Modelling and numerical analysis of two-phase flows in fractured porous media

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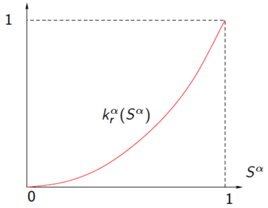
Outline

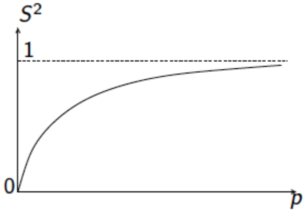
- Two phase hybrid dimensional Darcy flow
- Gradient Discretization
- VAG scheme
- Numerical Results

Equi-dimensional model in phase pressures formulation

u^α : phase pressure
$\alpha = 1$: wetting phase, $\alpha = 2$: non wetting phase

$p = u^2 - u^1$: capillary pressure
$S^2(\mathbf{x}, p)$: inverse of capillary pressure graph, $S^1(\mathbf{x}, p) = 1 - S^2(\mathbf{x}, p)$
$k^\alpha(\mathbf{x}, S^\alpha) = \frac{k_r^\alpha(\mathbf{x}, S^\alpha)}{\mu^\alpha}$: phase mobility





Equi-dimensional model in phase pressures formulation

- Rock properties:
  - $\phi(\mathbf{x})$: porosity
  - $\Lambda(\mathbf{x})$: absolute permeability
- Fluid model:
  - Incompressible flow (fixed densities ρ^α, $\alpha = 1, 2,$)
  - Immiscible flow

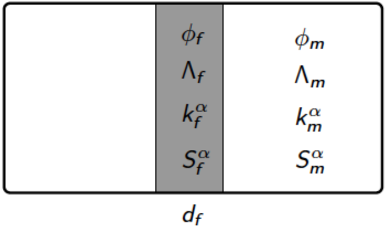
Darcy velocities:

$$\mathbf{q}^\alpha = -k^\alpha(\mathbf{x}, S^\alpha(\mathbf{x}, p))\ \Lambda(\mathbf{x})(\nabla u^\alpha - \rho^\alpha \mathbf{g}), \quad \alpha = 1, 2,$$

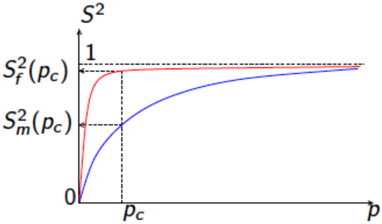
Volume conservation for each phase:

$$\phi(\mathbf{x})\partial_t S^\alpha(\mathbf{x}, p) + \mathrm{div}(\mathbf{q}^\alpha) = 0, \quad \alpha = 1, 2.$$

Equi-dimensional model: matrix and fracture domains



- Fracture: can act as drain or barrier
- Fracture width: $d_f << $ matrix size L
- Fracture rocktype (f): $\Lambda_f, \phi_f, k_f^\alpha, S_f^\alpha$
- Matrix rocktype (m): $\Lambda_m, \phi_m, k_m^\alpha, S_m^\alpha$

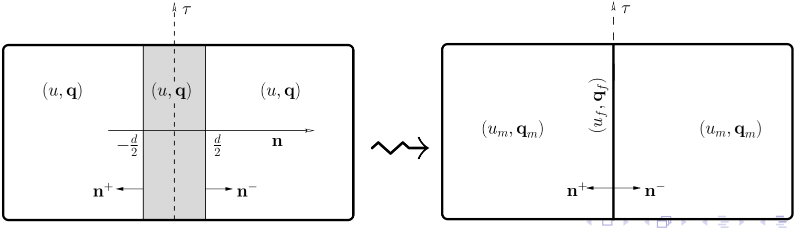


Dimensional Hybridizing (codimension 1 in the fracture)

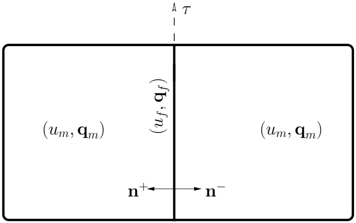
[Granet et al 2001], [Jaffré et al. 2002], [Bogdanov et al 2003], [Faille et al 2003], [Karimi Fard 2004], [Jaffré et al. 2005]

- **Dimensional hybridizing**: averaging the model equations over the fracture width
- **Objectives**: facilitate the mesh generation and lower the number of degrees of freedom

equi-dimensional model: hybrid-dimensional model:

