Hybrid dimensional two phase flows in fractured porous media

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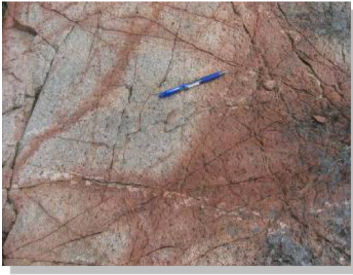
Outline

- Modelling Darcy flows in Discrete Fracture Networks
- The Vertex Approximate Gradient Discretization (VAG)
- Numerical Results

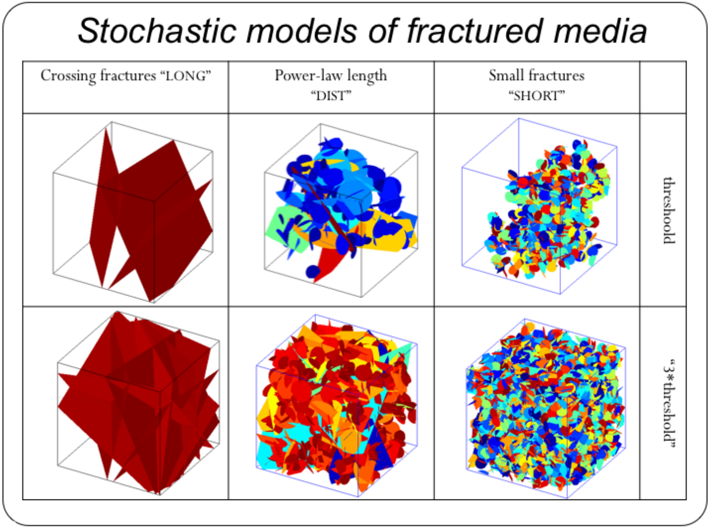
Fractured porous media: multiple scales (figures from J. R. de Dreuzy, Geosciences Rennes and team INRIA Sage)



Flow in Fractured porous media: two main approaches

- Double Continuum Media: 3D fracture medium coupled to 3D matrix medium
- Discrete fracture models (DFM): 2D fracture model coupled to 3D matrix medium
- Possibility to couple both approaches
  - Double Continuum media for small fractures coupled with DFM for large fractures
  - Numerical Homogeneization: parameters of the Double Continuum media computed by a DFM model

Discrete fracture networks (DFN) (figures from J. R. de Dreuzy, Geosciences Rennes and team INRIA Sage)

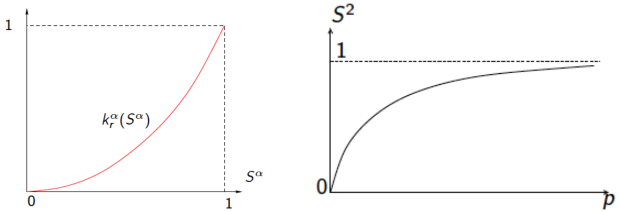


Modelling Darcy flows in Discrete Fracture Networks

Equi-dimensional model in phase pressures formulation

Two-phase Darcy law: $\mathbf{q}^\alpha = -k^\alpha(\mathbf{x}, S^\alpha(\mathbf{x}, p))\ \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g}).$

$\alpha = 1:$ wetting phase, $\alpha = 2:$ non wetting phase
$\phi(\mathbf{x}):$ porosity
$\Lambda(\mathbf{x}):$ absolute permeability
$u^\alpha:$ phase pressure
$p = u^2 - u^1:$ capillary pressure
$S^2(\mathbf{x}, p):$ inverse of capillary pressure graph, $S^1(\mathbf{x}, p) = 1 - S^2(\mathbf{x}, p)$
$k^\alpha(\mathbf{x}, S^\alpha) = \frac{k_r^\alpha(\mathbf{x}, S^\alpha)}{\mu^\alpha}:$ phase mobility



Equi-dimensional model in phase pressures formulation

Two-phase Darcy law:

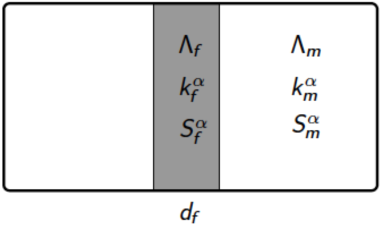
$$\mathbf{q}^\alpha = -k^\alpha(\mathbf{x}, S^\alpha(\mathbf{x}, p))\, \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g}), \quad \alpha = 1, 2,$$

