Hybrid dimensional two phase flows in fractured porous media

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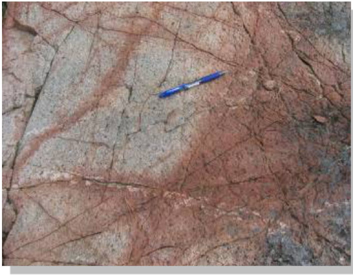
Outline

- Modelling Darcy flows in Discrete Fracture Networks
- The Vertex Approximate Gradient Discretization (VAG)
- Applications and Numerical Results

Fractured porous media: multiple scales (figures from J. R. de Dreuzy, Geosciences Rennes and team INRIA Sage)



Fractured porous media: applications

- Oil and gas exploration and production
- Hydrogeology
- Geothermal energy
- Geological storages
- Soil remediation
- ...

Flow in Fractured porous media: two main approaches

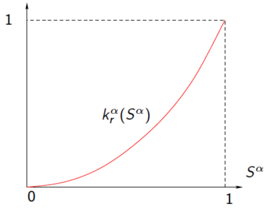
- Double Continuum Media: 3D fracture medium coupled to 3D matrix medium
- Discrete fracture models (DFM): 2D fracture model coupled to 3D matrix medium
- Possibility to couple both approaches
    - Double Continuum media for small fractures coupled with DFM for large fractures
    - Numerical Homogeneization: parameters of the Double Continuum media computed by a DFM model

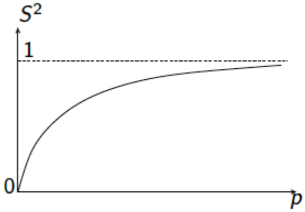
Modelling Darcy flows in Discrete Fracture Networks

Equi-dimensional model in phase pressures formulation

u^α : phase pressure
$\alpha = 1$: wetting phase, $\alpha = 2$: non wetting phase

$p = u^2 - u^1$: capillary pressure
$S^2(\mathbf{x}, p)$: inverse of capillary pressure graph, $S^1(\mathbf{x}, p) = 1 - S^2(\mathbf{x}, p)$
$k^\alpha(\mathbf{x}, S^\alpha) = \frac{k_r^\alpha(\mathbf{x}, S^\alpha)}{\mu^\alpha}$: phase mobility





Equi-dimensional model in phase pressures formulation

- Rock properties:
  - $\phi(\mathbf{x})$: porosity
  - $\Lambda(\mathbf{x})$: absolute permeability
- Fluid model:
  - Incompressible flow (fixed densities ρ^α, $\alpha = 1, 2,$)
  - Immiscible flow

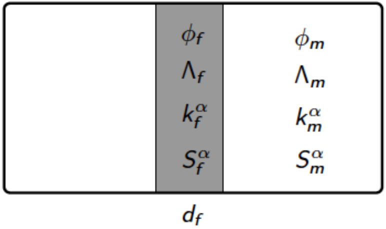
Darcy velocities:

$$\mathbf{q}^\alpha = -k^\alpha(\mathbf{x}, S^\alpha(\mathbf{x}, p))\ \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g}), \quad \alpha = 1, 2,$$

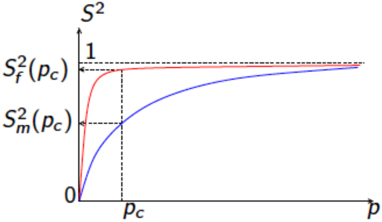
Volume conservation for each phase:

$$\phi(\mathbf{x})\partial_t S^\alpha(\mathbf{x}, p) + \operatorname{div}(\mathbf{q}^\alpha) = 0, \quad \alpha = 1, 2.$$

Equi-dimensional model: matrix and fracture domains



- Fracture: can act as drain or barrier
- Fracture width: $d_f << $ matrix size L
- Fracture rocktype (f): $\Lambda_f, \phi_f, k_f^\alpha, S_f^\alpha$
- Matrix rocktype (m): $\Lambda_m, \phi_m, k_m^\alpha, S_m^\alpha$