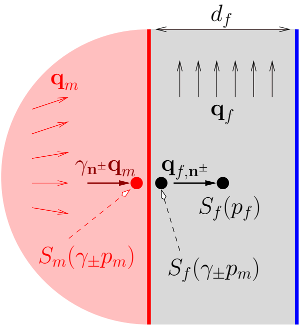


Hybrid dimensional models



$$
\begin{cases}
\text{Matrix Darcy Law:} & \mathbf{q}_m^\alpha = -k_m^\alpha(S_m^\alpha(p_m))\,\Lambda_m(\nabla u_m^\alpha - \rho^\alpha \mathbf{g}) \\
\text{Matrix Vol. Cons.:} & \phi_m \partial_t S_m^\alpha(p_m) + \mathrm{div}(\mathbf{q}_m^\alpha) = 0 \\
\text{Fracture Darcy Law:} & \mathbf{q}_f^\alpha = -d_f k_f^\alpha(S_f^\alpha(p_f))\,\Lambda_{f,\tau}(\nabla_\tau u_f^\alpha - \rho^\alpha \mathbf{g}_\tau) \\
\text{Fracture Vol. Cons.:} & \phi_f d_f \partial_t S_f^\alpha(p_f) + \mathrm{div}_\tau(\mathbf{q}_f^\alpha) + \gamma_{n^+}\mathbf{q}_m^\alpha + \gamma_{n^-}\mathbf{q}_m^\alpha = 0
\end{cases}
$$

Transmission conditions at the matrix fracture interface



- **Discontinuous pressure model**:

$$\gamma_{\mathbf{n}^\pm}\mathbf{q}_m^\alpha = \mathbf{q}^\alpha_{f,\mathbf{n}^\pm} \approx$$

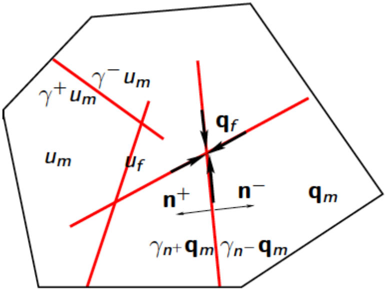
$$k_f^\alpha(S_f^\alpha(\gamma_\pm p_m))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m^\alpha - u_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^\pm\right)^+$$

$$+\, k_f^\alpha(S_f^\alpha(p_f))\Lambda_{f,\mathbf{n}}\left(\frac{\gamma_\pm u_m^\alpha - u_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^\pm\right)^-$$

- **Continuous pressure model** $\left(\frac{\Lambda_{f,\mathbf{n}}}{d_f} >> \frac{\Lambda_{m,\mathbf{n}}}{L}\right)$:

$$\gamma_+ u_m^\alpha = \gamma_- u_m^\alpha = u_f^\alpha.$$

Generalization to complex Discrete Fracture Network



- Pressure continuity and flux conservation is assumed at fracture intersections
- Zero flux is assumed at immersed fracture tips

Discretization: state of the art

- MFE or MHFE: Jaffré et al 2002, Firoozabadi 2008
- TPFA: Faille et al. 2003, Karimi-Fard et al 2004, Angot et al 2009
- CVFE: Bogdanov et al. 2003, Reichenberger et al 2006, Firoozabadi et al 2007, Matthai et al 2007
- XFEM type methods: Formaggia, Scotti et al 2012
- MPFA: Faille et al, Nordbotten et al, 2012, Edwards 2014
- HFV, MFD: Faille et al, Formaggia et al 2016

In this talk:

- Gradient Discretization (Eymard et al 2010, Droniou et al 2013)
- VAG scheme

Gradient Discretization

- Vector space of discrete unknowns: $X^0_{\mathcal{D}} = X^0_m \times X^0_f$
- Matrix and Fracture gradient and jump reconstruction operators:

$$\begin{array}{rcl} \nabla^m_{\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Omega)^d \\ \nabla^f_{\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Gamma)^{d-1} \\ [\![\cdot]\!]_{\mathfrak{a},\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Gamma),\ \mathfrak{a} = \pm \end{array}$$

- Matrix, Fracture function and trace piecewise constant reconstruction operators:

$$\begin{array}{rcl} \Pi^m_{\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Omega) \\ \Pi^f_{\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Gamma) \\ \mathrm{T}^{\mathfrak{a}}_{\mathcal{D}} & : & X^0_{\mathcal{D}} \to L^2(\Gamma),\ \mathfrak{a} = \pm \end{array}$$

with $\quad [\![u]\!]_{\mathfrak{a}} = \gamma_{\mathfrak{a}} u_m - u_f,\ \mathfrak{a} = \pm.$