Volume conservation for each phase:

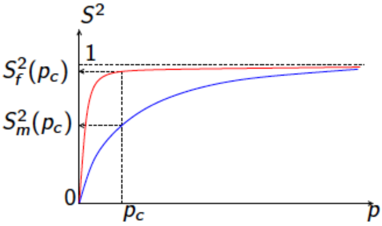
$$\phi(\mathbf{x})\partial_t S^\alpha(\mathbf{x}, p) + \mathrm{div}(\mathbf{q}^\alpha) = 0, \quad \alpha = 1, 2.$$

- Incompressible flow
- Immiscible flow

Equi-dimensional model: matrix and fracture domains

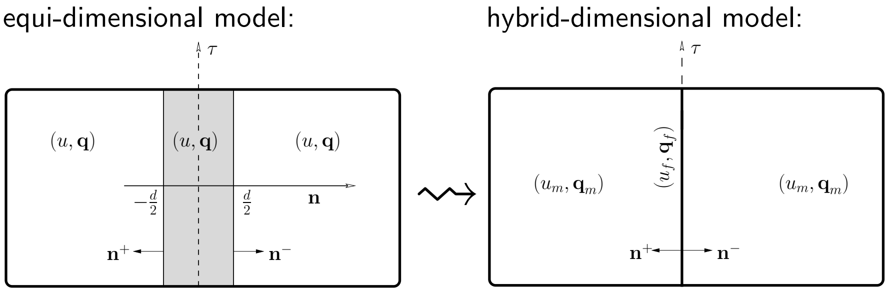


- Fracture: can act as drain or barrier
- Fracture width: $d_f << $ matrix size L
- Fracture rocktype (f): $\Lambda_f, k_f^\alpha, S_f^\alpha$
- Matrix rocktype (m): $\Lambda_m, k_m^\alpha, S_m^\alpha$



Dimensional Hybridizing (codimension 1 in the fracture)

- **Dimensional hybridizing**: averaging the model equations over the fracture width
- **Objectives**: facilitate the mesh generation and lower the number of degrees of freedom



Dimensional Hybridizing

- Continuous pressure models (pervious fractures):

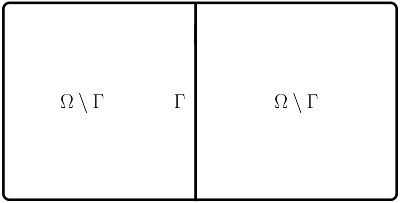
  [Granet et al 2001], [Jaffré et al. 2002], [Bogdanov et al 2003], [Karimi Fard 2004], [Bastian et al 2006], [Brenner et al 2014]

- Discontinuous pressure models (pervious fractures or barriers):

  [Faille et al 2003], [Jaffré et al. 2005], [Firoozabadi et al 2008], [Angot et al 2009], [Brenner et al 2016]

- Objectives:
  - Derive hybrid dimensional two phase flow models
  - Discretization using the VAG scheme
  - Compare the equi-dimensional and hybrid dimensional models

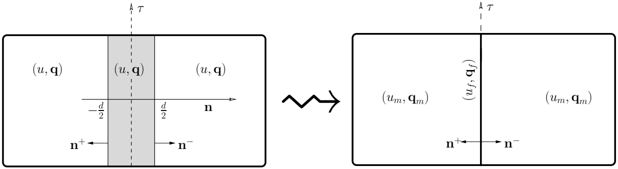
Dimensional Hybridizing: matrix domain



- Equations in the matrix domain $\Omega \setminus \Gamma$:

$$\begin{cases} \text{Darcy law with gravity:} & \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_m))\, \Lambda_m(\nabla u_m^\alpha - \rho^\alpha \mathbf{g}), \\ \text{Volume conservation:} & \phi_m \partial_t S_m^\alpha(p_m) + \mathrm{div}(\mathbf{q}_m^\alpha) = 0, \end{cases}$$

