MathStatNeuro workshop, Nice, France
September 8-10, 2015

Relating neural firings to epidemics and tweets.

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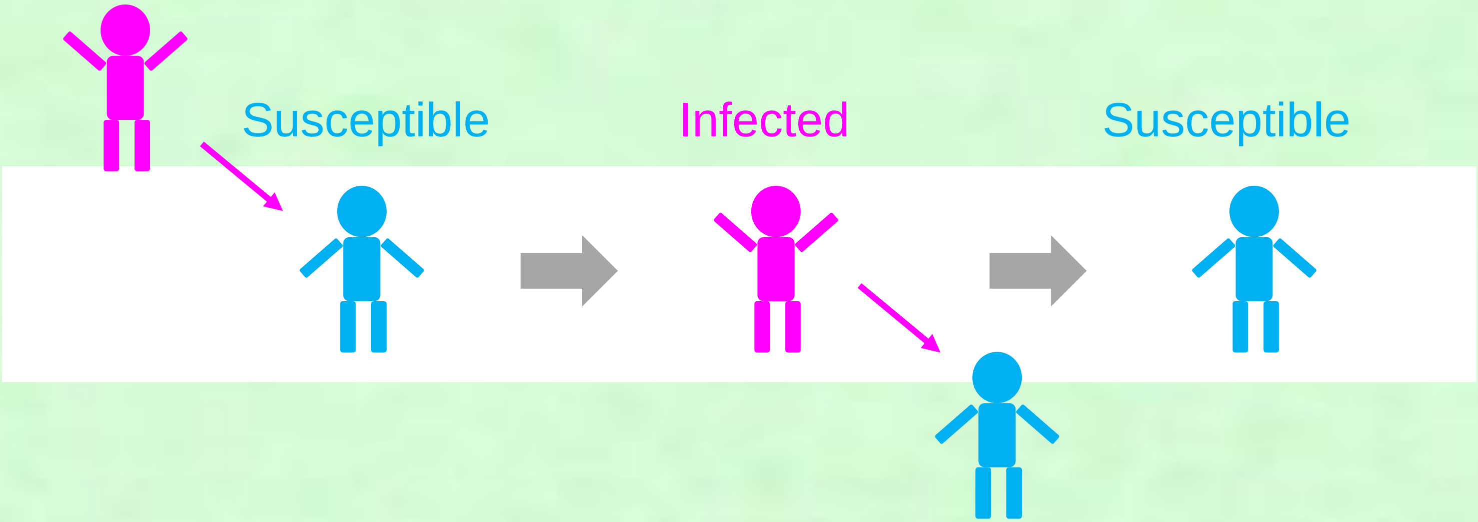
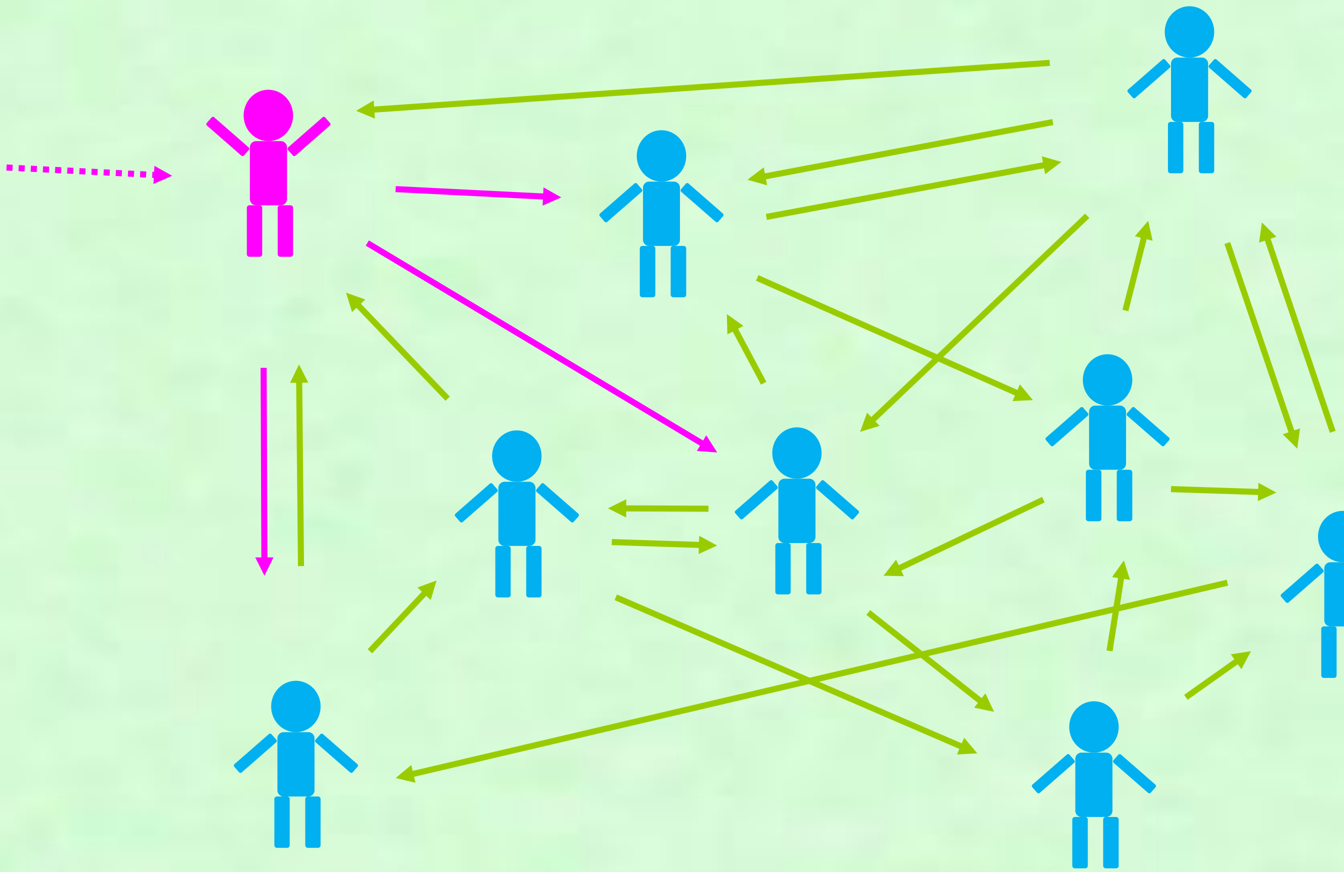
+ Tomokatsu Onaga

Epidemics: SIS model



Epidemics: SIS model

"One dog barks at something, and a hundred bark at the sound." --- *Chinese proverb*



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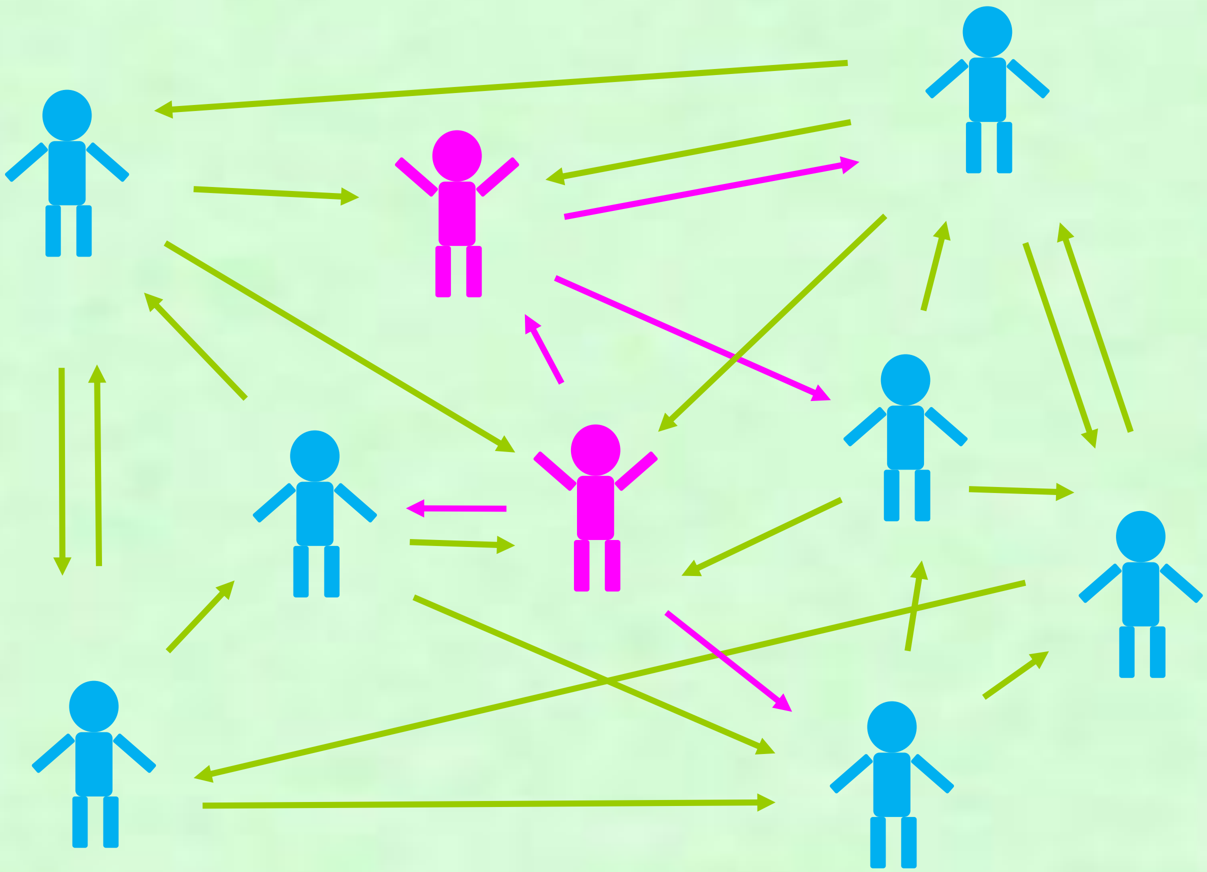