Discrete Model (no gravity)

Find $u_{\mathcal{D}}^{\alpha,n} = (u_m^{\alpha,n}, u_f^{\alpha,n}) \in X_{\mathcal{D}}^0$ such that for all $v_{\mathcal{D}}^{\alpha} = (v_m^{\alpha}, v_f^{\alpha}) \in X_{\mathcal{D}}^0$:

$$\begin{aligned}
\sum_{\mu=m,f} \Bigg\{ \int_\mu \Phi_\mu \frac{S_\mu^\alpha(\Pi_{\mathcal{D}}^\mu p_{\mathcal{D}}^n) - S_\mu^\alpha(\Pi_{\mathcal{D}}^\mu p_{\mathcal{D}}^{n-1})}{\Delta t} \Pi_{\mathcal{D}}^\mu v_{\mathcal{D}}^\alpha \\
+ \int_\mu k_\mu^\alpha(S_\mu^\alpha(\Pi_{\mathcal{D}}^\mu p_{\mathcal{D}}^n))\, \mathbf{K}_\mu \nabla_{\mathcal{D}}^\mu u_{\mathcal{D}}^{\alpha,n} \cdot \nabla_{\mathcal{D}}^\mu v_{\mathcal{D}}^\alpha \Bigg\} \\
+ \sum_{\mathfrak{a}=\pm} \int_\Gamma \frac{2\Lambda_{f,\mathbf{n}}}{d_f} \Big(k_f^\alpha(S_f^\alpha(\mathrm{T}_{\mathcal{D}}^{\mathfrak{a}} p_{\mathcal{D}}^n)) [\![u_{\mathcal{D}}^{\alpha,n}]\!]_{\mathfrak{a},\mathcal{D}}^+ \\
+ k_f^\alpha(S_f^\alpha(\Pi_{\mathcal{D}}^f p_{\mathcal{D}}^n)) [\![u_{\mathcal{D}}^{\alpha,n}]\!]_{\mathfrak{a},\mathcal{D}}^- \Big) [\![v_{\mathcal{D}}^\alpha]\!]_{\mathfrak{a},\mathcal{D}} \\
= \quad 0
\end{aligned}$$

with $\Phi_m = \phi_m$, $\Phi_f = d_f \phi_f$ and $\mathbf{K}_m = \Lambda_m$, $\mathbf{K}_f = d_f \Lambda_{f,\tau}$.

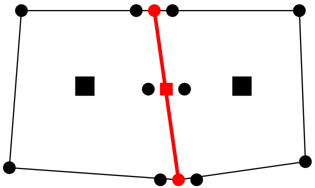
Convergence Result

- For coercive, consistant, limit conforming, compact gradient schemes:

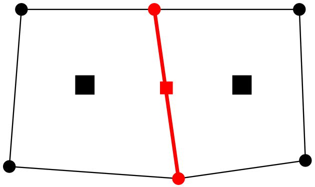
$$
\left\{
\begin{array}{ll}
(\Pi^m_{\mathcal{D}^l}, \Pi^f_{\mathcal{D}^l}) u_{\mathcal{D}^l} \rightharpoonup (u_m, u_f) & \text{in } L^2((0,T)\times\Omega) \times L^2((0,T)\times\Gamma) \\
(\nabla^m_{\mathcal{D}^l}, \nabla^f_{\mathcal{D}^l}) u_{\mathcal{D}^l} \rightharpoonup (\nabla u_m, \nabla_\tau u_f) & \text{in } L^2((0,T)\times\Omega)^d \times L^2((0,T)\times\Gamma)^{d-1} \\
\mathbb{T}^{\mathfrak{a}}_{\mathcal{D}^l} u_{\mathcal{D}^l} \rightharpoonup \gamma_{\mathfrak{a}} u_m & \text{in } L^2((0,T)\times\Gamma) \\
[\![u_{\mathcal{D}^l}]\!]_{\mathfrak{a},\mathcal{D}^l} \rightharpoonup [\![u]\!]_{\mathfrak{a}} & \text{in } L^2((0,T)\times\Gamma) \\
(S_m(\Pi^m_{\mathcal{D}^l} p_{\mathcal{D}^l}), S_f(\Pi^f_{\mathcal{D}^l} p_{\mathcal{D}^l})) & \\
\quad \to (S_m(p_m), S_f(p_f)) & \text{in } L^2((0,T)\times\Omega) \times L^2((0,T)\times\Gamma) \\
S_f(\mathbb{T}^{\mathfrak{a}}_{\mathcal{D}^l} p_{\mathcal{D}}) \to S_f(\gamma_{\mathfrak{a}} p_m) & \text{in } L^2((0,T)\times\Gamma)
\end{array}
\right.
$$

