional reduction: averaging over the fracture width

- $$u_f^\alpha = \frac{1}{d_f} \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} u^\alpha \, dn, \quad S_f^\alpha(p_{c,f}) \approx \frac{1}{d_f} \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} S_f^\alpha(p_c) \, dn$$
$$p_{c,f} = u_f^2 - u_f^1.$$
- $$\mathbf{q}_f^\alpha = \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} \mathbf{q}_\tau^\alpha \, dn$$
$$\approx -d_f k_f^\alpha(S_f^\alpha(p_{c,f})) \Lambda_{f,\tau} (\nabla_\tau u_f^\alpha - \rho^\alpha \mathbf{g}_\tau) \quad (\text{Fracture Darcy Law})$$
- $$\int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} \left(\phi_f \partial_t S_f(p_c) + \text{div}(\mathbf{q}^\alpha) \right) \, dn = 0$$
$$\Rightarrow d_f \phi_f \partial_t S_f^\alpha(p_{c,f}) + \text{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_{n^+} \mathbf{q}_m^\alpha + \gamma_{n^-} \mathbf{q}_m^\alpha = 0$$

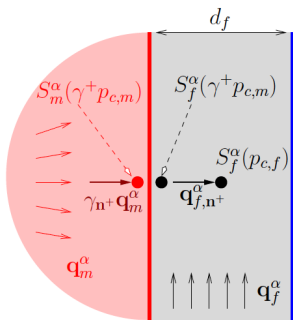
(Fracture conservation equation)

DFM models



$$\left\{ \begin{array}{l}
 \text{Matrix Darcy Law: } \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_{c,m})) \Lambda_m(\nabla u_m^\alpha - \rho^\alpha \mathbf{g}) \\
 \text{Matrix Vol. Cons.: } \phi_m \partial_t S_m^\alpha(p_{c,m}) + \text{div}(\mathbf{q}_m^\alpha) = 0 \\
 \text{Fracture Darcy Law: } \mathbf{q}_f^\alpha = -d_f k_f^\alpha(S_f^\alpha(p_{c,f})) \Lambda_{f,\tau}(\nabla_\tau u_f^\alpha - \rho^\alpha \mathbf{g}_\tau) \\
 \text{Fracture Vol. Cons.: } \phi_f d_f \partial_t S_f^\alpha(p_{c,f}) + \text{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_{n^+} \mathbf{q}_m^\alpha + \gamma_{n^-} \mathbf{q}_m^\alpha = 0
 \end{array} \right.$$

Transmission conditions at the matrix fracture interface



■ Discontinuous pressure DFM model:

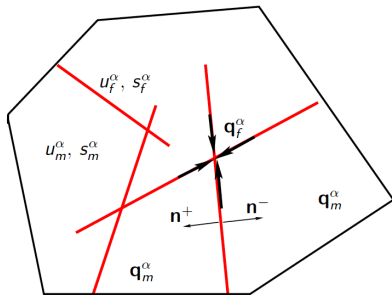
$$\gamma_{n^\pm} \mathbf{q}_m^\alpha = \mathbf{q}_{f,n^\pm}^\alpha \approx$$

$$k_f^\alpha(S_f^\alpha(\gamma_\pm p_{c,m})) \Lambda_{f,n} \left(\frac{u_f^\alpha - \gamma_\pm u_m^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g} \cdot \mathbf{n}^\pm \right)^- \\ + k_f^\alpha(S_f^\alpha(p_{c,f})) \Lambda_{f,n} \left(\frac{u_f^\alpha - \gamma_\pm u_m^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g} \cdot \mathbf{n}^\pm \right)^+$$

■ Continuous pressure DFM model $\left(\frac{\Lambda_{f,n}}{d_f} \gg \frac{\Lambda_{m,n}}{L} \right)$:

$$\gamma_+ u_m^\alpha = \gamma_- u_m^\alpha = u_f^\alpha.$$

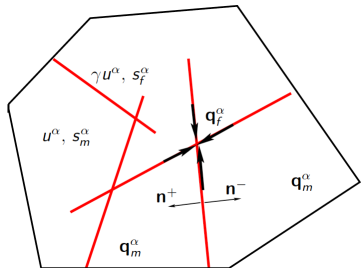
Generalization to complex Discrete Fracture Network