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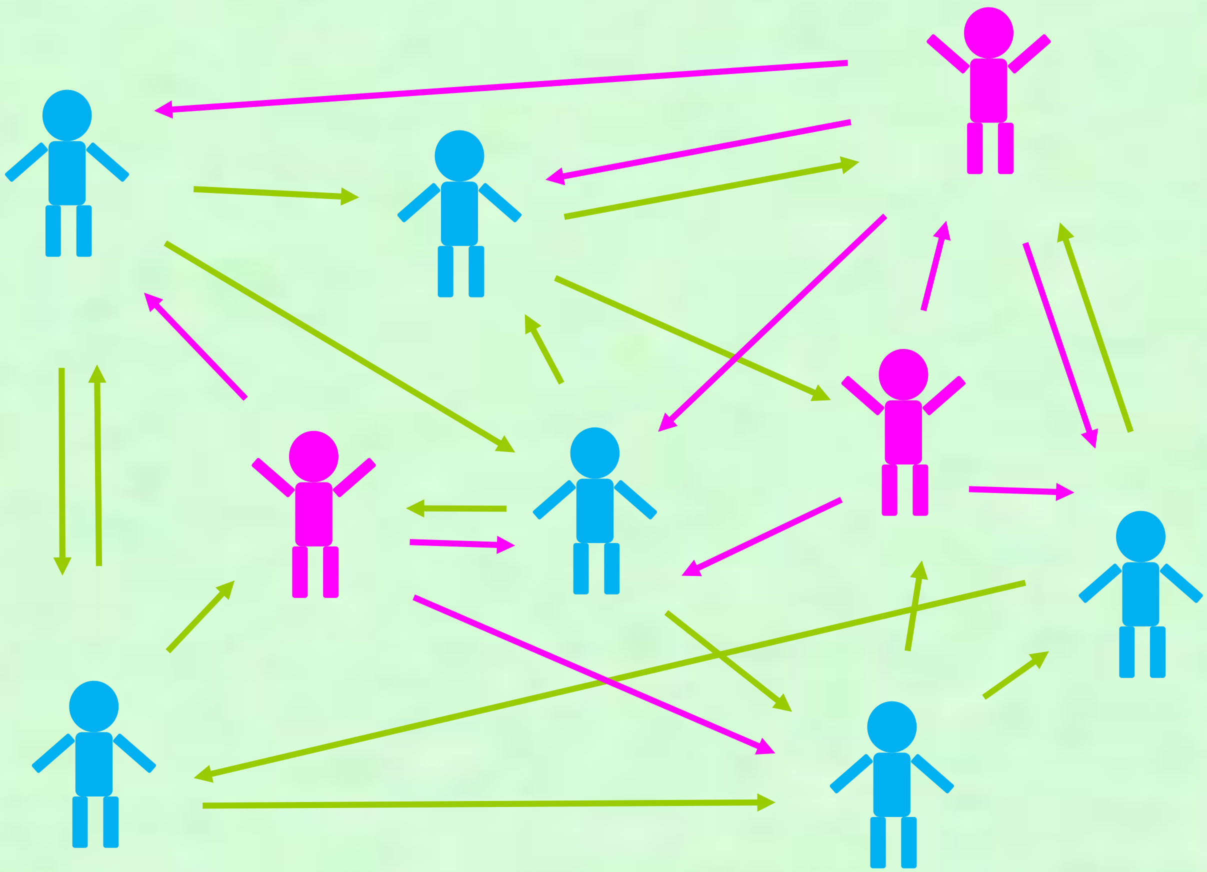

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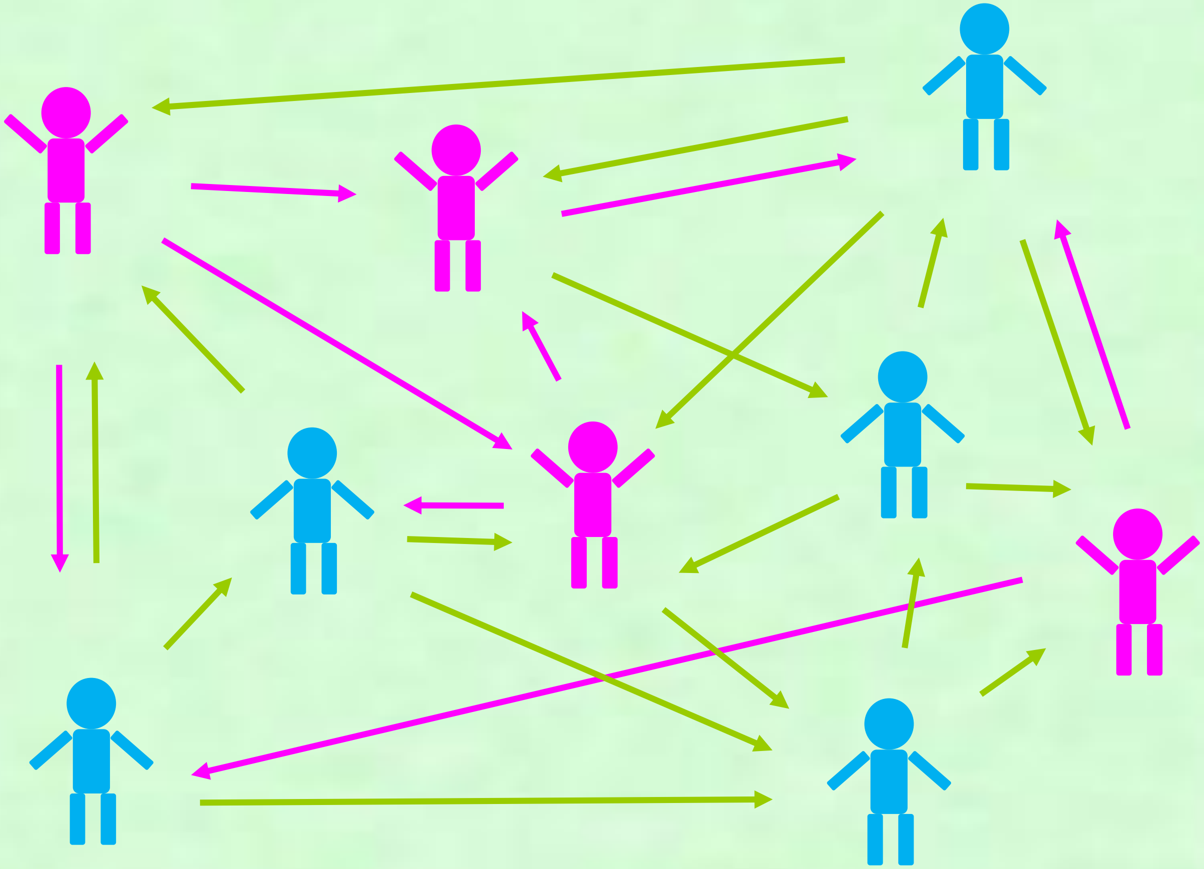

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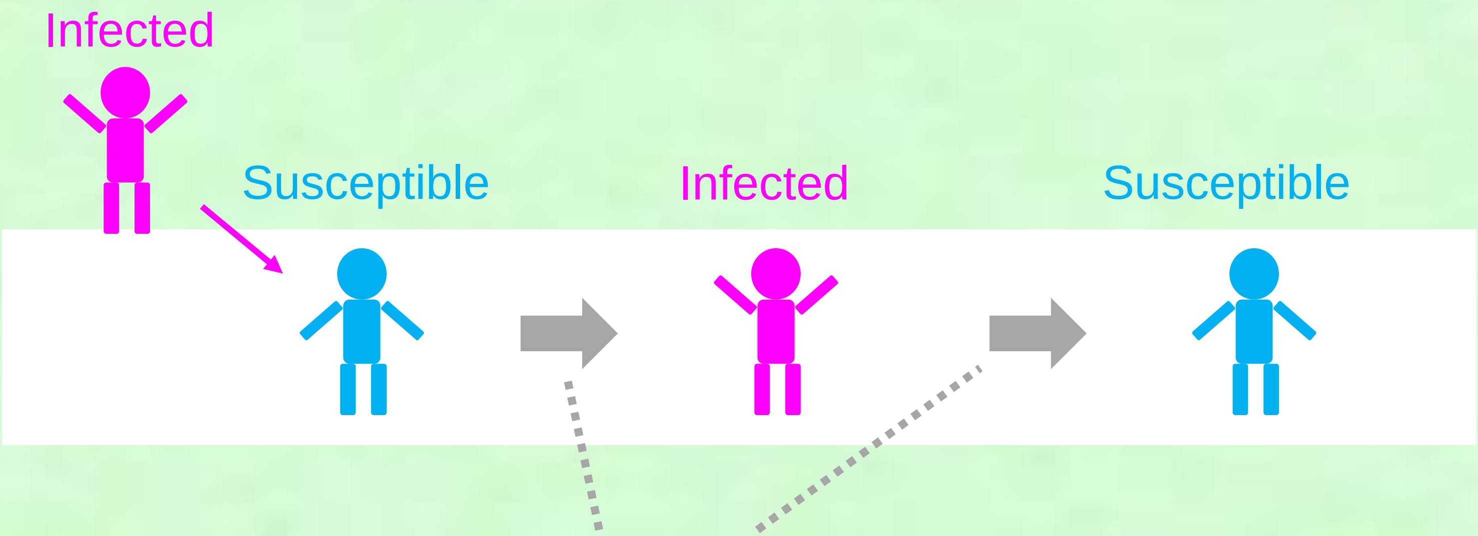


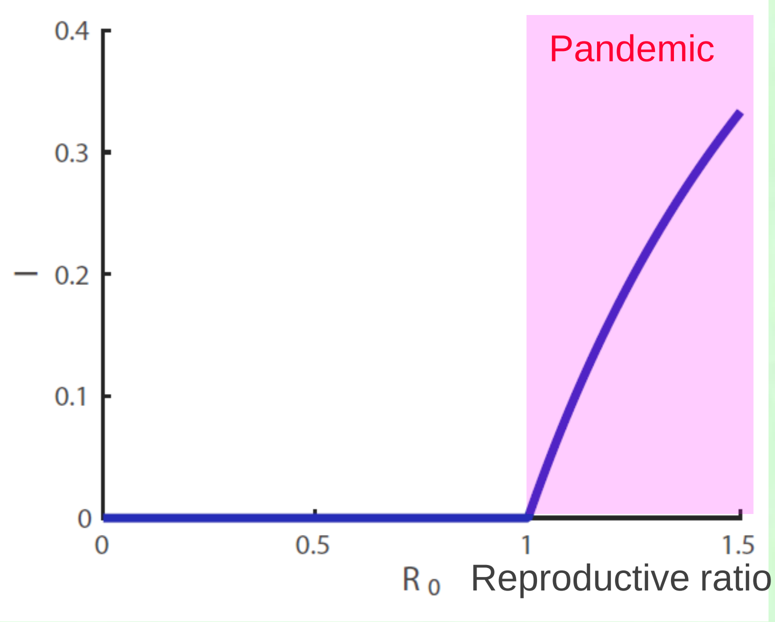
$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$I + S = 1$$

Epidemic transition

Basic reproductive ratio (R_0):
the avearge number of extra events induced by a single event.

The occurrence rate



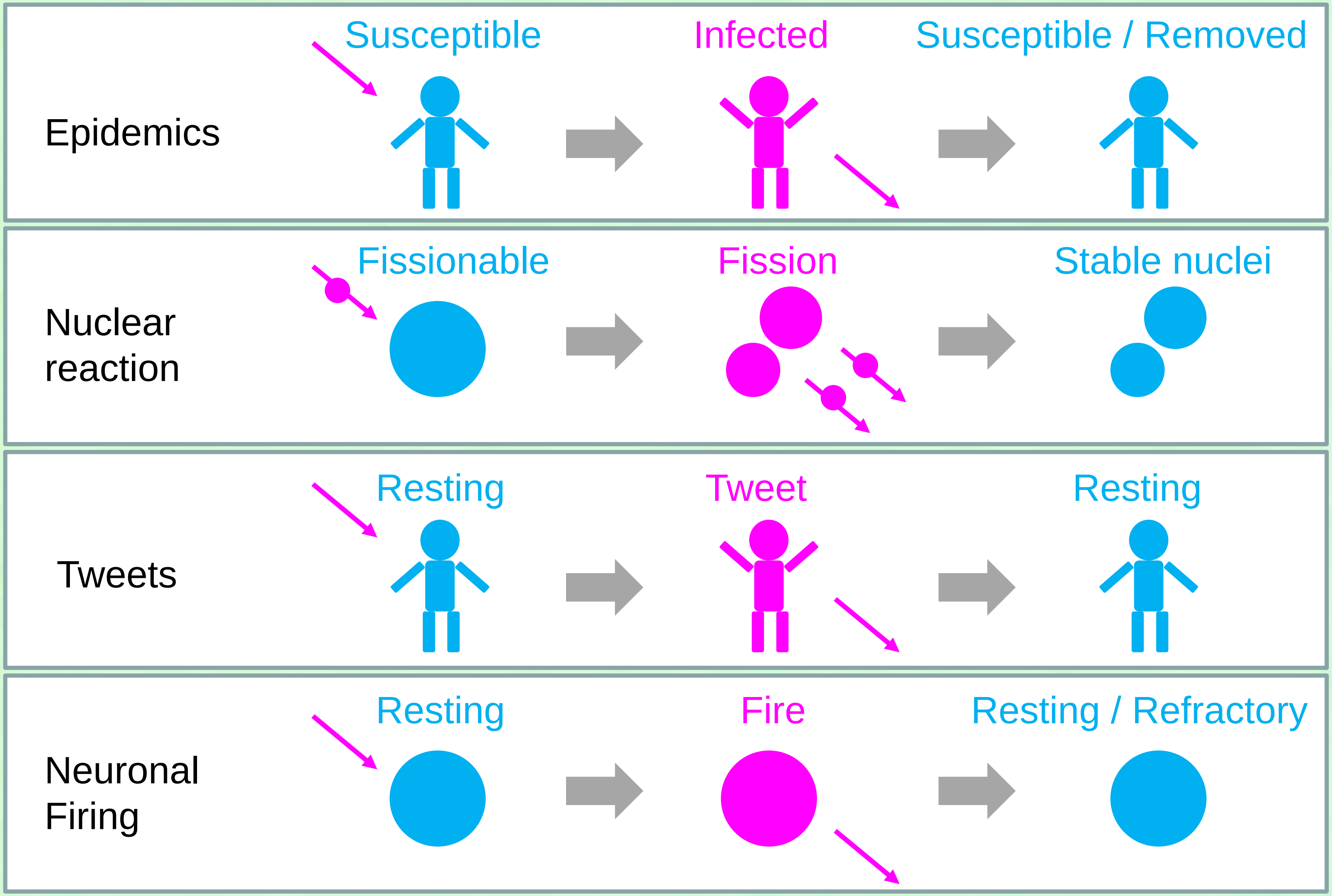
$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$I + S = 1$$

