Multiphase flow in porous media using the VAG scheme

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Abstract We present the use of the Vertex Approximate Gradient scheme for the simulation of multiphase flow in porous media. The porous volume is distributed to the natural grid blocks and to the vertices, hence leading to a new finite volume mesh. Then the unknowns in the control volumes may be eliminated, and a 27-point scheme results on the vertices unknowns for a hexahedral structured mesh. Numerical results show the efficiency of the scheme in various situations, including miscible gas injection.

1 Introduction

Simulation of multiphase flow in porous media is a complex task, which has been the object of several works over a long period of time, see the reference books [12] and [3]. Several types of numerical schemes have been proposed in the past decades. Those which are implemented in industrial codes are mainly built upon cell centred approximations and discrete fluxes, in a framework which is also that of the method we propose here. Let us briefly sketch this framework. The 3D simulation domain Ω is meshed by control volumes $X \in \mathscr{M}$. Let us denote by Λ the diffusion matrix (which is a possibly full matrix depending on the point of the domain).

For each control volume $X \in \mathscr{M}$, the set of neighbors $Y \in \mathscr{N}_X$ is the set of all control volumes involved in the mass balance in X, which means that the following approximation formula is used: $-\int_X \nabla \cdot \Lambda \nabla p \mathrm{d}x \simeq \sum_{Y \in \mathscr{N}_X} F_{X,Y}(P)$, where $P = (p_Z)_{Z \in \mathscr{M}}$ is the family of all pressure unknowns in the control volumes, and

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where the flux $F_{X,Y}(P)$, between control volumes X and Y, is a linear function of the components of P which ensures the following conservativity property:

$$F_{X,Y}(P) = -F_{Y,X}(P). \tag{1}$$

Such a linear function, which is expected to vanish on constant families, may be defined by

$$F_{X,Y}(P) = \sum_{Z \in \mathscr{M}_{X,Y}} a^Z_{X,Y} p_Z, \tag{2}$$

where the family $(a^Z_{X,Y})_{Z \in \mathscr{M}_{X,Y}}$ and $\mathscr{M}_{X,Y} \subset \mathscr{M}$ are such that $\sum_{Z \in \mathscr{M}_{X,Y}} a^Z_{X,Y} = 0$.

Assuming N_c constituents and N_α phases, the discrete balance laws then read

$$\begin{aligned} &\frac{\Phi_X}{\delta t}(A^{(n+1)}_{X,i} - A^{(n)}_{X,i}) + \sum_{\alpha=1}^{N_\alpha} \sum_{Y \in \mathscr{N}_X} M^{(n+1),\alpha}_{X,Y,i} F^{(n+1),\alpha}_{X,Y} = 0, \ \forall i = 1, \ldots, N_c, \\ &F^{(n+1),\alpha}_{X,Y} = F_{X,Y}(P^{(n+1),\alpha}) - \rho^{(n+1),\alpha}_{X,Y} g \cdot (x_Y - x_X), \ \forall \alpha = 1, \ldots, N_\alpha, \end{aligned} \tag{3}$$

where n is the time index, δt is the time step, Φ_X is the porous volume of the control volume $X \in \mathscr{M}$, $A_{X,i}$ represents the accumulation of constituent i in the control volume X per unit pore volume (assumed to take into account the dependence of the porosity with respect to the pressure), $M^\alpha_{X,Y,i}$ is the amount of constituent i transported by phase α from the control volume X to the control volume Y (generally computed by taking the upstream value with respect to the sign of $F_{X,Y}$), P^α is the family of the pressure unknowns of phase α, g is the gravity acceleration, $\rho^\alpha_{X,Y}$ is the bulk density of phase α between control volumes X and Y and x_X is the center of control volume X. In addition to these relations, the differences between the phase pressures are ruled by capillary pressure laws. Thermodynamical equilibrium and standard closure relations are used.

When applying scheme (3), one should be very wary of the use of conformal finite elements in the case of highly heterogenous media. Indeed, assuming that the control volumes are vertex centered with vertices located at the interfaces between different media, then the porous volume concerned by the flow of very permeable medium includes that of non permeable medium. This may lead to surprisingly wrong results on the components velocities. A possible interpretation of these poor results is that, when seen as a set of discrete balance laws, the finite element method provides the same amount of impermeable and permeable porous volume for the accumulation term for a node located at a heterogenous interface.

We present in this paper the use of a new scheme, called Vertex Approximate Gradient (VAG) scheme [8, 9], which can be implemented in (3) so that the components velocities are correctly approximated, thanks to a special choice of the control volumes and of the discrete fluxes, which respect the form (2). The purpose of respecting the form (3)-(2) is to be able to easily plug it into an existing reservoir code, say Multi-Point Flux Approximation (MPFA), by simply redefining the control volumes and the coefficients $a^Z_{X,Y}$ of the discrete flux.