- Pressure continuity and flux conservation is assumed at fracture intersections
- Zero flux is assumed at immersed fracture tips

Continuous pressure two-phase Discrete Fracture Matrix model

- Continuous pressures: $u_m^\alpha = u^\alpha$,
 $u_f^\alpha = \gamma u^\alpha$, $\alpha = 1, 2$
- Capillary pressure: $p_c = u^2 - u^1$
- Saturations:
 - $s_m^\alpha = S_m^\alpha(p_c)$
 - $s_f^\alpha = S_f^\alpha(\gamma p_c)$



$$\left\{ \begin{array}{l}
 \text{Matrix Darcy Law: } \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_c)) \Lambda_m(\nabla u^\alpha - \rho^\alpha \mathbf{g}) \\
 \text{Matrix Vol. Cons.: } \phi_m \partial_t S_m^\alpha(p_c) + \text{div}(\mathbf{q}_m^\alpha) = 0 \\
 \text{Fracture Darcy Law: } \mathbf{q}_f^\alpha = -d_f k_f^\alpha(S_f^\alpha(\gamma p_c)) \Lambda_{f,\tau}(\nabla_\tau \gamma u^\alpha - \rho^\alpha \mathbf{g}_\tau) \\
 \text{Fracture Vol. Cons.: } \phi_f d_f \partial_t S_f^\alpha(\gamma p_c) + \text{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_{n^+} \mathbf{q}_m^\alpha + \gamma_{n^-} \mathbf{q}_m^\alpha = 0
 \end{array} \right.$$

Finite Volume Discretizations

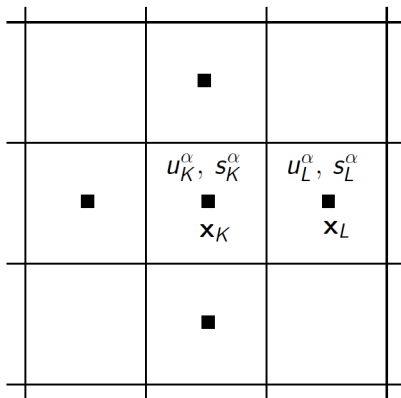
Discretization of DFM models: State of the art for two-phase Darcy flows

- **Continuous pressure models:**
 - Bogdanov et al. 2003 (CVFE)
 - Reichenberger et al 2006 (CVFE)
 - Firoozabadi et al 2007 (CVFE)
 - Matthai et al 2007 (CVFE)
 - Firoozabadi 2008 (MHFE + DG)
 - Groza et al 2013 (VAG)
 - Xing et al 2016 (VAG)
- **Discontinuous pressure models with harmonic average:**
 - Karimi-Fard et al 2004 (TPFA)
 - Faille et al, Nordbotten et al, 2012, Edwards et al 2014 (MPFA)
 - Gläser et al 2017 (MPFA)
- **Discontinuous pressure models with nonlinear transmission conditions:**
 - Elyes et al 2017 (MFE, splitting)
 - Hennicker et al 2017 (VAG, HFV)
 - [Aghili et al 2018 \(Two Point Flux Approximation \(TPFA\)\)](#)

Cell centred finite volume discretization of two phase Darcy flows [Peaceman 77, Aziz and Settari 79]

- Set of cells: $K \in \mathcal{M}$
- Cell center: \mathbf{x}_K
- Subset of cells sharing a face with K : \mathcal{M}_K
- Discrete unknowns: for all $K \in \mathcal{M}$:

$$u_K^\alpha, s_K^\alpha, \alpha = 1, 2.$$



Cell centred finite volume discretization of two phase Darcy flows

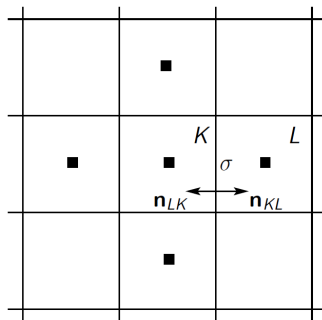
Conservation equation: $\phi \partial_t s^\alpha + \operatorname{div} \mathbf{q}^\alpha = 0$