Dimensional Hybridizing: averaging over the fracture width



- Phase superscript α dropped

$$\Lambda_f = \begin{pmatrix} \Lambda_{f,\tau} & 0 \\ 0 & \Lambda_{f,n} \end{pmatrix} \text{ in } (\tau, n) \text{ coordinates}$$

$$\begin{aligned} \mathbf{q} &= -k_f(S_f(p))\, \Lambda_f(\nabla u - \rho\mathbf{g}) \\ &= \underbrace{-k_f(S_f(p))\, \Lambda_{f,\tau}(\nabla_\tau u - \rho\mathbf{g}_\tau)}_{\text{tangential flux } \mathbf{q}_\tau} \quad \underbrace{-k_f(S_f(p))\, \Lambda_{f,n}(\partial_n u - \rho\mathbf{g}\cdot\mathbf{n})\mathbf{n}}_{\text{normal flux } (\mathbf{q}\cdot\mathbf{n})\mathbf{n}} \end{aligned}$$

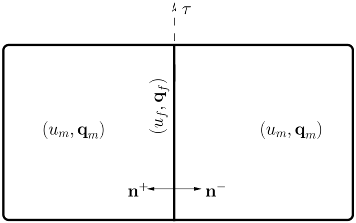
Dimensional Hybridizing: Fracture Equations

- $$u_f = \frac{1}{d_f}\int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} u\ dn; \qquad S_f(p_f) \approx \frac{1}{d_f}\int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} S_f(p)\ dn$$

- $$\mathbf{q}_f = \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} \mathbf{q}_\tau\ dn \approx -d_f k_f(S_f(p_f))\ \Lambda_{f,\tau}(\nabla_\tau u_f - \rho \mathbf{g}_\tau) \quad \text{(Darcy Law)}$$

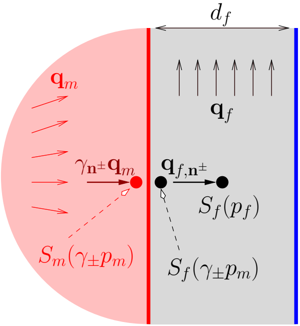
- $$-d_f \phi_f \partial_t S_f(p_f) \approx \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} \mathrm{div}(\mathbf{q})\ dn = \mathrm{div}_\tau(\mathbf{q}_f) + \left[\mathbf{q}\cdot\mathbf{n}\right]_{-\frac{d_f}{2}}^{\frac{d_f}{2}}$$
$$= \mathrm{div}_\tau(\mathbf{q}_f) + \gamma_{n^+}\mathbf{q}_m + \gamma_{n^-}\mathbf{q}_m \quad \text{(Conservation Equation)}$$

Hybrid dimensional models: Summary (α dropped)



$$
\left\{
\begin{array}{ll}
\text{Matrix Darcy Law:} & \mathbf{q}_m = -k_m(S_m(p_m))\,\Lambda_m(\nabla u_m - \rho\mathbf{g}) \\
\text{Matrix Conserv. Eq.:} & \phi_m\partial_t S_m(p_m) + \mathrm{div}(\mathbf{q}_m) = 0 \\
\text{Fracture Darcy Law:} & \mathbf{q}_f = -d_f k_f(S_f(p_f))\,\Lambda_{f,\tau}(\nabla_\tau u_f - \rho\mathbf{g}_\tau) \\
\text{Fracture Conserv. Eq.:} & \phi_f\partial_t S_f(p_f) + \mathrm{div}_\tau(\mathbf{q}_f) + \gamma_{n^+}\mathbf{q}_m + \gamma_{n^-}\mathbf{q}_m = 0
\end{array}
\right.
$$

Transmission conditions at the matrix fracture interface



- **Discontinuous pressure model**:

$$\gamma_{\mathbf{n}^\pm}\mathbf{q}_m = \mathbf{q}_{f,\mathbf{n}^\pm} \approx$$

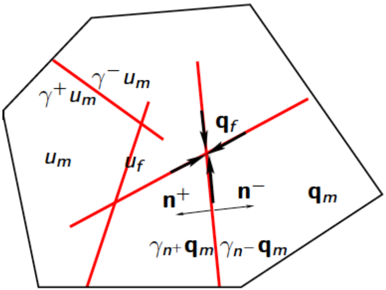
$$k_f(S_f(\gamma_\pm p_m))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m - u_f}{\frac{d_f}{2}} - \rho\mathbf{g}\cdot\mathbf{n}^\pm\right)^+$$

$$+k_f(S_f(p_f))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m - u_f}{\frac{d_f}{2}} - \rho\mathbf{g}\cdot\mathbf{n}^\pm\right)^-$$