Although part of this scheme is vertex centered, we show that the solution obtained on a very heterogeneous medium with a coarse mesh remains accurate. This is a great advantage of this scheme, which is also always coercive, symmetric, and leads to a 27-stencil on hexahedral structured meshes. In addition the VAG scheme is very efficient on meshes with tetrahedra since the scheme can then be written with the nodal unknowns only, thus inducing a reduction of the number of degrees of freedom by a factor 5 compared with cell centered finite volume schemes such as MPFA schemes [1, 2, 4, 5].

2 Presentation of the scheme

The VAG scheme is described in [8, 9], and its gradient scheme properties are related to those presented in [7] ; therefore we focus here on the specifities of the use of this scheme for a multiphase flow simulation of the form (3). Let $\mathscr{M}$ be a general mesh of Ω, defined by a set $\mathscr{G}$ of grid blocks and the set $\mathscr{V}$ of their vertices; this is a mesh of control volumes in the sense of the preceding section: a control volume is either a grid block $K \in \mathscr{G}$ or a vertex $v \in \mathscr{V}$. In particular, a porous volume must be associated to each control volume, *i.e.* to each grid block and to each vertex. Finally a flux $F_{X,Y}$ from the control volume X to the control volume Y must be specified.

Any given grid block $K \in \mathscr{G}$ has, say, N_K vertices; let us denote by $\mathscr{V}_K \subset \mathscr{V}$ the set of these vertices. We wish to define a flux between neighbouring control volumes $X = K$ and $Y = v \in \mathscr{V}_K$, and between neighbouring control volumes $X = v \in \mathscr{V}_K$ and $Y = K \in \mathscr{G}_v = \{Y = K \in \mathscr{G}$ such that $v \in \mathscr{V}_K\}$; to this purpose, we introduce a local discrete gradient $\nabla_{K,v}(P_K) \in \mathbb{R}^3$ (see [8, 9] for the precise definitions), which only depends on the values $P_K = (P_{K,v})_{v\in\mathscr{V}_K} = (p_v - p_K)_{v\in\mathscr{V}_K}$. We then introduce the matrices $(A_K^{v,v'})_{v,v'\in\mathscr{V}_K}$, which are defined by the following relation

$$\frac{|K|}{N_K} \sum_{v\in\mathscr{V}_K} \Lambda_K \nabla_{K,v} P_K \cdot \nabla_{K,v} Q_K = \sum_{v\in\mathscr{V}_K} \sum_{v'\in\mathscr{V}_K} A_K^{v,v'} P_{K,v'} Q_{K,v},\ \forall P_K, Q_K \in \mathbb{R}^{\mathscr{V}_K}.$$

The flux from control volume $X = K$ to control volume $Y = v$ is then given by

$$F_{X,Y}(P) = F_{K,v}(P) = -\sum_{v'\in\mathscr{V}_K} A_K^{v,v'}(p_{v'} - p_K),$$

which is of the same form as (2) ; using (1), we get $F_{Y,X}(P) = -F_{X,Y}(P)$. Let us now turn to the definition of porous volumes for all $X \in \mathscr{M}$. The question is to associate to each vertex a porous volume in such a way that the components velocities are well approximated. Let us denote by $\widetilde{\Phi}_K = \int_K \Phi(x)\mathrm{d}x$ the total porous volume of each grid block $K \in \mathscr{G}$. We shall then take out a little bit of this porous volume of each grid block to associate it with the control volumes of the vertices. In order to obtain a systematic way to redistribute the porous volume between the grid blocks and the vertices, we define a first indicator of the transmissivity between K and v by

$B_{K,v} = \sum_{v' \in \mathscr{V}_K} A_K^{v,v'} > 0, \forall K \in \mathscr{G}_v$, and then, for a small value $\mu \in]0,1[$ (for example, $\mu = 0.05$), a weighted relative transmissivity (which is larger for permeable regions than for impermeable ones):

$$\widetilde{B}_{K,v} = \mu \frac{B_{K,v}}{\sum_{L \in \mathscr{G}_v} B_{L,v}}, \ \forall v \in \mathscr{V}, \ \forall K \in \mathscr{G}_v, \tag{4}$$

The total porous volume can then be redistributed between all control volumes $X \in \mathscr{M}$, that is between the grid blocks and the vertices, by the following relations:

$$\Phi_X = \begin{cases} \sum_{K \in \mathscr{G}_v} \widetilde{B}_{K,v} \widetilde{\Phi}_K & \text{if } X = v \in \mathscr{V}, \\ \widetilde{\Phi}_K (1 - \sum_{v \in \mathscr{V}_K} \widetilde{B}_{K,v}) & \text{if } X = K \in \mathscr{G}. \end{cases}$$

Hence, we distribute a small amount of the porous volume of K to its vertices, in a conservative way; indeed, by construction, we get that

$$\sum_{X \in \mathscr{M}} \Phi_X = \sum_{K \in \mathscr{G}} \widetilde{\Phi}_K,$$

with all $\Phi_X > 0$, provided that the value μ be chosen sufficiently small. We can remark that:

1. the porous volume of a vertex $v \in \mathscr{V}$ located at the interface between high and low permeability regions is mainly extracted from the higher permeability region,
2. the part of the lower permeability region distributed to the vertices is reduced by the factor μ.