- Discrete conservation equations on each cell K :

$$\frac{1}{t^n - t^{n-1}} \int_{t^{n-1}}^{t^n} \left(\int_K \phi \partial_t s^\alpha d\mathbf{x} + \sum_{L \in \mathcal{M}_K} \int_{\sigma=KL} \mathbf{q}^\alpha \cdot \mathbf{n}_{KL} d\sigma \right) dt = 0$$

- Conservative approximation of the fluxes

$$F_{K,L}^\alpha = -F_{L,K}^\alpha \approx \int_{\sigma=KL} \mathbf{q}^\alpha \cdot \mathbf{n}_{KL} d\sigma$$



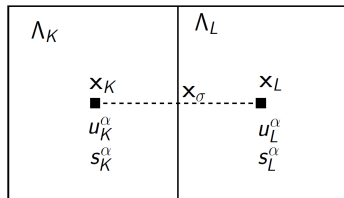
Two-Point Flux Approximation (orthogonal mesh)

Conservative approximation of the fluxes:

$$F_{K,L}^\alpha = -F_{L,K}^\alpha \approx \int_{\sigma=KL} -k^\alpha(s^\alpha)\Lambda(\mathbf{x})\nabla u^\alpha \cdot \mathbf{n}_{KL}d\sigma$$

$$V_{K,L}^\alpha = T_{KL}(u_K^\alpha - u_L^\alpha) \\ \approx \int_{\sigma=KL} -\Lambda(\mathbf{x})\nabla u^\alpha \cdot \mathbf{n}_{KL}d\sigma$$

$$\text{with } T_{KL} = \frac{|\sigma|}{\frac{|\mathbf{x}_K\mathbf{x}_\sigma|}{\Lambda_K} + \frac{|\mathbf{x}_L\mathbf{x}_\sigma|}{\Lambda_L}}$$



$$F_{K,L}^\alpha = k^\alpha(s_K^{\alpha,n}) (V_{KL}^{\alpha,n})^+ + k^\alpha(s_L^{\alpha,n}) (V_{KL}^{\alpha,n})^-$$

(two point monotone flux for the saturation equation)

Discrete conservation equations in each cell K

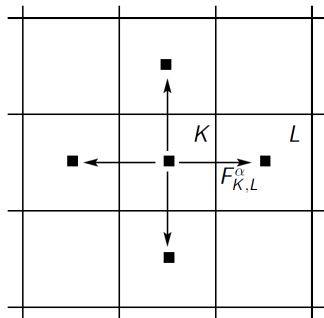
- $\phi_K = \int_K \phi(\mathbf{x}) d\mathbf{x}$

- Euler implicit time integration

$$\phi_K \frac{s_K^{\alpha,n} - s_K^{\alpha,n-1}}{t^n - t^{n-1}} + \sum_{L \in \mathcal{M}_K} F_{K,L}^{\alpha,n} = 0$$

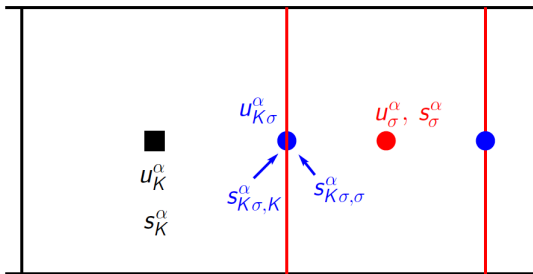
- $p_{c,K} = u_K^2 - u_K^1$

- $s_K^\alpha = S^\alpha(p_{c,K})$

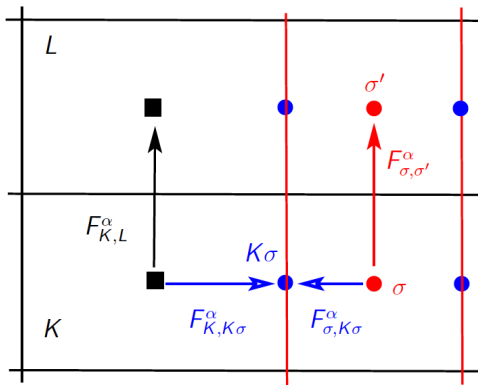


Discontinuous pressure DFM model: discrete unknowns

$$\begin{cases} s_K^\alpha = S_m^\alpha(u_K^2 - u_K^1), \\ s_\sigma^\alpha = S_f^\alpha(u_\sigma^2 - u_\sigma^1), \\ s_{K\sigma,K}^\alpha = S_m^\alpha(u_{K\sigma}^2 - u_{K\sigma}^1), \\ s_{K\sigma,\sigma}^\alpha = S_f^\alpha(u_{K\sigma}^2 - u_{K\sigma}^1) \end{cases}$$