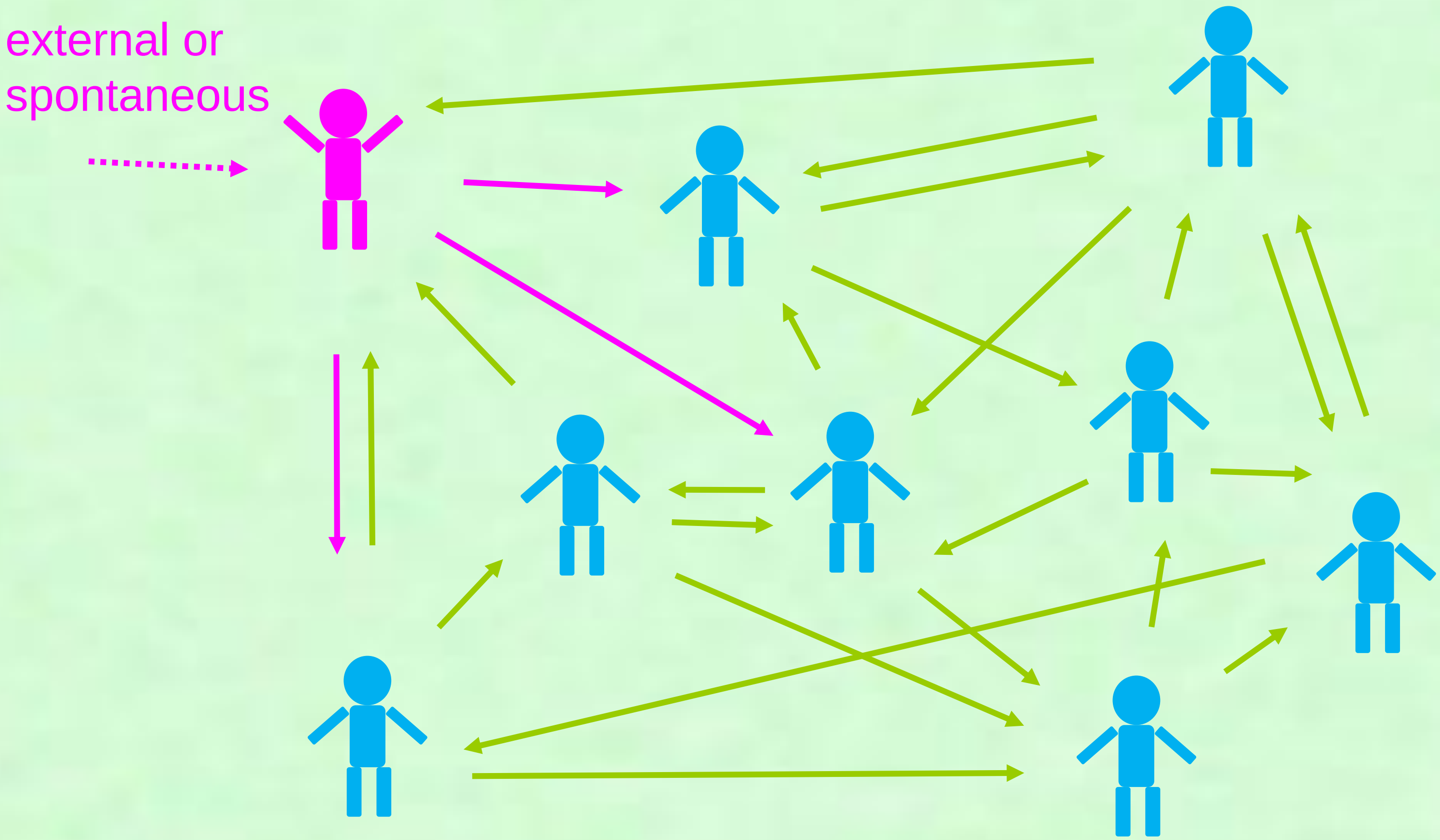
$$R_0 = \beta / \gamma$$

cf: Keeling & Rohani, *Modeling Infectious Diseases* 2008

Self-exciting processes



nuclear, tweets, neural



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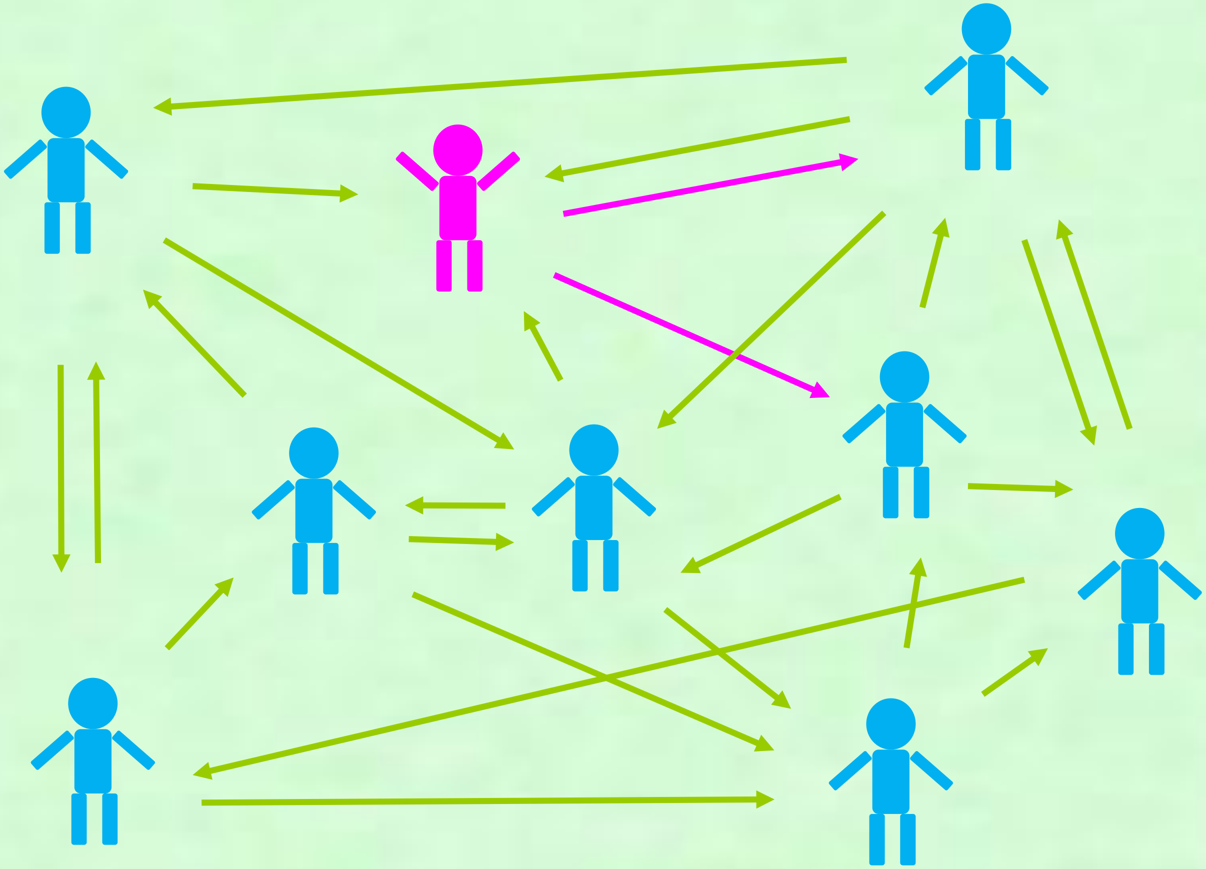
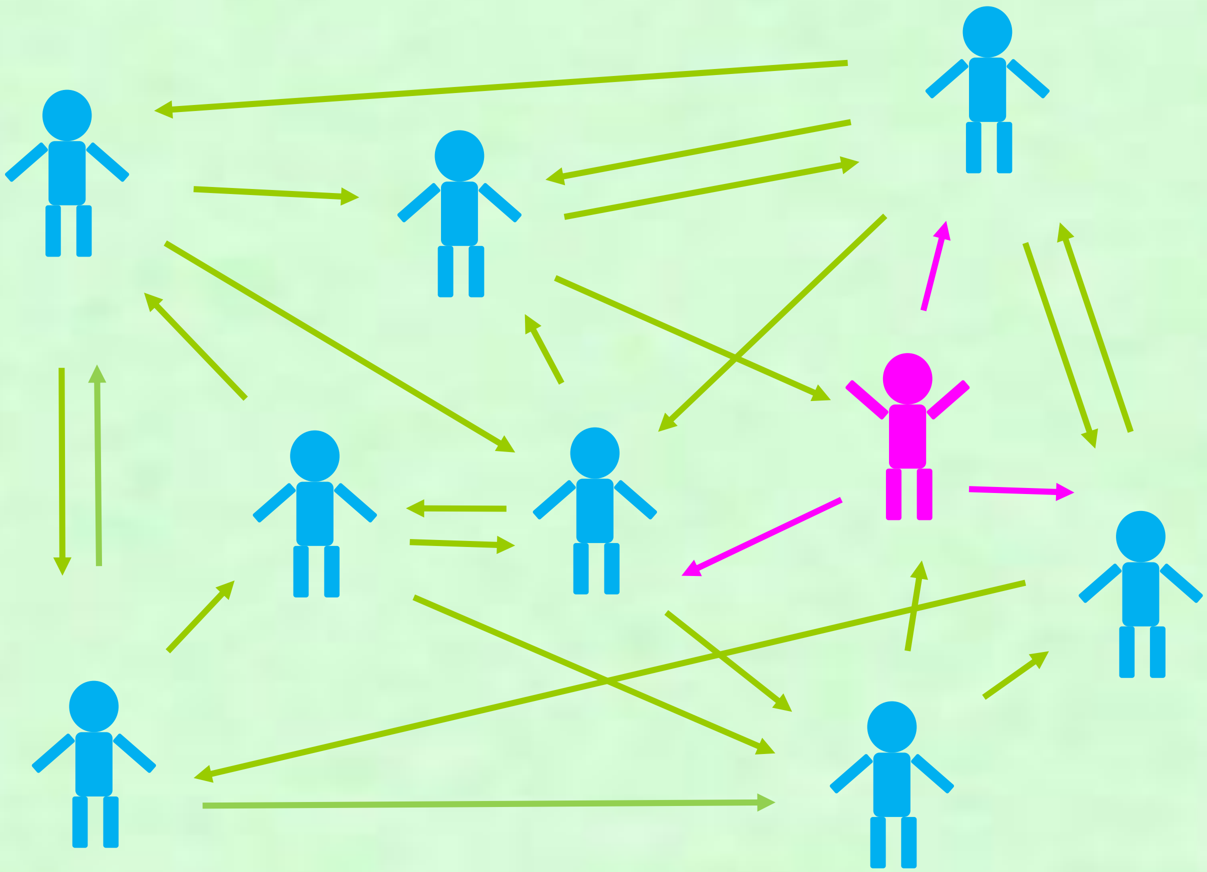