We recall that we keep the property ensured in the monophasic case on the full system: indeed, the linear systems issued from Newton's method may be solved by first eliminating all unknowns $K \in \mathscr{G}$, and then solve a 27-point system on $v \in \mathscr{V}$.

3 Numerical applications

3.1 Heterogeneous case

The first example is the injection of CO_2, considered as immiscible with the liquid phase, at the middle point of an isotropic and heterogeneous reservoir, with size $[-100,100] \times [0,50] \times [0,45]$ m^3. The reservoir includes three 15 m-thick layers. The top and bottom layers are assumed to be weakly permeable ($|\Lambda| = 10^{-16}$ m^2) and the medium layer is much more permeable ($|\Lambda| = 10^{-12}$ m^2). A regular coarse $100 \times 10 \times 15$ mesh is used for the simulation (depicted in Figure 1). The values $\mu = 0.01$ and $\mu = 0.05$ have been tested in (4), without significant influence on the results. The results of the VAG scheme are compared to those obtained using the two-point flux approximation (TPFA) scheme, which is available on such a regular

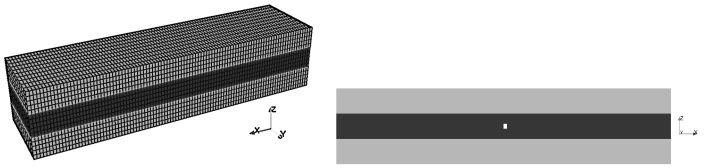

Fig. 1 First example. Left: mesh and layers. Right: the well is depicted at the center of the section $y = 25\ m$, illustrated by the white block

mesh. We observe in Figure 2 that the numerical diffusion along the axes of the mesh leads, after a short injection time, to a distorted profile of the gas saturation in the case of the TPFA scheme, known as Grid Orientation Effect (GOE), see also [10]. This phenomenon is clearer in the profile of the saturation at the end of the gas

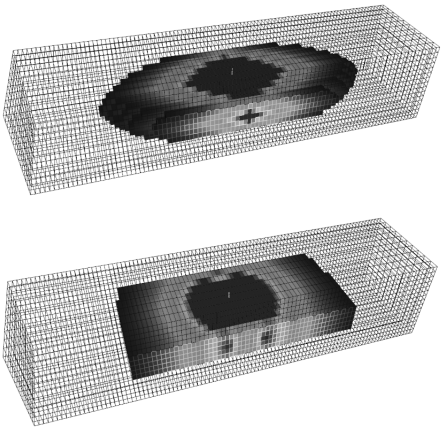

Fig. 2 View of the gas saturation in the reservoir, after a short injection time. Farthest to the well : $S = 0.001$. Closest to the well : $S = 0.042$. Top: TPFA scheme. Bottom: VAG scheme.

injection. We see in Figure 3 the important GOE due to the TPFA scheme, whereas this effect is nearly invisible in the results obtained with the VAG scheme.

Fig. 3 Gas saturation at the end of the gas injection. Section $z = 22.5\ m$. Farthest to the well : $S = 0$. Closest to the well : $S = 1$. Right: TPFA scheme. Left: VAG scheme.

Moreover, this distortion, also visible in the vertical section (Figure 4), is again corrected using the VAG scheme.

Fig. 4 Gas saturation at the end of the gas injection. Section $y = 25\ m$. Farthest to the well : $S = 0$. Closest to the well : $S = 1$. Right: TPFA scheme. Left: VAG scheme.

3.2 Near-Well case

In the second example, we consider the numerical simulation of the injection of CO_2 in near-well regions for a deviated well. A hexahedral radial part is connected to the outside boundary either by a hexahedral mesh or a hybrid mesh (using both pyramids and tetrahedra) as illustrated in Figure 5. The number of cells is roughly the same for both types of grids. This family of meshes is also used in the 3D benchmark on monophasic diffusion [11]. The medium is homogeneous, but anisotropic.