Discontinuous pressure DFM model: fluxes



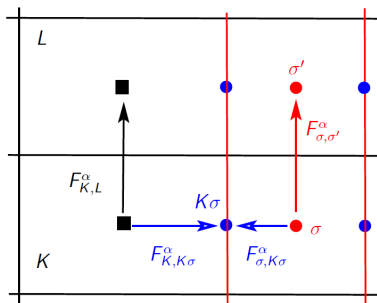
$$F_{K,K\sigma}^{\alpha} = k_m^{\alpha}(s_K^{\alpha})T_{K,K\sigma}(u_K^{\alpha} - u_{K\sigma}^{\alpha})^{+} + k_m^{\alpha}(s_{K\sigma,K}^{\alpha})T_{K,K\sigma}(u_K^{\alpha} - u_{K\sigma}^{\alpha})^{-},$$

$$F_{\sigma,K\sigma}^{\alpha} = k_f^{\alpha}(s_{\sigma}^{\alpha})T_{\sigma,K\sigma}(u_{\sigma}^{\alpha} - u_{K\sigma}^{\alpha})^{+} + k_f^{\alpha}(s_{K\sigma,\sigma}^{\alpha})T_{\sigma,K\sigma}(u_{\sigma}^{\alpha} - u_{K\sigma}^{\alpha})^{-}.$$

Discontinuous pressure DFM model: discrete conservation equations

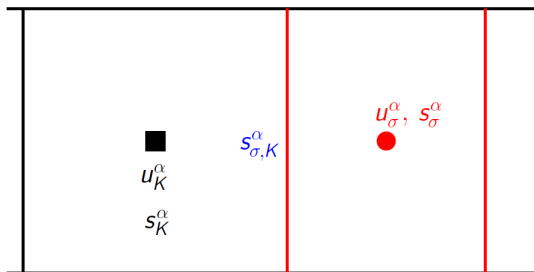
$$\left\{ \begin{array}{l} \phi_K \frac{S_K^\alpha - S_K^{\alpha,n-1}}{t^n - t^{n-1}} + \sum_L F_{K,L}^\alpha + \sum_\sigma F_{K,K\sigma}^\alpha = 0 \\ \phi_\sigma \frac{S_\sigma^\alpha - S_\sigma^{\alpha,n-1}}{t^n - t^{n-1}} + \sum_{\sigma'} F_{\sigma,\sigma'}^\alpha + \sum_K F_{\sigma,K}^\alpha = 0, \end{array} \right.$$

$$F_{K,K\sigma}^\alpha + F_{\sigma,K}^\alpha = 0.$$

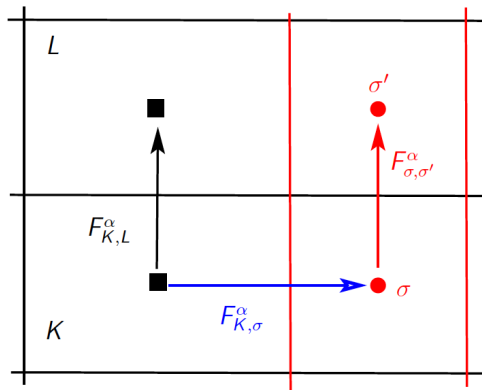


Continuous pressure DFM model: discrete unknowns

$$\begin{cases} s_K^\alpha = S_m^\alpha(u_K^2 - u_K^1), \\ s_\sigma^\alpha = S_f^\alpha(u_\sigma^2 - u_\sigma^1), \\ s_{\sigma,K}^\alpha = S_m^\alpha(u_\sigma^2 - u_\sigma^1), \end{cases}$$