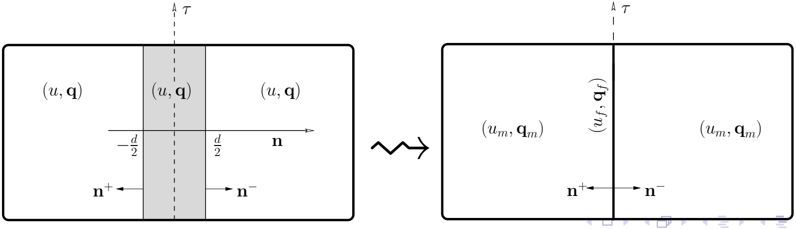


Dimensional Hybridizing (codimension 1 in the fracture)

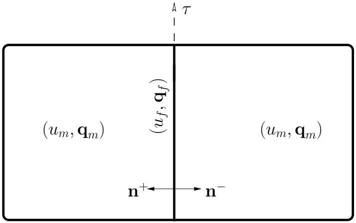
[Granet et al 2001], [Jaffré et al. 2002], [Bogdanov et al 2003], [Faille et al 2003], [Karimi Fard 2004], [Jaffré et al. 2005]

- **Dimensional hybridizing**: averaging the model equations over the fracture width
- **Objectives**: facilitate the mesh generation and lower the number of degrees of freedom

equi-dimensional model: hybrid-dimensional model:

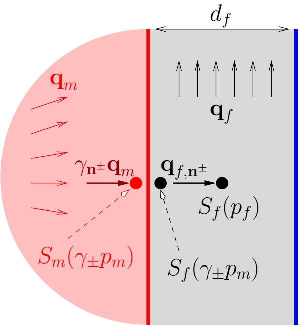


Hybrid dimensional models



$$\begin{cases}
\text{Matrix Darcy Law:} & \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_m))\, \Lambda_m(\nabla u_m^\alpha - \rho^\alpha \mathbf{g}) \\
\text{Matrix Vol. Cons.:} & \phi_m \partial_t S_m^\alpha(p_m) + \mathrm{div}(\mathbf{q}_m^\alpha) = 0 \\
\text{Fracture Darcy Law:} & \mathbf{q}_f^\alpha = -d_f k_f^\alpha(S_f^\alpha(p_f))\, \Lambda_{f,\tau}(\nabla_\tau u_f^\alpha - \rho^\alpha \mathbf{g}_\tau) \\
\text{Fracture Vol. Cons.:} & \phi_f d_f \partial_t S_f^\alpha(p_f) + \mathrm{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_{n^+}\mathbf{q}_m^\alpha + \gamma_{n^-}\mathbf{q}_m^\alpha = 0
\end{cases}$$

Transmission conditions at the matrix fracture interface



- **Discontinuous pressure model**:

$$\gamma_{\mathbf{n}^\pm}\mathbf{q}_m^\alpha = \mathbf{q}_{f,\mathbf{n}^\pm}^\alpha \approx$$

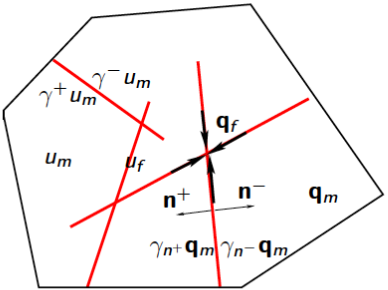
$$k_f^\alpha(S_f^\alpha(\gamma_\pm p_m))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m^\alpha - u_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^\pm\right)^+$$

$$+k_f^\alpha(S_f^\alpha(p_f))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m^\alpha - u_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^\pm\right)^-$$