- **Continuous pressure model** $\left(\frac{\Lambda_{f,\mathbf{n}}}{d_f} >> \frac{\Lambda_{m,\mathbf{n}}}{L}\right)$:

$$\gamma_+ u_m = \gamma_- u_m = u_f.$$

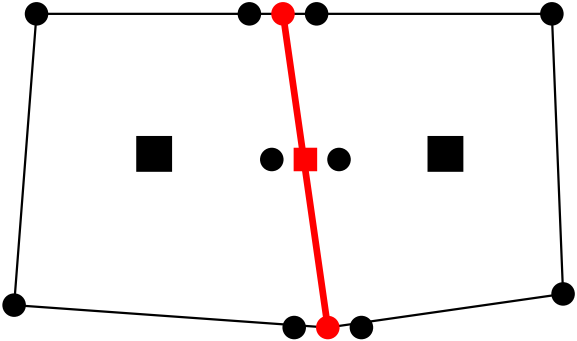
Generalization to complex Discrete Fracture Network



- Pressure continuity and flux conservation is assumed at fracture intersections
- Zero flux is assumed at immersed fracture tips

The Vertex Approximate Gradient Discretization (VAG)

Degrees of Freedom (discontinuous pressure model)



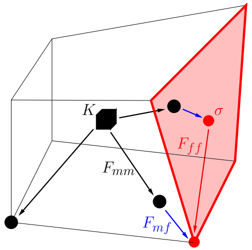
- Two matrix cells touching a fracture face
  - Matrix d.o.f. (black)
  - Fracture d.o.f. (red)

VAG Matrix-Matrix and Fracture-Fracture Fluxes

- **mm (ff) fluxes:**
  - upwind (w.r.t. the phase mobility)
  - MPFA
  - local stencil to each cell K (fracture face σ)

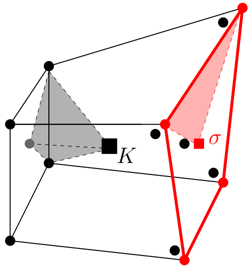
$$F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^-$$

$$f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_K^{\nu\nu'}(u_K - u_{\nu'} - \rho(\mathbf{x}_K - \mathbf{x}_{\nu'}) \cdot \mathbf{g})$$



- **mm (ff) transmissivities:**
  - conforming $\mathbb{P}^1$ FE gradient on a tetrahedral (triangular) submesh ($\{\mathfrak{e}_\nu\}_\nu = \mathbb{P}^1$ FE Basis Functions)

$$T_K^{\nu\nu'} = \int_K \Lambda \nabla \mathfrak{e}_\nu \nabla \mathfrak{e}_{\nu'} \mathrm{d}\mathbf{x}$$



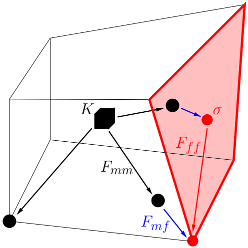
VAG Matrix-Fracture Fluxes

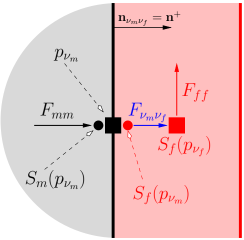
- **mf fluxes:**
  - upwind (w.r.t. the phase mobility)
  - TPFA
  - Saturation jump at mf interfaces
  - Gravity in normal direction

$$F_{\nu_m \nu_f}(u) = k_f(S_f(p_{\nu_m})) f_{\nu_m \nu_f}(u)^+ + k_f(S_f(p_{\nu_f})) f_{\nu_m \nu_f}(u)^-$$

$$f_{\nu_m \nu_f}(u) = T_{\nu_m \nu_f}(u_{\nu_m} - u_{\nu_f} - \frac{\rho d_f}{2} \mathbf{g} \cdot \mathbf{n}_{\nu_m \nu_f})$$

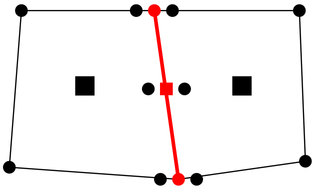
- **mf transmissivities:**
  - Mass Lumping of $\mathbb{P}^1$ FE basis function traces



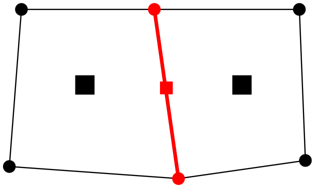


Hybrid Continuous Pressure and Hybrid Discontinuous Pressure degrees of freedom

- Hybrid Discontinuous Pressure



- Hybrid Continuous Pressure



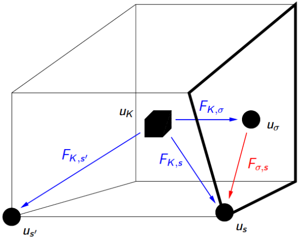
Hybrid Continuous Pressure model: Fluxes