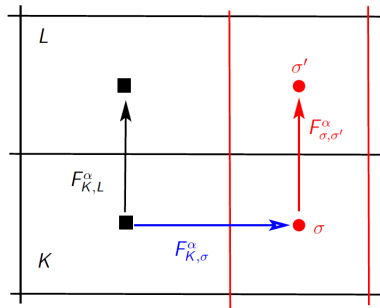
Continuous pressure DFM model: fluxes



$$F_{K,\sigma}^\alpha = k_m^\alpha (s_K^\alpha) T_{K,\sigma} (u_K^\alpha - u_\sigma^\alpha)^+ + k_m^\alpha (s_{\sigma,K}^\alpha) T_{K,\sigma} (u_K^\alpha - u_\sigma^\alpha)^-,$$

Continuous pressure DFM model: discrete conservation equations

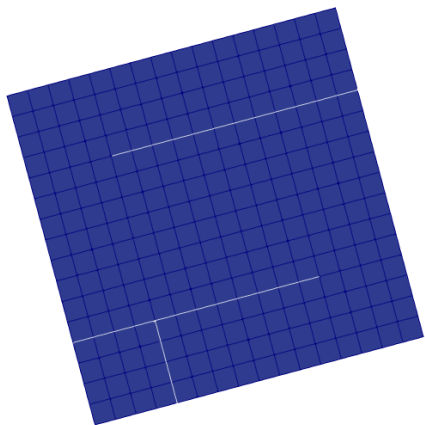
$$\begin{cases} \phi_K \frac{s_K^\alpha - s_K^{\alpha,n-1}}{t^n - t^{n-1}} + \sum_L F_{K,L}^\alpha + \sum_\sigma F_\sigma^\alpha = 0, \\ \phi_\sigma \frac{s_\sigma^\alpha - s_\sigma^{\alpha,n-1}}{t^n - t^{n-1}} + \sum_{\sigma'} F_{\sigma,\sigma'}^\alpha - \sum_K F_{K,\sigma}^\alpha = 0, \end{cases}$$



Numerical examples

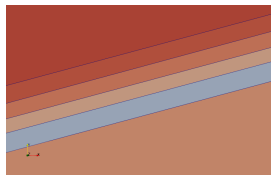
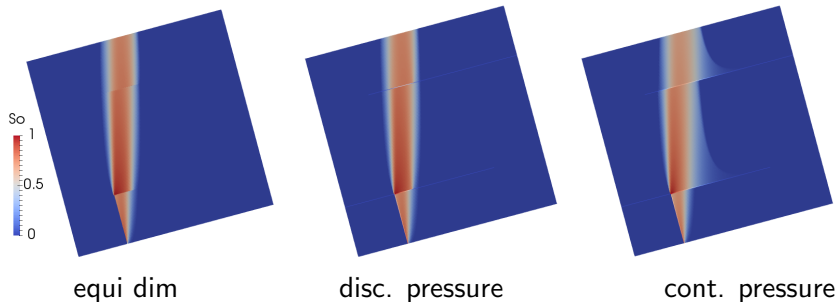
Comparisons between DFM and equi-dimensional models

- $\Omega = (0, 100 \text{ m})^2$ rotated
- $d_f = 1 \text{ cm}$, $\Lambda_f = \lambda_{f,n} = 10^3 \Lambda_m$,
 $\phi_f = 0.35$.
- $\Lambda_m = 0.1 \text{ Darcy}$, $\phi_m = 0.2$.
- $\rho^w = 1000$ and $\rho^o = 800 \text{ Kg.m}^{-3}$.
- Initially water saturated
- Oil injection in the bottom fracture



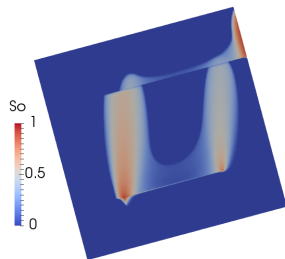
Comparisons between DFM and equi-dimensional models: gravity dominant flow, homogeneous $P_c(s^2)$

$$P_{c,m}(s^2) = P_{c,f}(s^2) = -b \log(1 - s^2) \text{ with } b = 10 \text{ Pa.}$$