- **Continuous pressure model** $\left(\frac{\Lambda_{f,\mathbf{n}}}{d_f} >> \frac{\Lambda_{m,\mathbf{n}}}{L}\right)$:

$$\gamma_+ u_m^\alpha = \gamma_- u_m^\alpha = u_f^\alpha.$$

Generalization to complex Discrete Fracture Network



- Pressure continuity and flux conservation is assumed at fracture intersections
- Zero flux is assumed at immersed fracture tips

The Vertex Approximate Gradient Discretization (VAG)

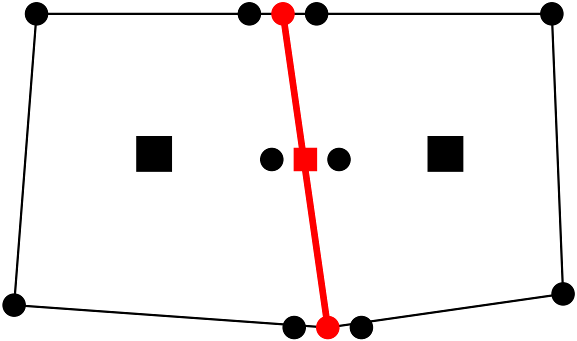
Discretization: state of the art

- MFE or MHFE: Jaffré et al 2002, Firoozabadi 2008
- TPFA: Faille et al. 2003, Karimi-Fard et al 2004
- CVFE: Bogdanov et al. 2003, Bastian et al 2006, Firoozabadi et al 2007
- XFEM type methods: Formaggia, Scotti et al 2012
- MPFA: Faille et al, Nordbotten et al, 2012, Edwards 2014
- HFV, MFD: Faille et al, Formaggia et al 2016
- ...

Our contributions:

- For both continuous and discontinuous of models
  - Extension of the Gradient scheme framework (Eymard et al 2010)
  - VAG and HFV discretizations
  - Two phase flows

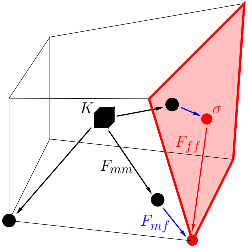
VAG scheme: degrees of Freedom (discontinuous pressure model)



- Two matrix cells touching a fracture face
  - Matrix d.o.f. (black)
  - Fracture d.o.f. (red)

VAG Matrix-Matrix and Fracture-Fracture Fluxes

- **mm (ff) fluxes:**
  - upwind (w.r.t. the phase mobility)
  - MPFA
  - local stencil to each cell K (fracture face σ)

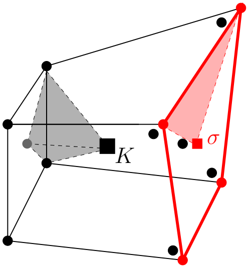


$$F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^-$$

$$f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_K^{\nu\nu'}(u_K - u_{\nu'} - \rho(\mathbf{x}_K - \mathbf{x}_{\nu'}) \cdot \mathbf{g})$$

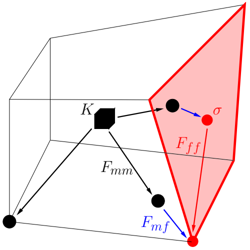
- **mm (ff) transmissivities:**
  - conforming $\mathbb{P}^1$ FE gradient on a tetrahedral (triangular) submesh ($\{\mathfrak{e}_\nu\}_\nu = \mathbb{P}^1$ FE Basis Functions)

$$T_K^{\nu\nu'} = \int_K \Lambda \nabla \mathfrak{e}_\nu \nabla \mathfrak{e}_{\nu'} \mathrm{d}\mathbf{x}$$



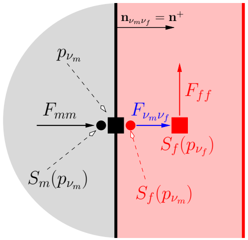
VAG Matrix-Fracture Fluxes

- **mf fluxes:**
  - upwind (w.r.t. the phase mobility)
  - TPFA
  - Saturation jump at mf interfaces
  - Gravity in normal direction



$$F_{\nu_m\nu_f}(u) = k_f(S_f(p_{\nu_m}))f_{\nu_m\nu_f}(u)^+ + k_f(S_f(p_{\nu_f}))f_{\nu_m\nu_f}(u)^-$$