nuclear, tweets, neural



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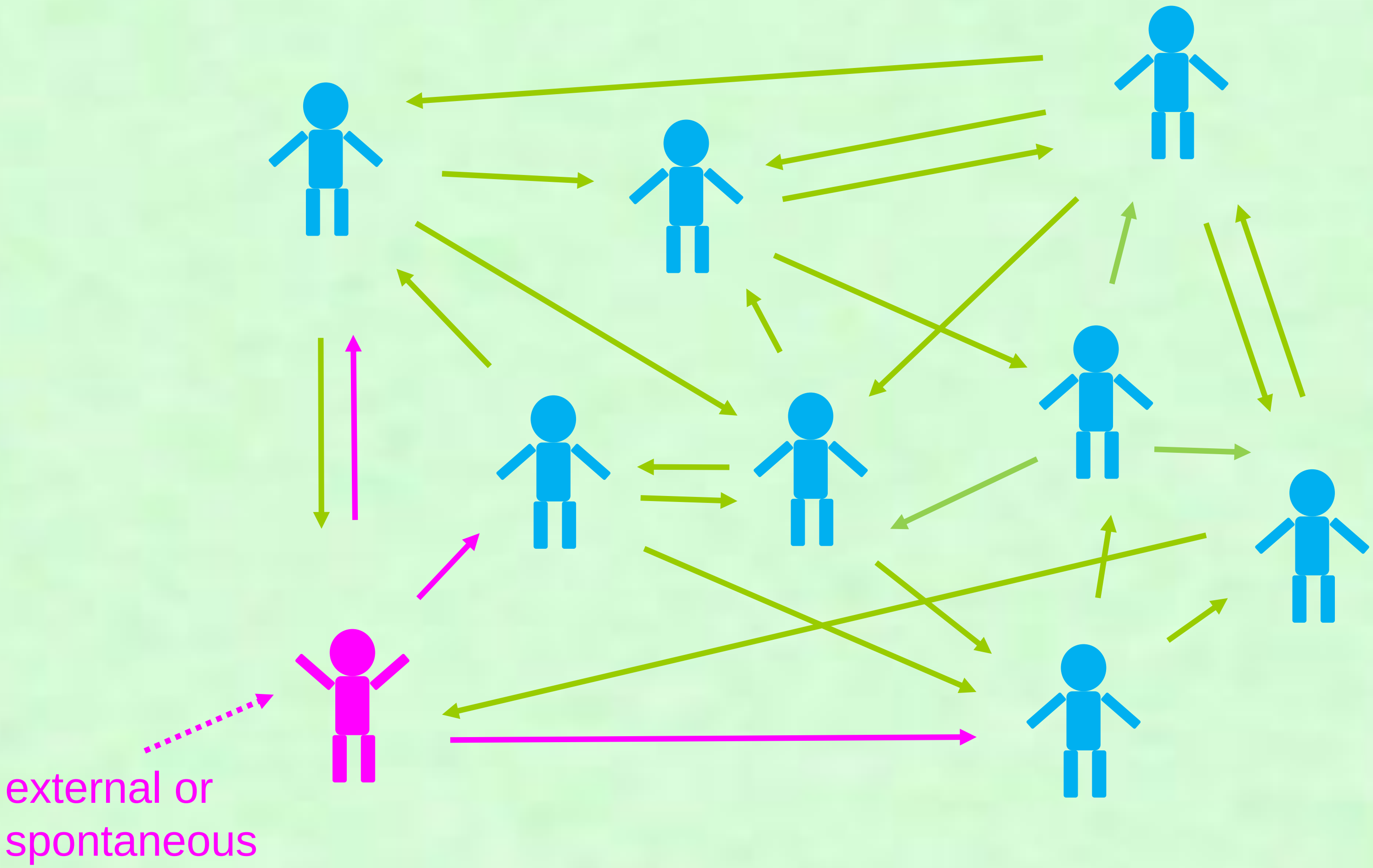

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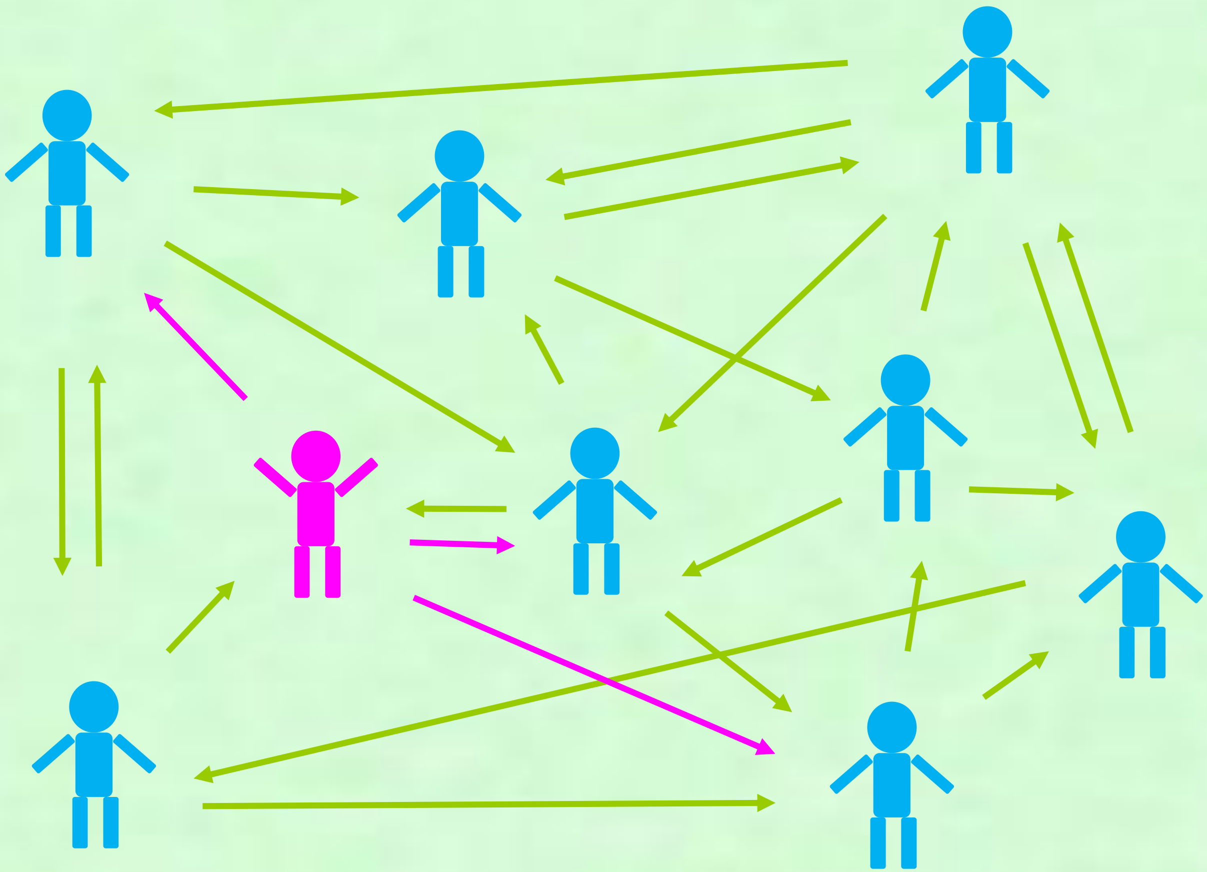

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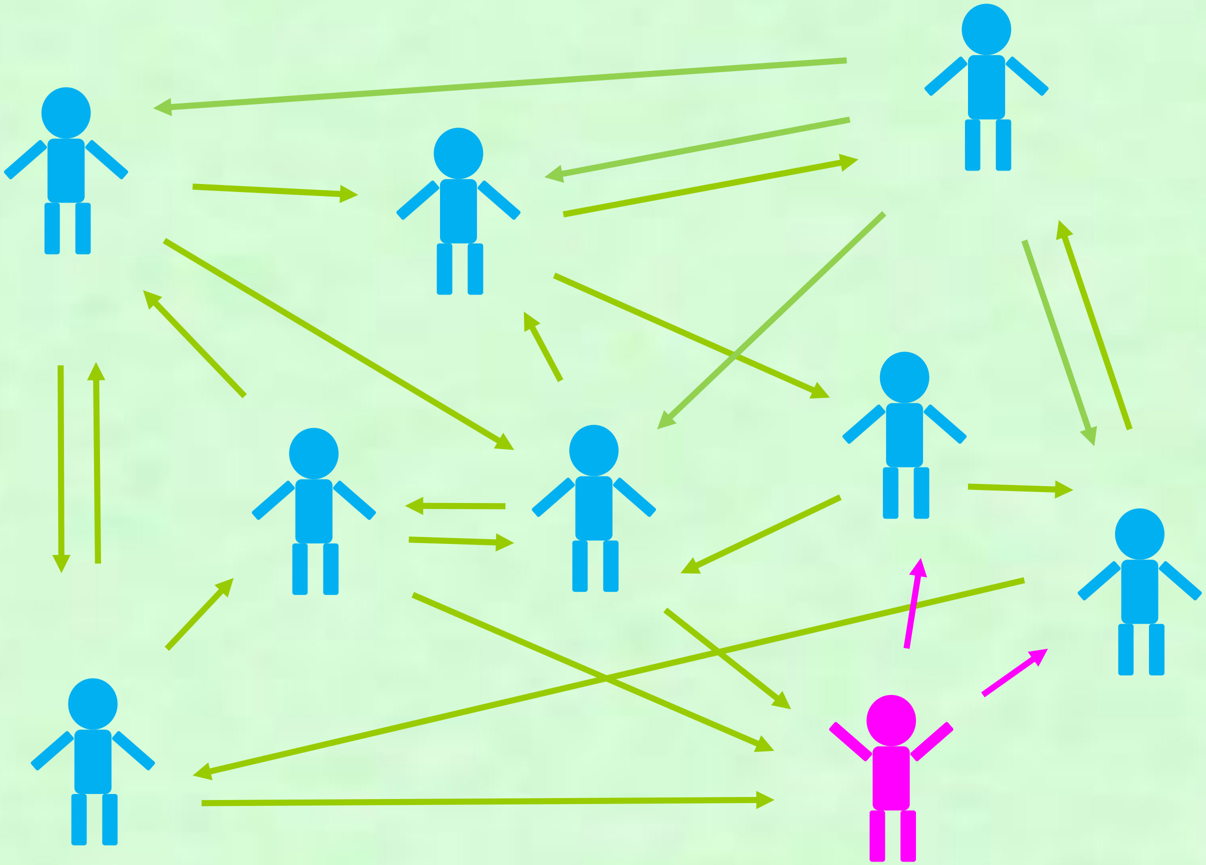