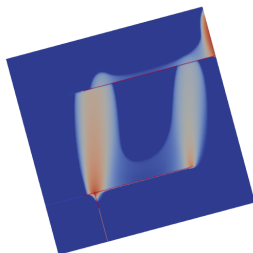


Comparisons between DFM and equi-dimensional models: gravity dominant flow, matrix acting as a capillary barrier

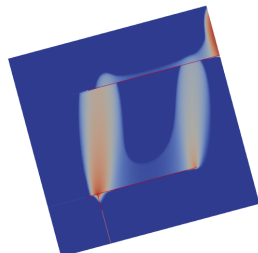
$$P_{c,m}(s^2) = 10^3(1 - \log(1 - s^2)) \text{ Pa}, \quad P_{c,f}(s^2) = -10 \log(1 - s^2) \text{ Pa}.$$



equi dim



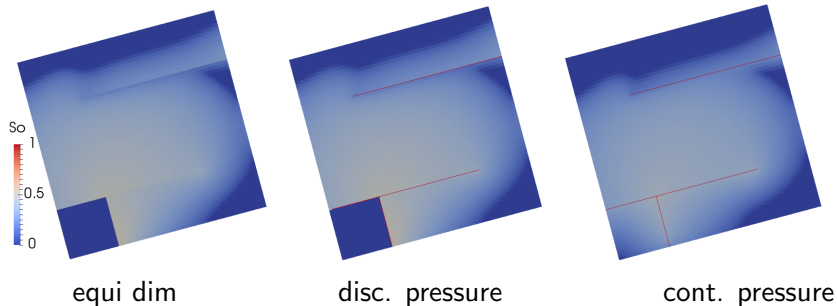
disc. pressure



cont. pressure

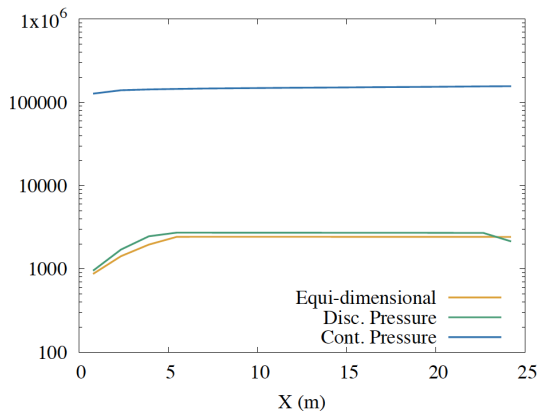
Comparisons between DFM and equi-dimensional models: viscous dominant flow, matrix acting as a capillary barrier

$$P_{c,m}(s^2) = 10^5(1 - \log(1 - s^2)) \text{ Pa}, \quad P_{c,f}(s^2) = -100 \log(1 - s^2) \text{ Pa}.$$



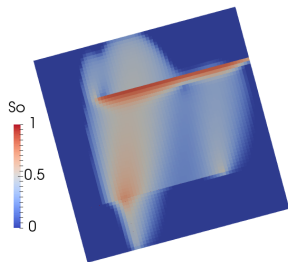
Comparisons between DFM and equi-dimensional models: viscous dominant flow, matrix acting as a capillary barrier

$$P_{c,m}(s^2) = 10^5(1 - \log(1 - s^2)) \text{ Pa}, \quad P_{c,f}(s^2) = -100 \log(1 - s^2) \text{ Pa}.$$

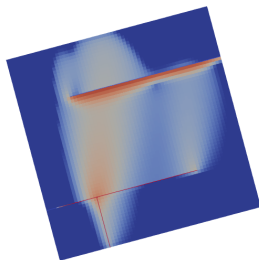


Capillary pressure along the vertical fracture.