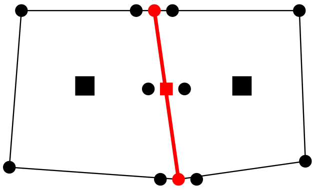
$$f_{\nu_m\nu_f}(u) = T_{\nu_m\nu_f}(u_{\nu_m} - u_{\nu_f} - \frac{\rho d_f}{2}\mathbf{g}\cdot\mathbf{n}_{\nu_m\nu_f})$$



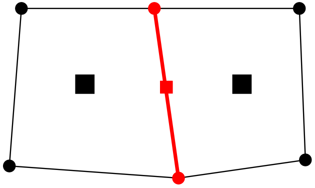
- **mf transmissivities:**
  - Mass Lumping of $\mathbb{P}^1$ FE basis function traces

Hybrid Continuous Pressure and Hybrid Discontinuous Pressure degrees of freedom

- Hybrid Discontinuous Pressure



- Hybrid Continuous Pressure



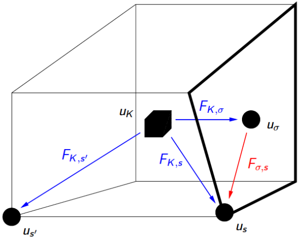
Hybrid Continuous Pressure model: Fluxes

- Matrix mm fluxes

$$F_{K\nu}(u) = k_m(S_m(p_K))f_{K\nu}(u)^+ + k_m(S_m(p_\nu))f_{K\nu}(u)^-$$
$$f_{K\nu}(u) = \sum_{\nu' \in \partial K} T_K^{\nu\nu'}(u_K - u_{\nu'} - \rho(\mathbf{x}_K - \mathbf{x}_{\nu'}) \cdot \mathbf{g})$$

- Fracture ff fluxes

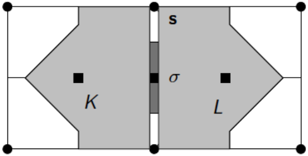
$$F_{\sigma s}(u) = k_f(S_f(p_\sigma))f_{\sigma s}(u)^+ + k_f(S_f(p_s))f_{\sigma s}(u)^-$$
$$f_{\sigma s}(u) = \sum_{s' \in \partial\sigma} T_\sigma^{ss'}(u_\sigma - u_{s'} - \rho(\mathbf{x}_\sigma - \mathbf{x}_{s'}) \cdot \mathbf{g})$$



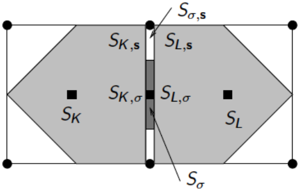
- No mf fluxes

Hybrid Continuous Pressure model

Control Volumes



Saturations at matrix fracture interfaces



$$\begin{cases} S^\alpha_{K,\nu} = S^\alpha_m(p_\nu), \\ S^\alpha_K = S^\alpha_m(p_K), \end{cases} \qquad \begin{cases} S^\alpha_{\sigma,\mathbf{s}} = S^\alpha_f(p_\mathbf{s}), \\ S^\alpha_\sigma = S^\alpha_f(p_\sigma). \end{cases}$$

Applications and Numerical Results

Comparison of equi- and hybrid-dimensional models: with J. Hennicker, K. Brenner and P. Samier