Vertex Approximate Gradient (VAG) scheme [Eymard et al 2010]

- Hybrid Discontinuous Pressure



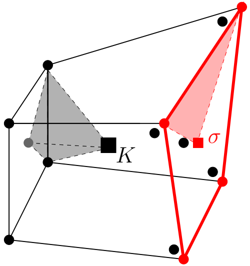
- Hybrid Continuous Pressure

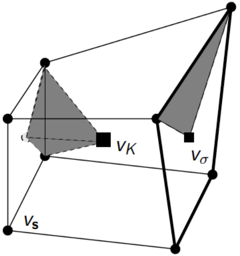


VAG operators

- **Discontinuous pressure model:**
  - $\nabla^m_{\mathcal{D}}$ (resp. $\nabla^f_{\mathcal{D}}$): conforming $\mathbb{P}^1$ FE on a tetrahedral (resp. triangular) submesh
  - $[\![\cdot]\!]_{\mathfrak{a},\mathcal{D}}$: piecewise constant using lumping
  - $\Pi^m_{\mathcal{D}}$, $\Pi^f_{\mathcal{D}}$, $\mathbb{T}^{\mathfrak{a}}_{\mathcal{D}}$: piecewise constant

- **Continuous pressure model:**
  - $\nabla^m_{\mathcal{D}}$ (resp. $\nabla^f_{\mathcal{D}}$): conforming $\mathbb{P}^1$ FE on a tetrahedral (triangular) submesh
  - $\Pi^m_{\mathcal{D}}$, $\Pi^f_{\mathcal{D}}$: piecewise constant





Comparison of equi- and hybrid-dimensional models: two phase flow

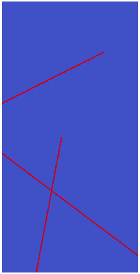
- $\Omega = (0,4) \times (0,8)$ m
- Equi-dimensional mesh: 22500 triangles
- Hybrid dimensional mesh: 16900 triangles
- Matrix:

$$\phi_m = 0.2, \quad \Lambda_m \text{ isotropic}$$

- Fractures:

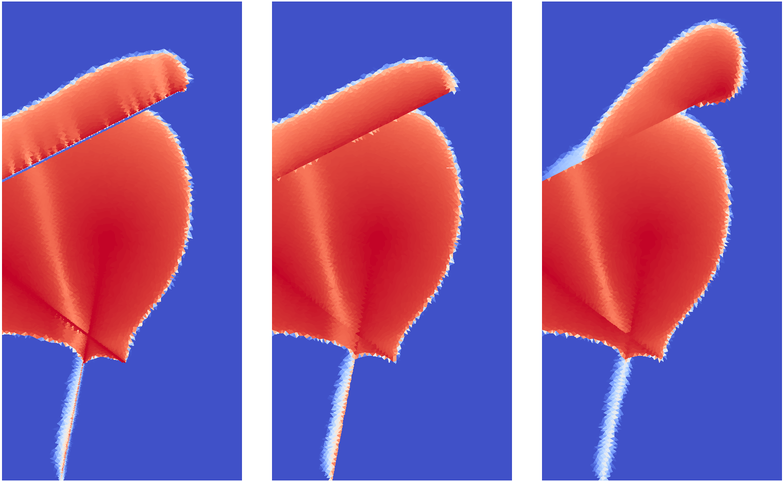
$$d_f = 0.04m, \quad \phi_f = 0.4, \quad \Lambda_f \text{ isotropic}$$