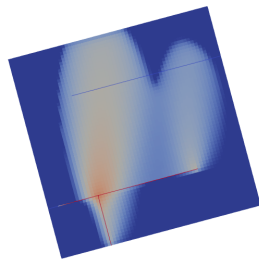
Comparisons between DFM and equi-dimensional models: drains and barrier



equi dim



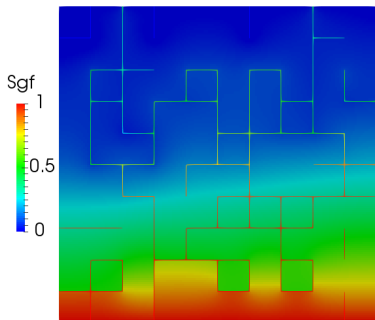
disc. pressure



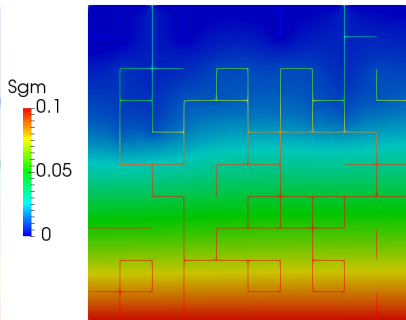
cont. pressure

Drains acting as a barrier for water

Desaturation by suction of a water saturated low permeability rock



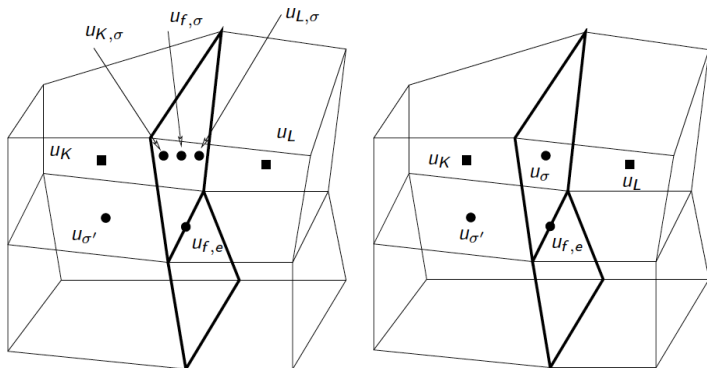
disc. pressure



cont. pressure

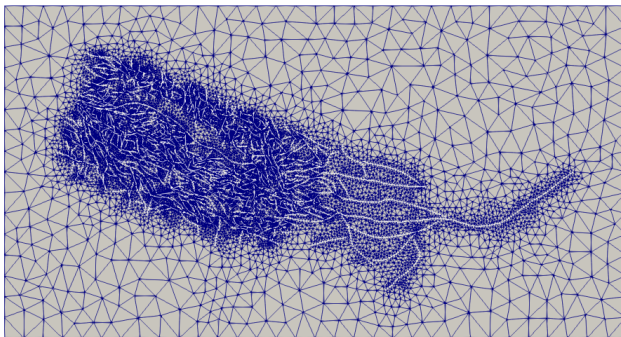
Discretization: extension to more complex meshes and geometries

- **Face based approaches:** Hybrid Finite Volume (HFV) discretization
 - adapted to discontinuous pressure models
 - expensive on 3D meshes



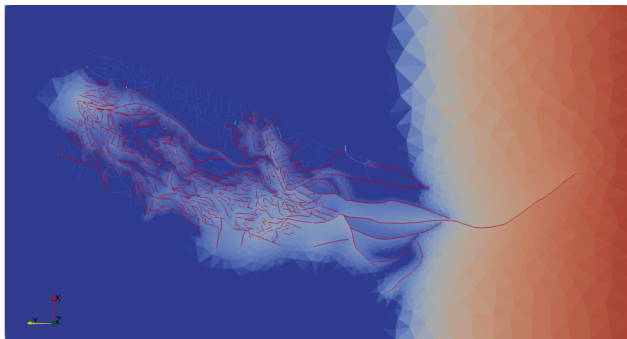
2D simulation of the discontinuous pressure DFM model with the HFV scheme

Data set: $P_{c,m}(s^w) = 10^5(1 - \log(s^w))$, $P_{c,f}(s^w) = -100 \log(s^w)$ Pa.
 $\Lambda_m = 10^{-13}$ m², $\Lambda_f = \lambda_{f,n} = 10^{-10}$ m², $d_f = 1$ cm, $t_f = 1600$ days.



Mesh: (Courtesy of M. Karimi-Fard, Stanford, and A. Lapène, Total):
 $\Omega = (0, 100 \text{ m}) \times (0, 186.5 \text{ m})$, 32340 cells, 48558 faces, 5344 fracture faces.