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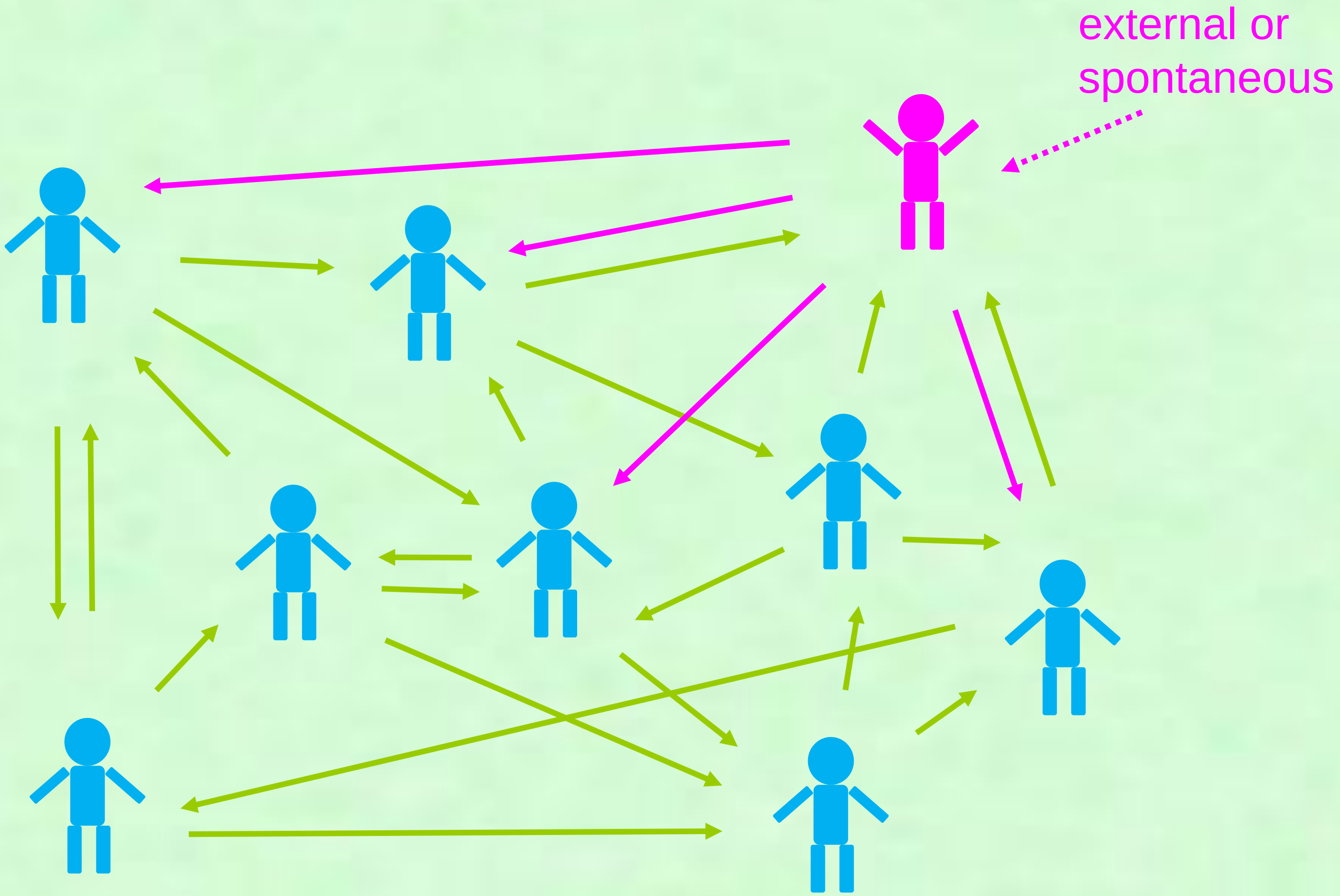

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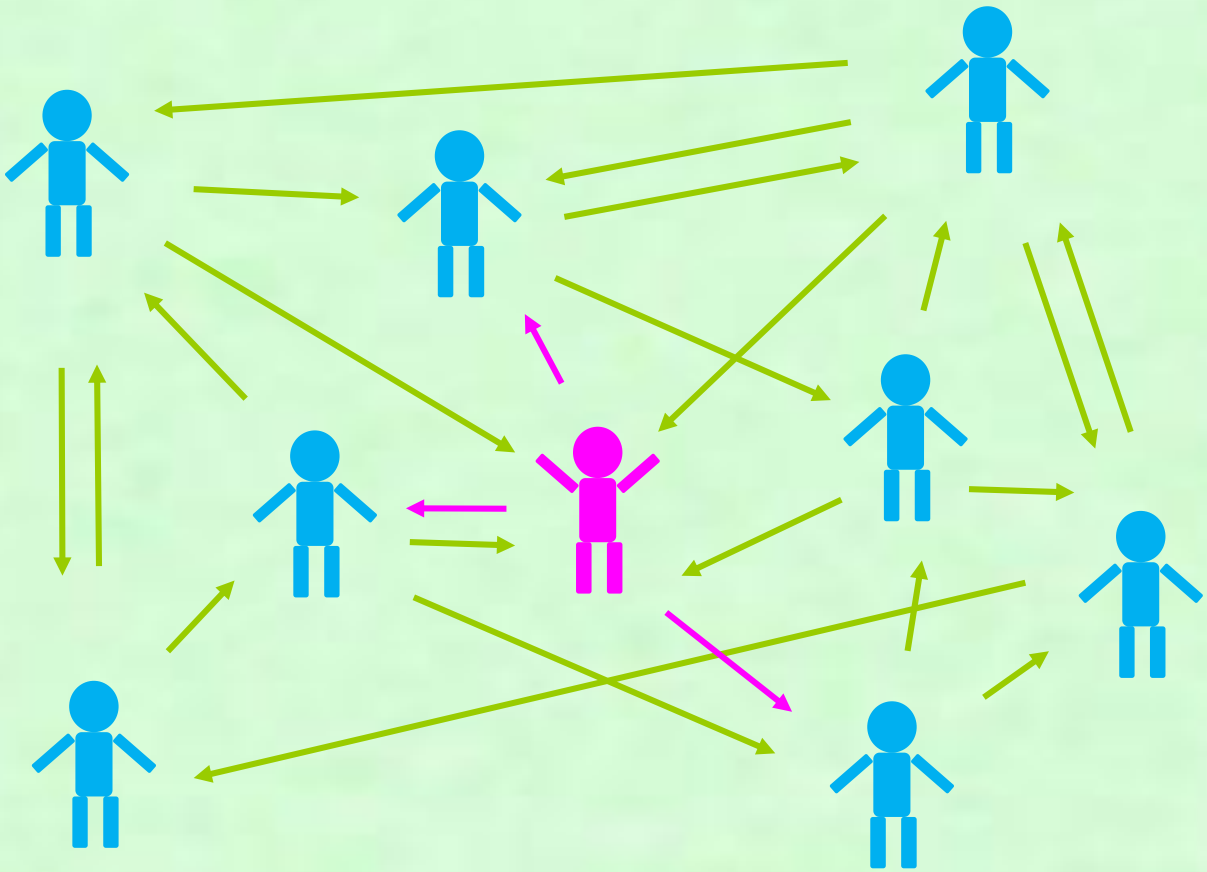
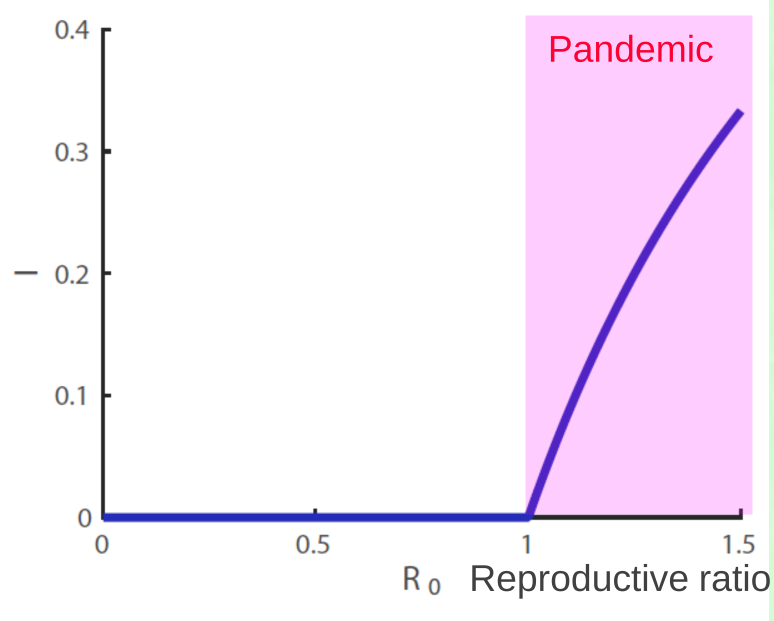


Revise the SIS model

by taking account of external stimulation or spontaneous activity.

The occurrence rate



The (original) SIS model

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$I + S = 1$$

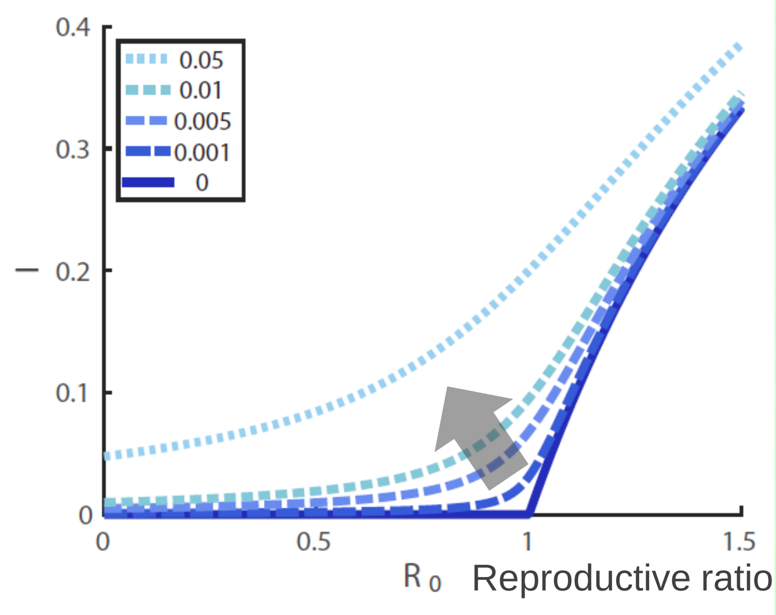
$$R_0 = \beta/\gamma$$

Revise the SIS model

by taking account of external stimulation or spontaneous activity.

>>> Epidemic transition disappears.