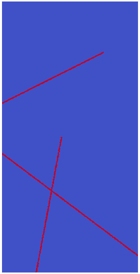
- $\Omega = (0, 400) \times (0, 800)$ m
- Equi-dimensional mesh: 22500 triangles
- Hybrid dimensional mesh: 16900 triangles
- Matrix:

$$\phi_m = 0.2, \quad \Lambda_m \text{ isotropic}$$

- Faults:

$$d_f = 4m, \quad \phi_f = 0.4, \quad \Lambda_f \text{ isotropic}$$

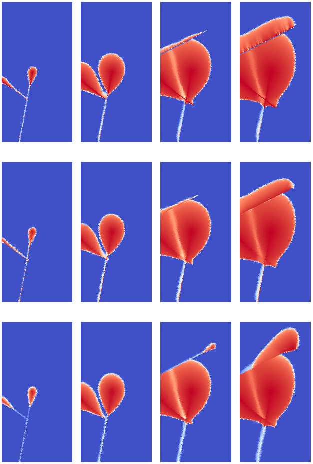
- Injection of oil in the bottom fault
- Initially saturated with water



Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(S_m^o) = p_{c,f}(S_f^o) = 0$

Zero Capillary Pressure: $p_{c,m}(S_m^o) = p_{c,f}(S_f^o) = 0$

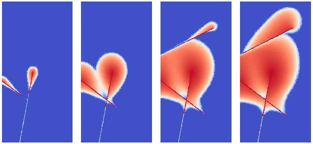
- Equi dim
- Hybrid Disc.
- Hybrid Cont.



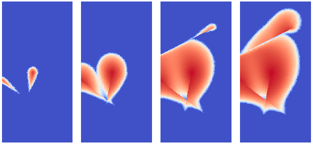
Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m} \neq 0$; $p_{c,f} = 0$

Capillary Pressure: $p_{c,m}(S_m^o) = -10^5 \ln(S_m^o)$; $p_{c,f} = 0$

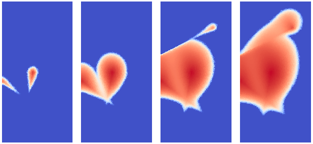
- Equi dim



- Hybrid Disc.

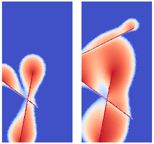


- Hybrid Cont.

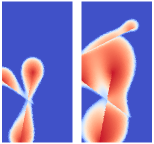


Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} = -10^5 \ln(S_m^o)$, $p_{c,f} = -10^4 \ln(S_m^o)$

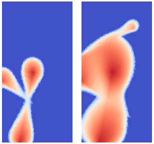
- Equi dim



- Hybrid Disc.

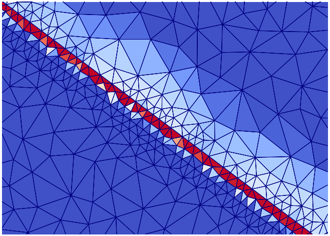
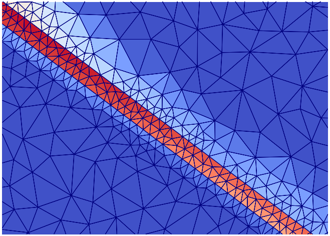


- Hybrid Cont.



Drains: $\Lambda_f/\Lambda_m = 100$; $p_{c,m} = -10^5 \ln(S_m^o)$

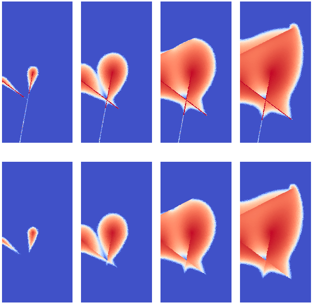
Equi-Dimensional Model: Saturation Stratification in the fault Network

$p_{c,f} = 0$	$p_{c,f} = -10^4 \ln(S_m^o)$
	

Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

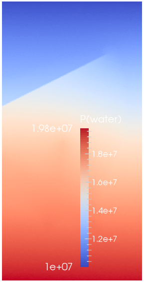
Capillary Pressure: $p_{c,m} = -10^5 \ln(S_m^o)$; $p_{c,f} = 0$

- Equi dim
- Hybrid Disc.