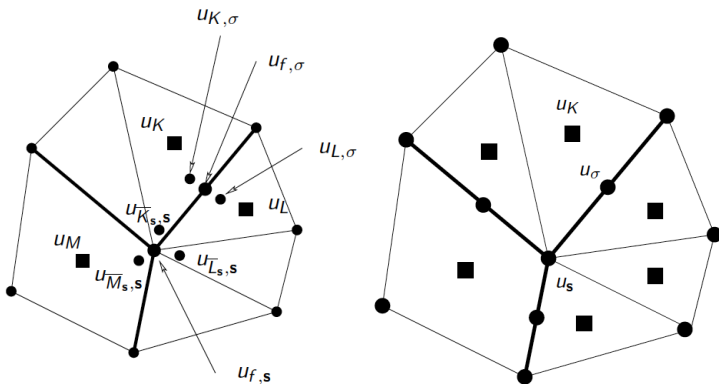
2D simulation of the discontinuous pressure DFM model with the HFV scheme



- 1442 time steps
- 19 Newton it. per time step
- 19 GMRes it. per Newton step (CPR-AMG preconditioner)
- total CPU time with 1 core Intel i7 2.6 GHz: 8h.

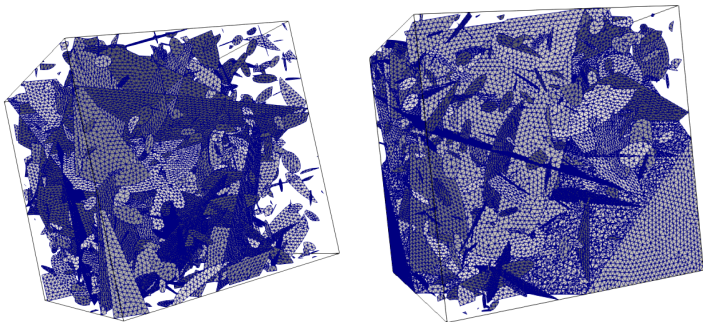
Discretization: extension to more complex meshes and geometries

- **Vertex based approaches:** Vertex Approximate Gradient (VAG) discretization
 - adapted to continuous pressure models
 - efficient on tetrahedral meshes for large fracture networks
 - difficulties to solve the nonlinear systems for discontinuous pressure models

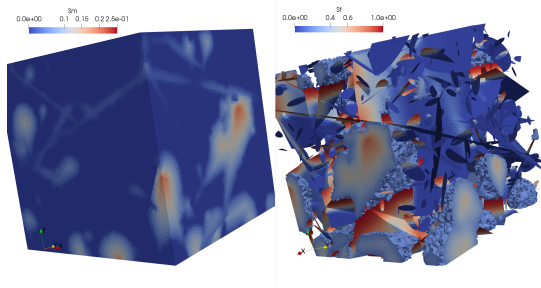


Example: Large 3D DFM simulation

- $\Omega = (0, 300)^3$ m, 1000 fractures, $d_f = 1$ cm.
- tetrahedral mesh with $1.7 \cdot 10^6$ cells, $2.9 \cdot 10^5$ nodes, 10^5 fracture faces (P. Laug, G. Pichot, Inria)
- $\frac{\Lambda_f}{\Lambda_m} = 10^3$, $p_{c,m} = -10^5 \log(s_m^1)$, $p_{c,f} = -10^3 \log(s_f^1)$.
- oil injected during 15 years in the bottom fractures, initially water saturated.



Large 3D DFM simulation: continuous pressure model



- Vertex Approximate Gradient Scheme
- Total Velocity formulation (with El Houssaine Quenjel)
- 87 time steps for $t_f = 15$ years
- 10.8 Newton it. per time step
- 18 GMRes it. per Newton step (CPR-AMG preconditioner)
- total CPU time with 1 core Intel i7 2.6 GHz: **10.7 hours.**

Conclusions and perspectives

- Complex geometries
- Highly nonlinear transmission conditions at matrix fracture interfaces
- Robustness challenges for large fracture networks in 3D
- The continuous pressure model is not always a good approximation for two-phase flows even for drains
- More physics: thermodynamics, thermics, chemistry, poromechanics

Acknowledgements

Thanks for your attention and to Total, Storengy, BRGM and Andra for their support.