The occurrence rate



The (original) SIS model

$$\frac{dI}{dt} = \beta S I - \gamma I$$

with spontaneous activity

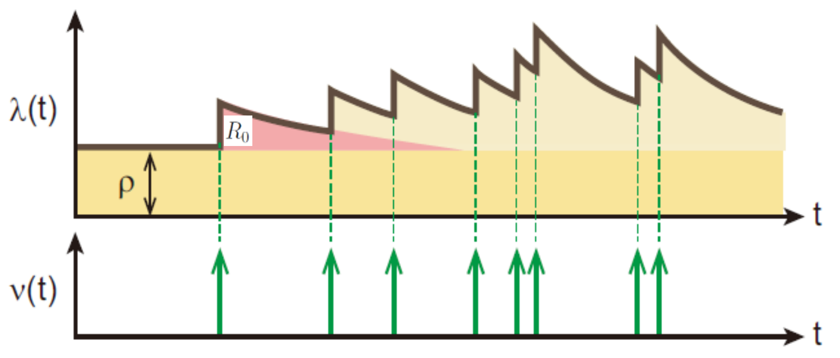
$$\frac{dI}{dt} = \beta S I - \gamma I + \rho$$

$I + S = 1$ $R_0 = \beta / \gamma$

Hawkes process

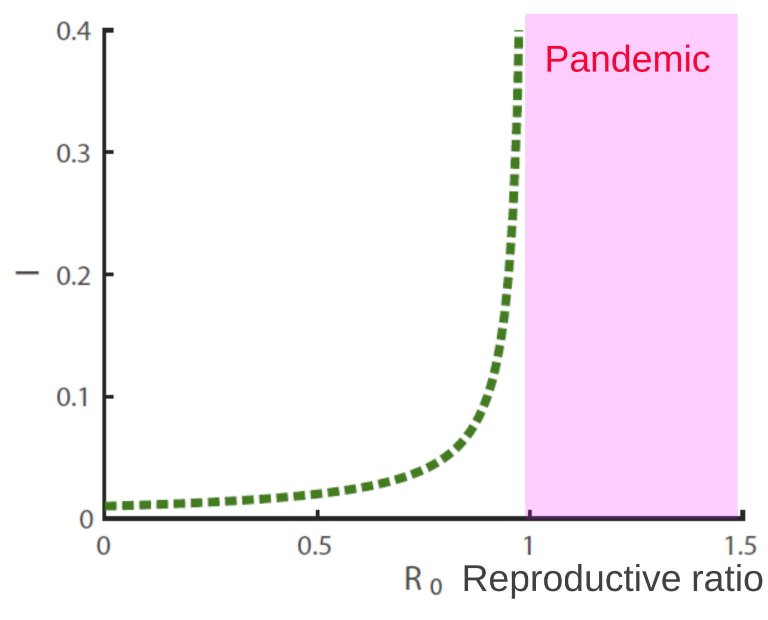
$$\lambda(t) = \rho + R_0 \sum_{t_k} f(t - t_k)$$

$$\int_0^\infty f(t)dt = 1$$



the self-exciting process

The occurrence rate



$$\langle \lambda(t) \rangle = \frac{\rho}{1 - R_0}$$

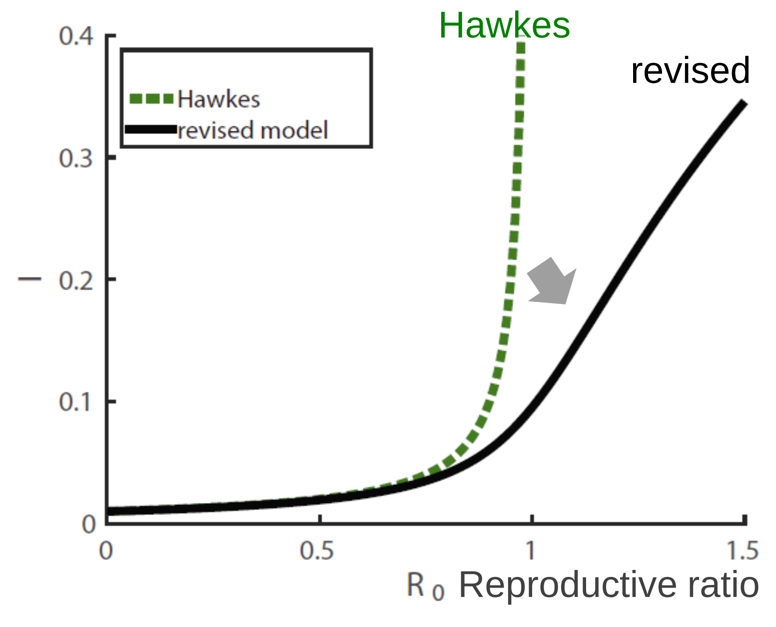
cf: Pernice, Staude, Cardanobile, and Rotter, PLoS Comput Biol 2011

Revise the Hawkes process

by taking account of refractory effect.

>>> pandemic divergence disappears.

The occurrence rate



The Hawkes process

$$\lambda(t) = \rho + R_0 \sum_{t_k} f(t - t_k)$$

+ refractory effect

$$\lambda(t) = \rho + \left(1 - \frac{\lambda(t)}{\gamma}\right) R_0 \sum_{t_k} f(t - t_k)$$

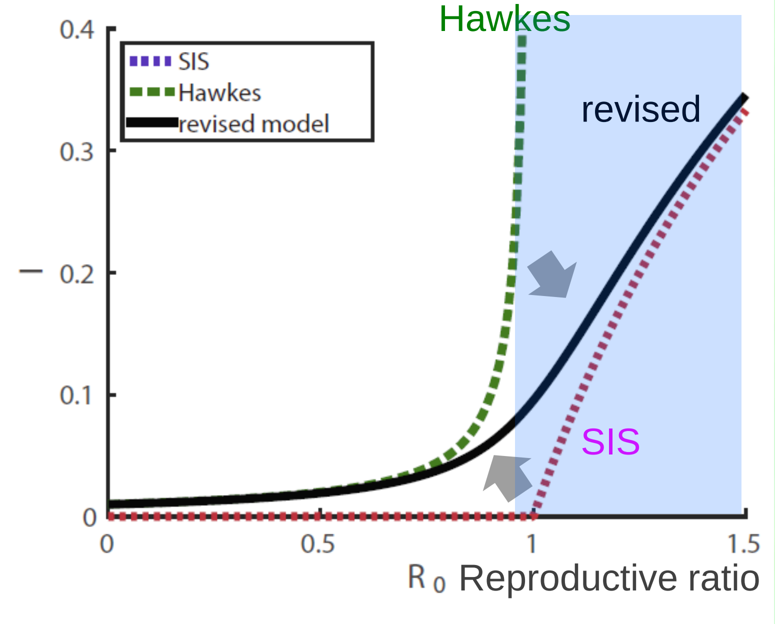
$$I \leftrightarrow \frac{\langle \lambda(t) \rangle}{\gamma}$$

Hawkes and SIS revised

Hawkes process + refractory effect = SIS model + spontaneous activity

$$\lambda(t) = \rho + \left(1 - \frac{\lambda(t)}{\gamma}\right) R_0 \sum_{t_k} f(t - t_k) \quad \Leftrightarrow \quad \frac{dI}{dt} = \beta S I - \gamma I + \rho$$

The occurrence rate

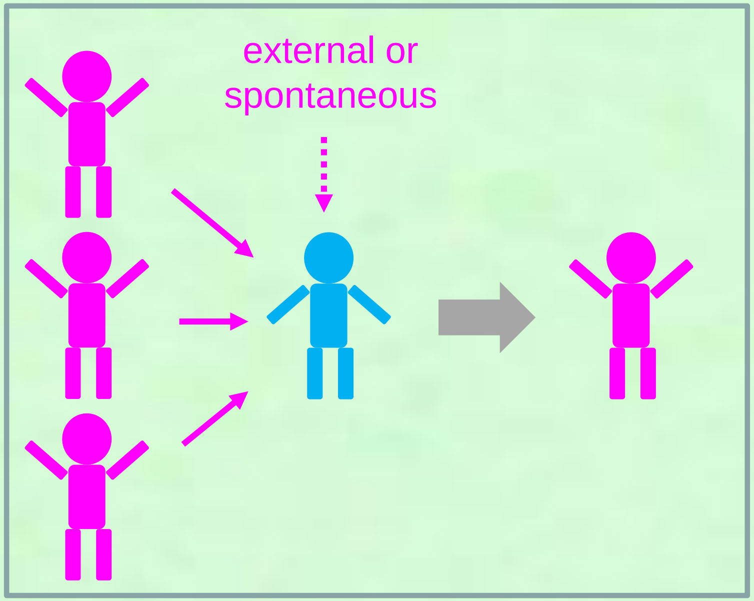


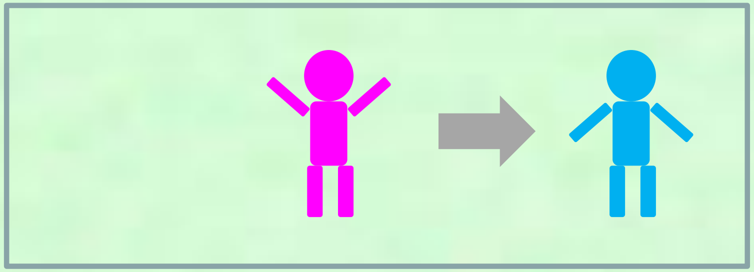
Epidemic transition disappears.

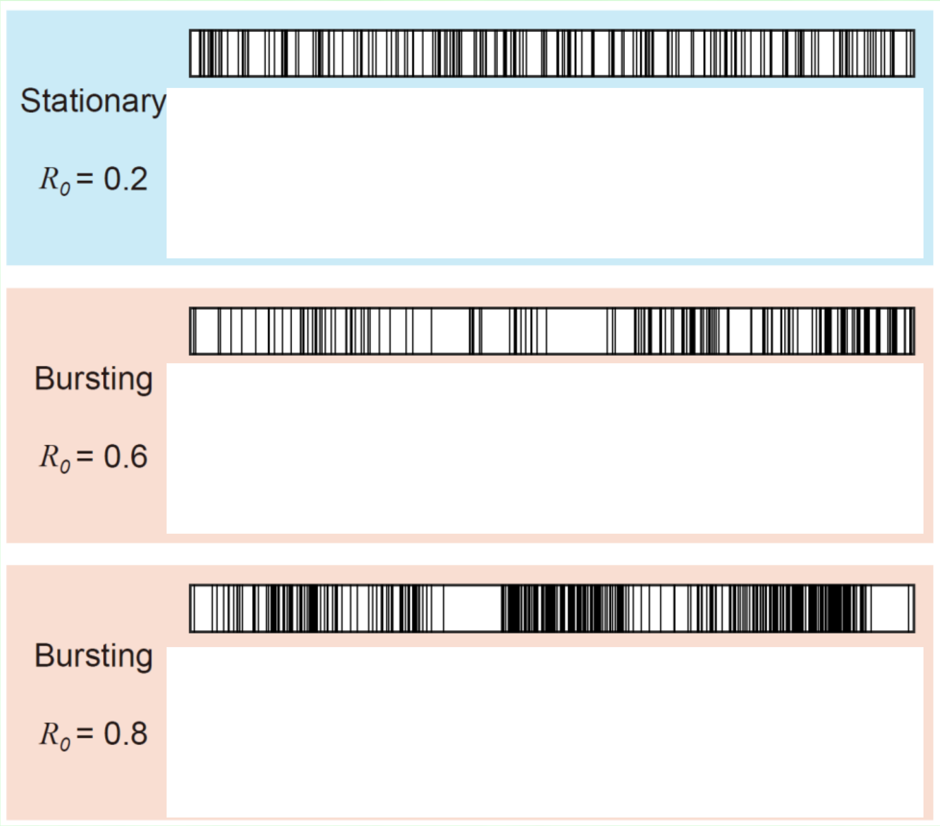
Bursting


Onaga

Elementary process for the SIS model.