- Injection of oil in the bottom fracture
- Initially saturated with water

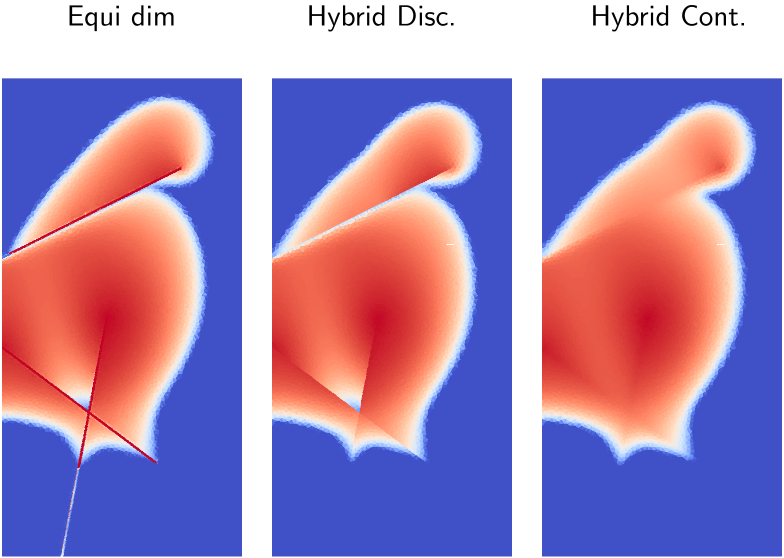


Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(s^o) = p_{c,f}(s^o) = 0$

Equi dim — Hybrid Disc. — Hybrid Cont.

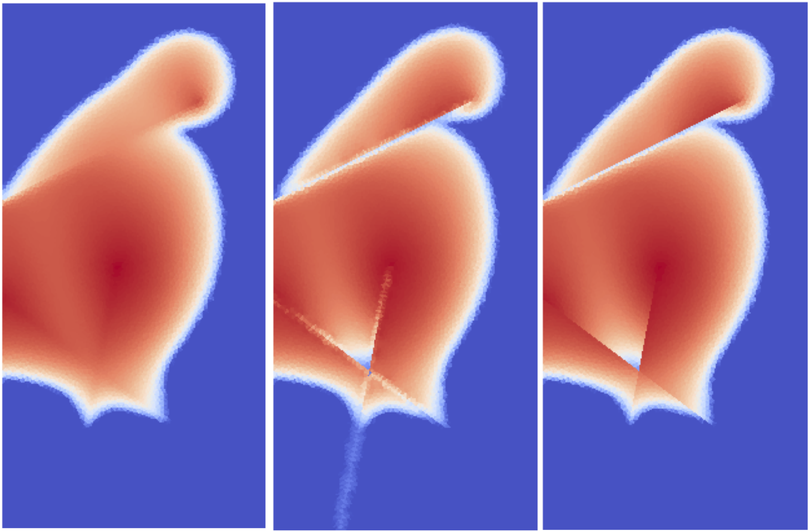


Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(s^o) = -10^3 \ln(s^o)$; $p_{c,f}(s^o) = 0$



Drains: $\Lambda_f/\Lambda_m = 1000$; $p_{c,m}(s^o) = -10^3 \ln(s^o)$; $p_{c,f}(s^o) = 0$

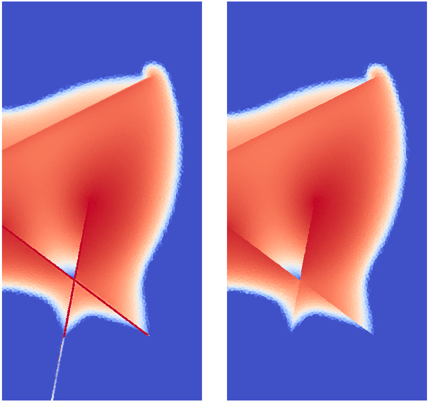
Hybrid Cont. Hybrid Cont. v2 Hybrid Disc.



Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

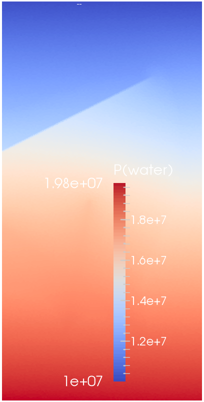
Capillary Pressure: $p_{c,m}(s^o) = -10^3 \ln(s^o)$; $p_{c,f}(s^o) = 0$

Equi dim Hybrid Disc.

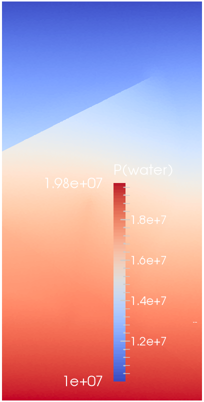


Drain-Barrier: $\Lambda_f^{drain}/\Lambda_m = 1000$; $\Lambda_f^{barrier}/\Lambda_m = 0.01$

Equi dim.



Hybrid Disc.



Conclusion and Perspectives

- Two phase hybrid dimensional model taking into account
  - networks of fractures
  - drains and barriers
  - discontinuous capillary pressures
  - gravity

- On going work
  - Large networks
  - Compositional models

Acknowledgements

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