- Matrix mm fluxes

$$F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^-$$
$$f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_K^{\nu\nu'}(u_K - u_{\nu'} - \rho(\mathbf{x}_K - \mathbf{x}_{\nu'}) \cdot \mathbf{g})$$

- Fracture ff fluxes

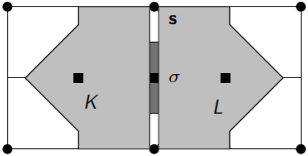
$$F_{\sigma s}(u) = k_f(S_f(p_\sigma))f_{\sigma s}(u)^+ + k_f(S_f(p_s))f_{\sigma s}(u)^-$$
$$f_{\sigma s}(u) = \sum_{s' \in \partial\sigma} T_\sigma^{ss'}(u_\sigma - u_{s'} - \rho(\mathbf{x}_\sigma - \mathbf{x}_{s'}) \cdot \mathbf{g})$$



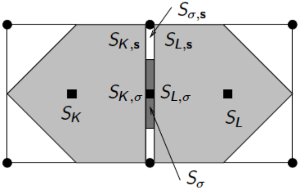
- No mf fluxes

Hybrid Continuous Pressure model

Control Volumes



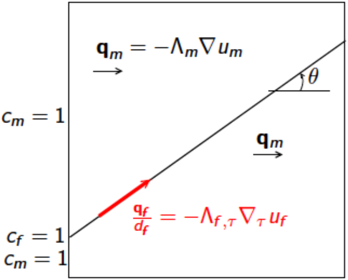
Saturations at matrix fracture interfaces

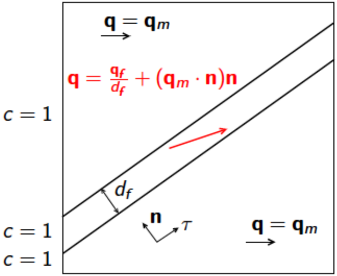


$$\begin{cases} S^\alpha_{K,\nu} = S^\alpha_m(p_\nu), \\ S^\alpha_K = S^\alpha_m(p_K), \end{cases} \qquad \begin{cases} S^\alpha_{\sigma,\mathbf{s}} = S^\alpha_f(p_\mathbf{s}), \\ S^\alpha_\sigma = S^\alpha_f(p_\sigma). \end{cases}$$

Numerical Results

Comparison of equi- and hybrid-dimensional models: tracer equation





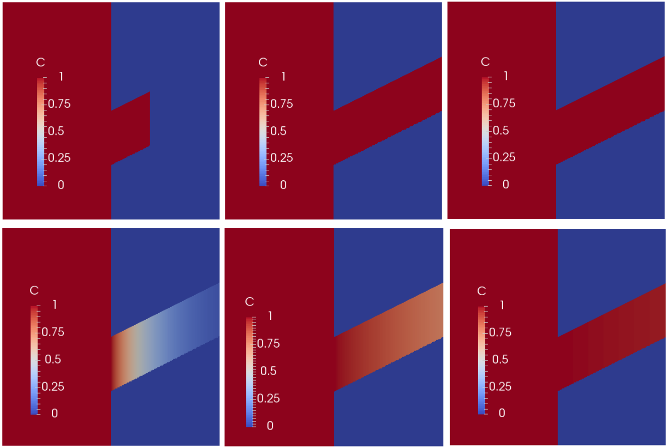
Hyb. dim.: $\begin{cases} \partial_t c_m + \mathrm{div}(c_m \mathbf{q}_m) = 0, \\ d_f \partial_t c_f + \mathrm{div}_\tau(c_f \mathbf{q}_f) + \gamma_{n^+}(c_m \mathbf{q}_m) + \gamma_{n^-}(c_m \mathbf{q}_m) = 0, \\ \gamma_{n^\pm}(c_m \mathbf{q}_m) = c_f (\gamma_{n^\pm} \mathbf{q}_m)^+ + (\gamma_\pm c_m)(\gamma_{n^\pm} \mathbf{q}_m)^-. \end{cases}$

Equi dim.: $\partial_t c + \mathrm{div}(c\mathbf{q}) = 0.$

Comparison of equi- and hybrid-dimensional tracer solutions in the matrix at $t = 0.5$

$d_f = 0.001$, $\Lambda_m = 1$, and $\Lambda_{f,\tau} = 100$, 1000 and 10000.