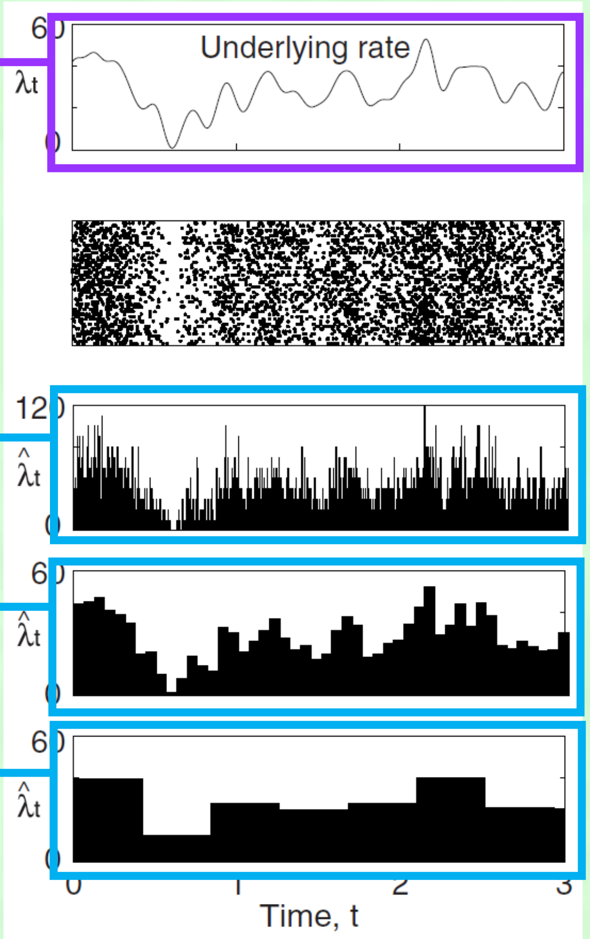


Estimating the rate

Shimazaki

Mean **I**ntegrated **S**quared **E**rror

$$\mathrm{MISE} \equiv \frac{1}{T}\int_0^T E\left(\hat{\lambda}_t - \lambda_t\right)^2 dt$$



rate

spikes

histogram 1

histogram 2

histogram 3

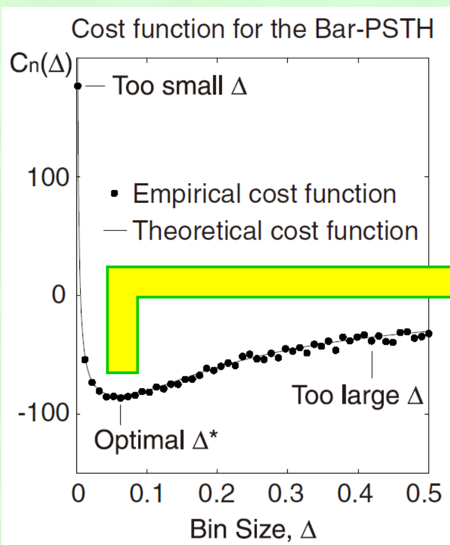
Shimazaki & Shinomoto, Neural Computation 2007. Cf., Rudemo 1982.

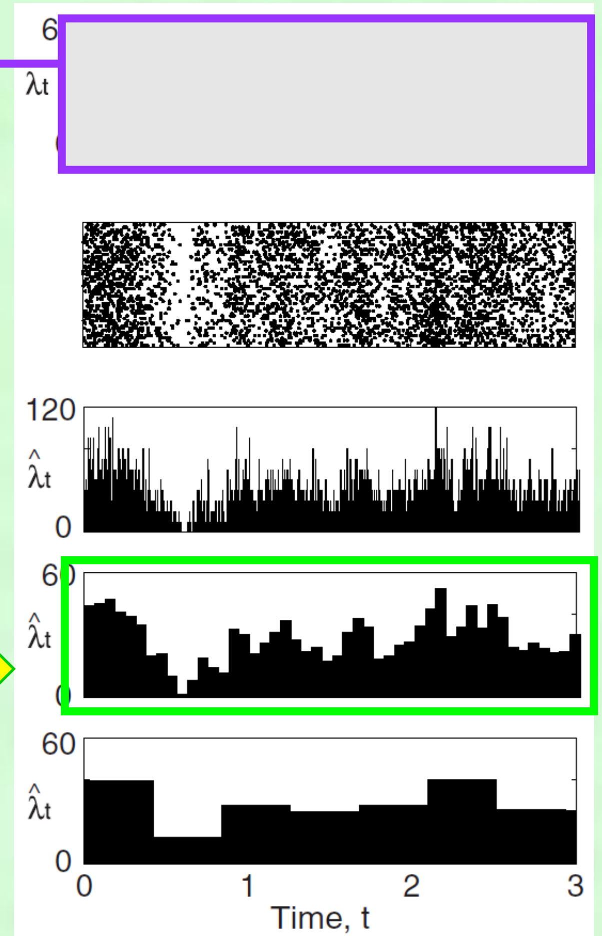
Estimating the rate

Mean **I**ntegrated **S**quared **E**rror

$$\mathrm{MISE} \equiv \frac{1}{T}\int_0^T E\left(\hat{\lambda}_t - \square\right)^2 dt$$

rigorous inference





rate

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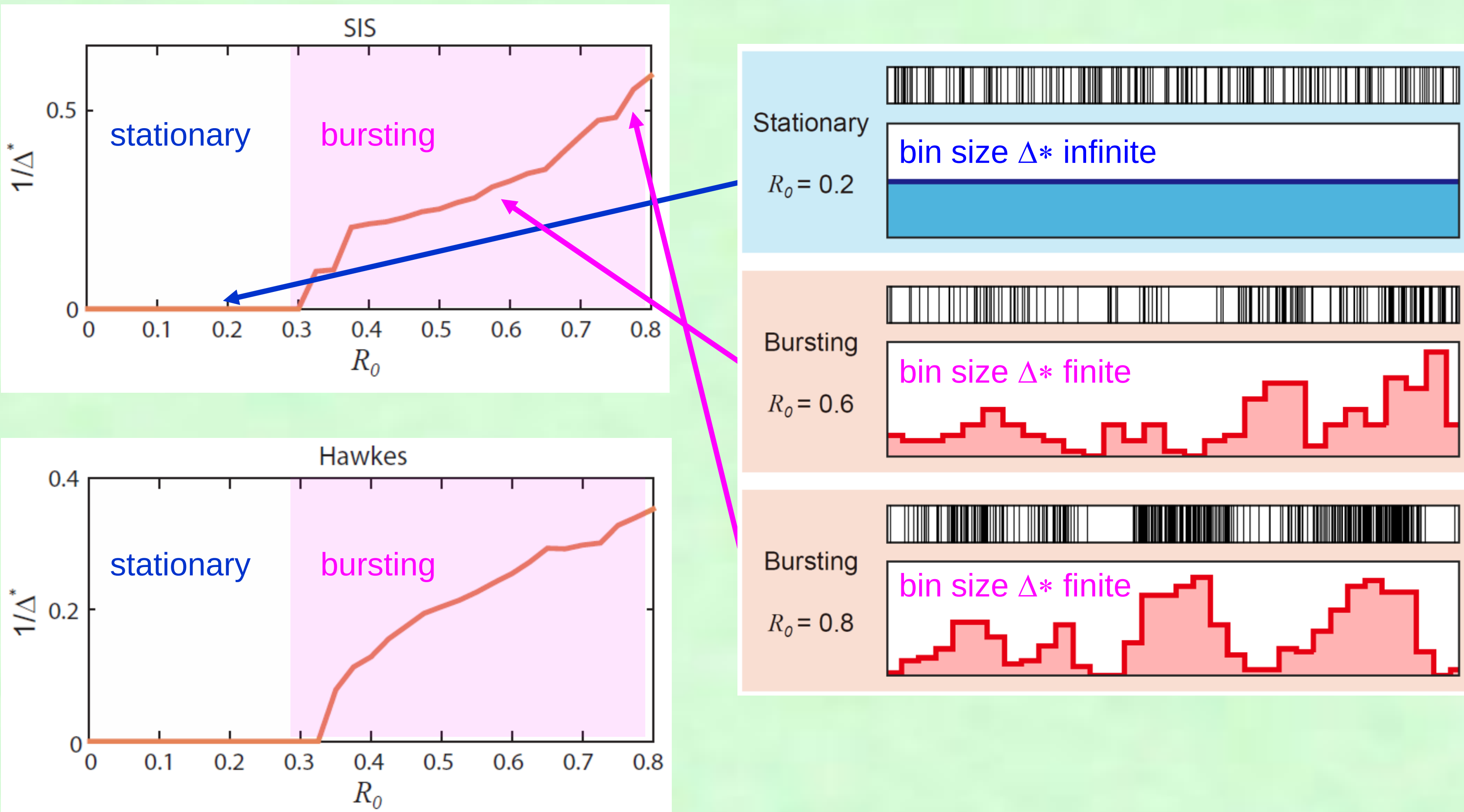
Shimazaki & Shinomoto, Neural Computation 2007. Cf., Rudemo 1982.

Fluctuation in the occurrence

Rate fluctuation or burst of events suggested by a finite optimal bin size $\Delta*$



Hawkes process

Onaga

Hawkes process:

$$\lambda(t) = \rho + R_0 \sum_{t_k} f(t - t_k)$$

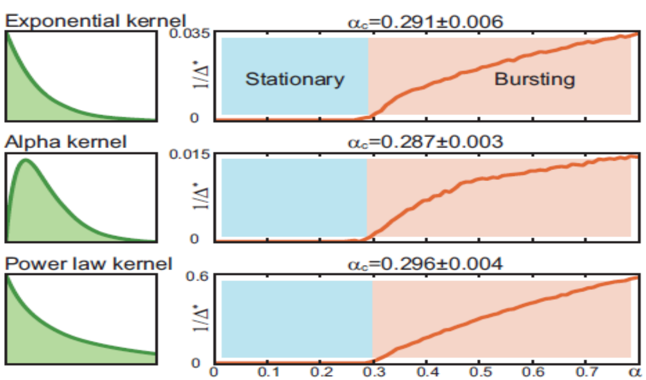
$$\int_0^\infty f(t)\,dt = 1$$

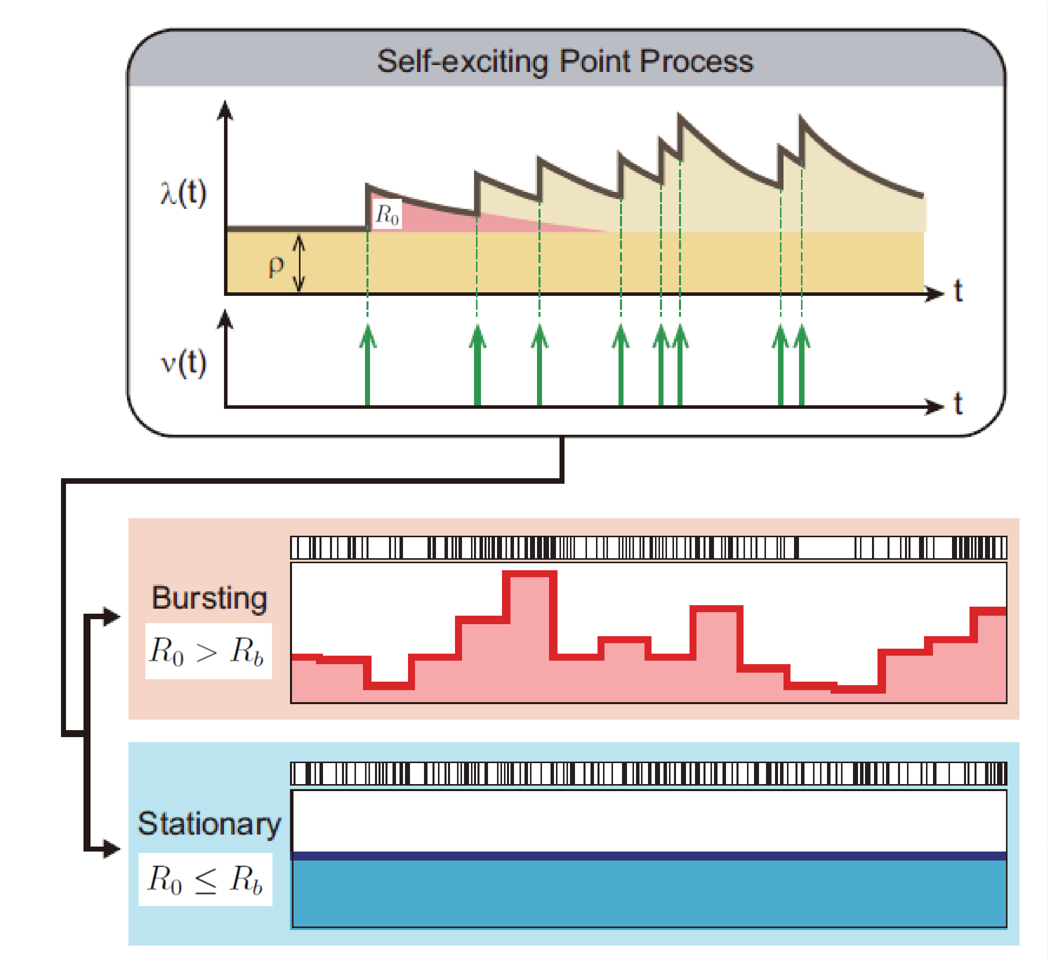
detectability condition

$$\frac{1}{\langle \lambda \rangle} \int_{-\infty}^{\infty} \langle \delta\lambda(t+s)\delta\lambda(t) \rangle \, ds > 1$$