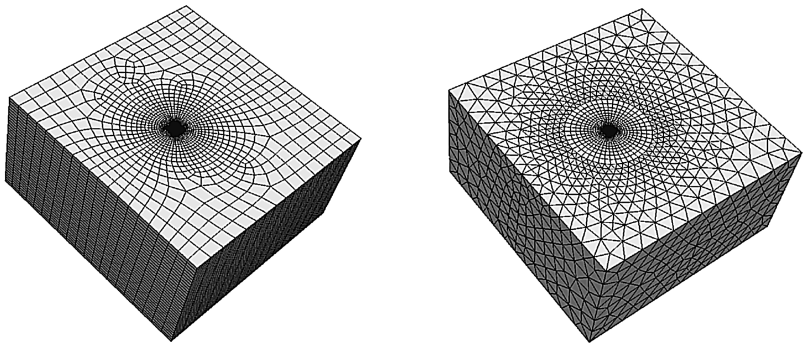

Fig. 5 Near-well grid : the hexahedral mesh (left) and the hybrid mesh (right).

We consider that the CO_2 can be dissolved in the aqueous phase.

We consider in Figures 7 and 6 the mass outflow rate of CO_2 in the two phases at the outer boundary using the VAG scheme and the MPFA O-scheme on both types of grids. The values 0.01 and 0.05 have been tested for the parameter μ used in (4) and the results are very close. In order to keep the output clearer, the curves are only plotted for $\mu = 0.05$. We observe that the VAG scheme produces results which are not very sensitive to the type of the grid. On the contrary, the MPFA O-scheme

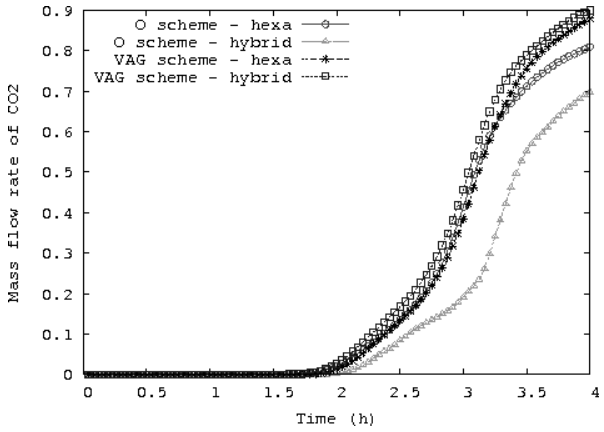

Fig. 6 Outflowing mass flow rate of CO_2 in the water phase at the outer boundary.

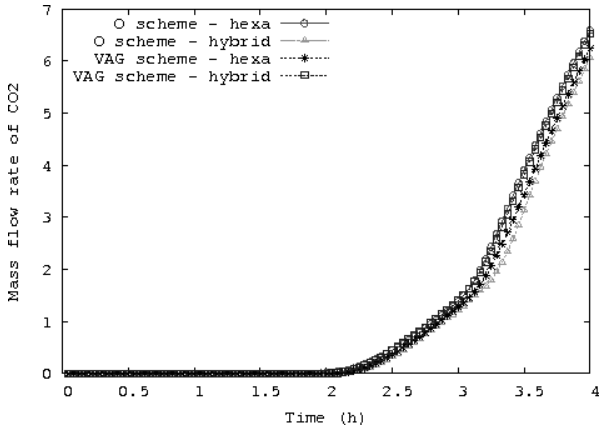

Fig. 7 Outflowing mass flow rate of CO_2 in the gas phase at the outer boundary.

shows a significant sensitivity to the type the grid, since the production of CO_2 is slowed down by the use of the tetrahedral mesh.

	cell unknowns	nodal unknowns
hexahedral mesh	74 679	74 800
hybrid mesh	77 599 - *with 28 704 tetrahedra*	37 883

Table 1 Numbers of cell and nodal unknowns for both near-well meshes.

Finally, Table 1 shows the numbers of cell and nodal unknowns for both families of meshes. As mentioned in the introduction, a vertex centered VAG scheme is very efficient on meshes with tetrahedra since the degrees of freedom can be reduced in comparison with cell centered schemes.

4 Conclusion

The above numerical results show that the VAG scheme seems to be an efficient scheme for multiphase flow simulation of a heterogeneous anisotropic reservoir; it features the following properties:

1. it may be implemented, without any additional cost, into an MPFA industrial code;
2. it leads to a 27-point compact stencil and a symmetric and coercive operator for the treatment of the diffusion terms, even in the case of distorted meshes and heterogeneous and anisotropic diffusion,
3. its cost is considerably reduced in the case of meshes with tetrahedra compared with cell centered MPFA schemes;
4. it remains accurate on coarse meshes thanks to a well-chosen distribution of the porous volume between the center of the control volumes and the vertices;
5. since a pore volume is assigned to the Neumann boundary nodes, the Neumann conditions are obtained by writing the conservation and closure equations as in the inner control volumes.

Full scale reservoir simulations will be performed in order to confirm the efficiency of the method.

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