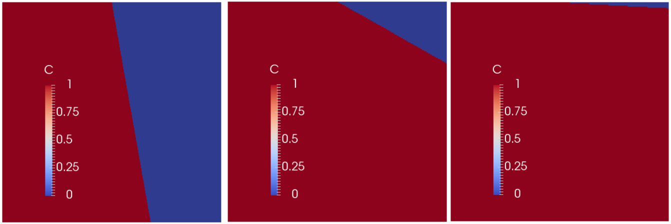
$\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 0.1$ $\qquad$ $\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 1$ $\qquad$ $\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 10$



Equi dimensional solution in the fracture at time $t = 0.5$

$d_f = 0.001$, $\Lambda_m = 1$, and $\Lambda_{f,\tau} = 100$, 1000 and 10000.

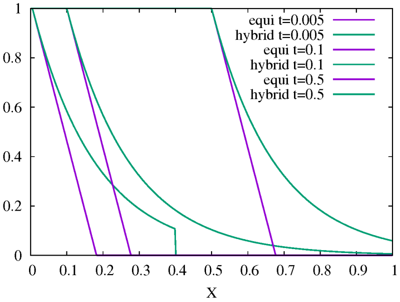
$\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 0.1$ $\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 1$ $\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 10$



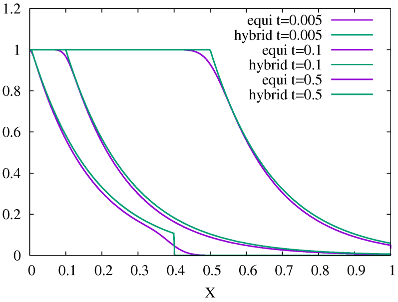
Equi dimensional solutions in the fracture with and without normal diffusion in the fracture

$$\partial_t c + \mathrm{div}\Big(c\mathbf{q} - D_{f,n}\mathbf{1}_{\Omega_f}\mathbf{n}\otimes\mathbf{n}\nabla c\Big) = 0.$$

$$\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 1, \quad \frac{D_{f,n}}{d_f|q_m\cdot\mathbf{n}|} = 0$$



$$\frac{\Lambda_{f,\tau} d_f}{\Lambda_m L} = 1, \quad \frac{D_{f,n}}{d_f|q_m\cdot\mathbf{n}|} = \sqrt{5}$$



Comparison of equi- and hybrid-dimensional models: two phase flow

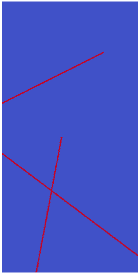
- $\Omega = (0, 400) \times (0, 800)$ m
- Equi-dimensional mesh: 22500 triangles
- Hybrid dimensional mesh: 16900 triangles
- Matrix:

$$\phi_m = 0.2, \quad \Lambda_m \text{ isotropic}$$

- Faults:

$$d_f = 4m, \quad \phi_f = 0.4, \quad \Lambda_f \text{ isotropic}$$

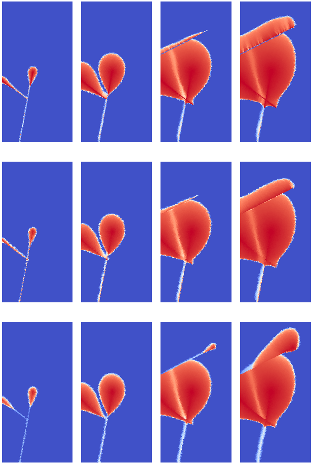
- Injection of oil in the bottom fault
- Initially saturated with water



Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(S_m^o) = p_{c,f}(S_f^o) = 0$

Zero Capillary Pressure: $p_{c,m}(S_m^o) = p_{c,f}(S_f^o) = 0$

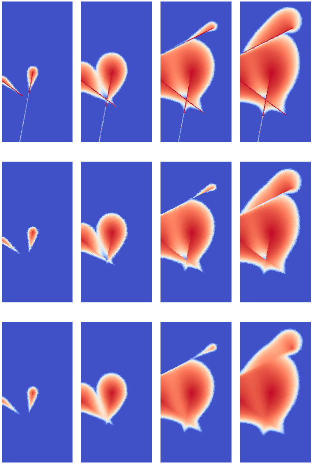
- Equi dim
- Hybrid Disc.
- Hybrid Cont.



Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m} \neq 0$; $p_{c,f} = 0$

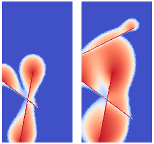
Capillary Pressure: $p_{c,m}(S_m^o) = -10^5 \ln(S_m^o)$; $p_{c,f} = 0$

- Equi dim
- Hybrid Disc.
- Hybrid Cont.

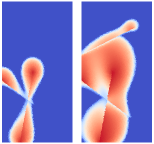


Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} = -10^5 \ln(S_m^o)$, $p_{c,f} = -10^4 \ln(S_m^o)$

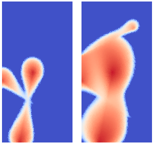
- Equi dim



- Hybrid Disc.



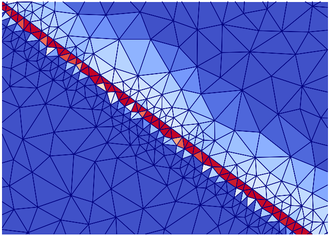
- Hybrid Cont.



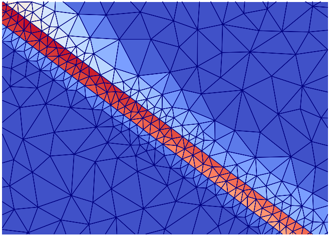
Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} = -10^5 \ln(S_m^o)$

Equi-Dimensional Model: Saturation Stratification in the fault Network

$p_{c,f} = 0$



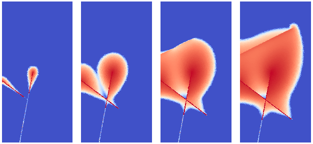
$p_{c,f} = -10^4 \ln(S_m^o)$



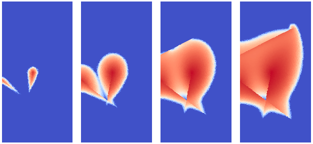
Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

Capillary Pressure: $p_{c,m} = -10^5 \ln(S_m^o)$; $p_{c,f} = 0$

- Equi dim

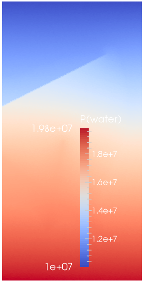


- Hybrid Disc.

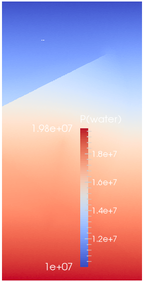


Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

Equi dim.



Hybrid Disc.



Computational Performance

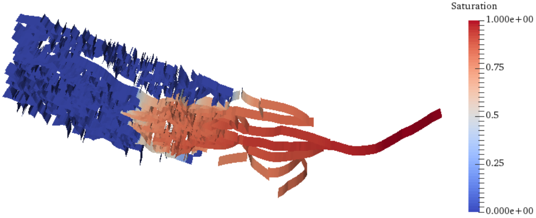
Model	CPU
Test Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m} = p_{c,f} = 0$	
equi dim.	9024
disc. hybrid	2951
cont. hybrid	960
Test Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m} \neq 0$; $p_{c,f} = 0$	
equi dim.	30697
disc. hybrid	4123
cont. hybrid	1022
Test Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} \neq 0$; $p_{c,f} \neq 0$	
equi dim.	2213
disc. hybrid	876
cont. hybrid	602
Test Drain-Barrier	
equi dim.	24199
disc. hybrid	3546

Conclusion

- Two phase flow hybrid discontinuous dimensional model taking into account
  - networks of fractures
  - drains and barriers
  - discontinuous capillary pressures
  - gravity
- Numerical experiments and comparisons with the hybrid continuous model show that
  - the model takes into account barriers
  - the model is more accurate even for pervious fractures, especially for gravity dominant flows
  - the model is more expensive due to smaller control volumes at the matrix fracture interfaces

Perspectives

- Better compromise accuracy - CPU time: "remove" the small control volumes at the mf interfaces
- Convergence analysis using the gradient scheme framework (with J. Droniou)
- Large 3D networks: HPC with F. Xing and BRGM
- More complex physics: geothermal flows with F. Xing and BRGM



Acknowledgements

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