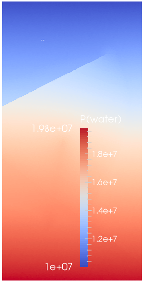


Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

Equi dim.



Hybrid Disc.

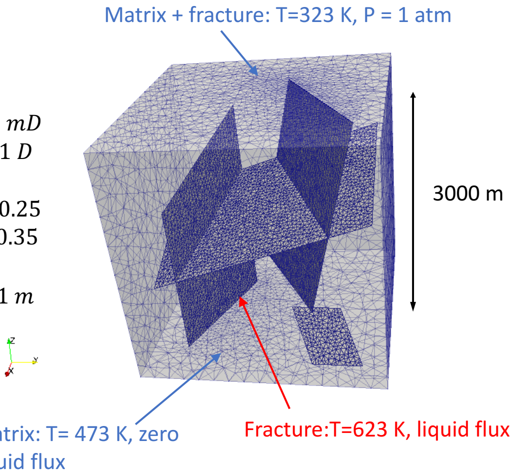


High energy geothermal test case: with F. Xing and S. Lopez (BRGM)

$$\Lambda_m = 10\ mD$$
$$\Lambda_f = 1\ D$$

$$\phi_m = 0.25$$
$$\phi_f = 0.35$$

$$d_f = 1\ m$$



Thermal two phase flow

Model: H_2O component in liquid or vapor phase
Unknowns: P, T, S^l, S^g
Continuous Pressure model
P, T, S formulation.

Conservation of mass, energy and volume:

$$\begin{cases} \phi\partial_t\left(\sum_{\alpha=l,g}\rho^\alpha S^\alpha\right)+\mathrm{div}\left(\sum_{\alpha=l,g}\rho^\alpha\mathbf{q}^\alpha\right)=0,\\ \phi\partial_t\left(\sum_{\alpha=l,g}\rho^\alpha e^\alpha S^\alpha\right)+(1-\phi)\partial_t E_r+\mathrm{div}\left(\sum_{\alpha=l,g}\rho^\alpha h^\alpha\mathbf{q}^\alpha-\lambda\nabla T\right)=0,\\ S^l+S^g=0, \end{cases}$$

$+$ liquid vapor thermodynamical equilibrium:

$$\begin{cases} T=T_{sat}(P) & \text{if} \quad S^g>0 \text{ and } S^l>0,\\ S^g=0 & \text{if} \quad T<T_{sat}(P),\\ S^l=0 & \text{if} \quad T>T_{sat}(P). \end{cases}$$

High energy geothermy: temperature and gas saturation

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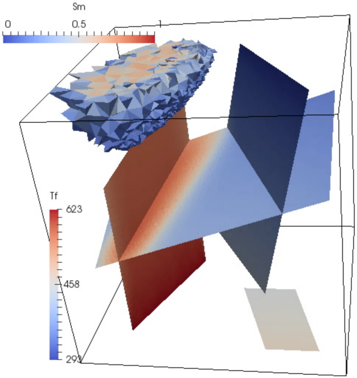
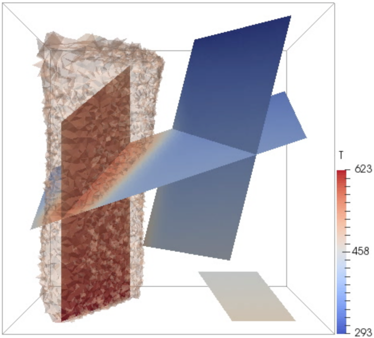
High energy geothermy: temperature and gas saturation

S_m^g and T_f



T_f and $T_m > 500$ K



Conclusion and Perspectives

- Two phase hybrid dimensional model taking into account
  - networks of fractures
  - drains and barriers
  - discontinuous capillary pressures
  - gravity

- On going work
  - Convergence analysis using the gradient scheme framework (with J. Droniou)
  - Geothermal systems: ANR project with BRGM, Storengy, MdS, LJLL, LJAD
  - Large networks

Acknowledgements

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