critical excitability

$$R_b = 1 - 1/\sqrt{2} \approx 0.2929$$





Onaga & Shinomoto, Phys Rev E 2014

Hawkes process


Onaga

Hawkes process:

$$\lambda(t) = \rho + \alpha \sum_k f(t - t_k)$$

$R_0 \Rightarrow \alpha$

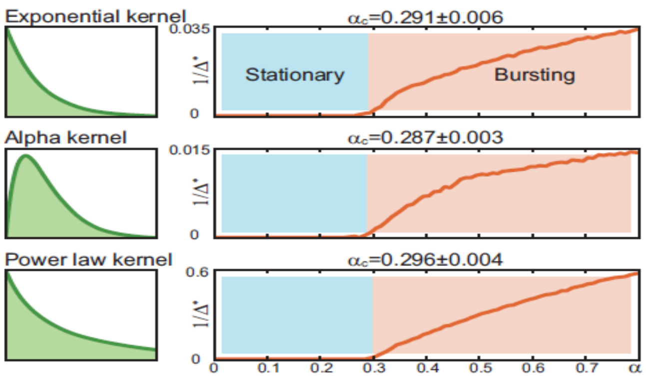
$$\int_0^\infty f(t)\,dt = 1$$

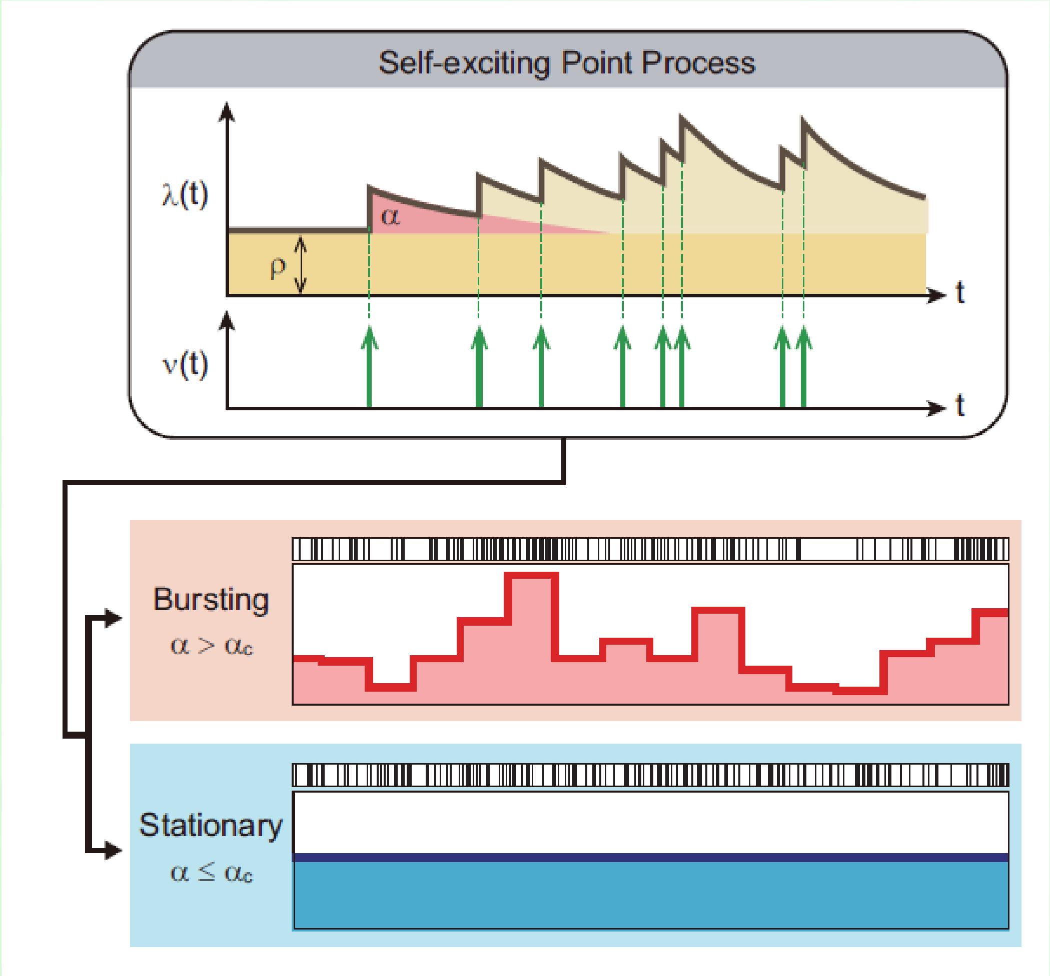
detectability condition

$$\frac{1}{\langle\lambda\rangle}\int_{-\infty}^{\infty} \langle \delta\lambda(t+s)\delta\lambda(t)\rangle \, ds > 1$$

critical excitability

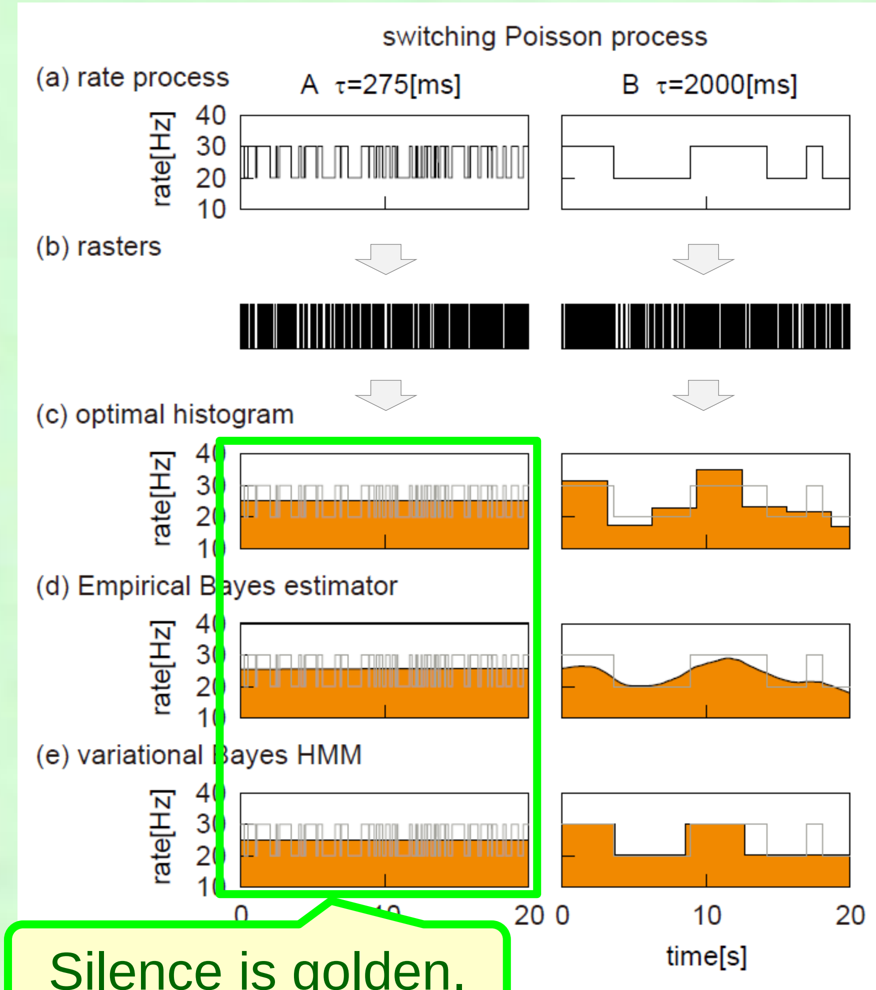
$$\alpha_c = 1 - 1/\sqrt{2} \approx 0.2929$$





Onaga & Shinomoto, Phys Rev E 2014

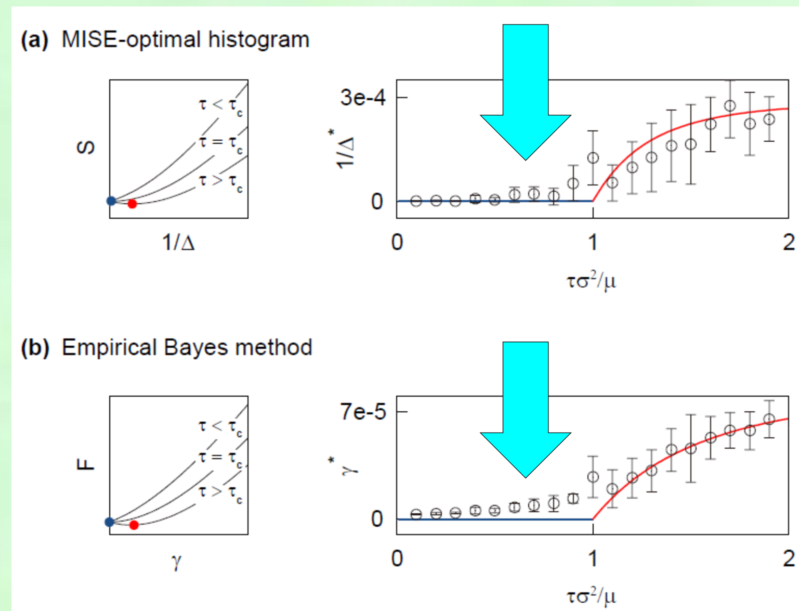
Limit of detecting fluctuation



Silence is golden.

Rate fluctuation is undetectable, if

$$C = \frac{1}{\langle \lambda \rangle} \int_{-\infty}^{\infty} dt \langle \delta\lambda(t) \delta\lambda(0) \rangle \leq 1$$



Koyama Shimokawa & Shinomoto, J Phys A 2004; 2007; Shintani & Shinomoto, Phys Rev E 2012

Optimal histogram binsize

$$S(\Delta) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \langle (\lambda(t) - \hat{\lambda}_\Delta(t))^2 \rangle dt$$

binsize underlying rate histogram

Mean **I**ntegrated **S**quared **E**rror

$$S(\Delta) = \left\langle \frac{1}{\Delta}\int_0^\Delta \left[\lambda^2(t) - \frac{2K}{\Delta}\lambda(t)\right] dt + \frac{K^2}{\Delta^2} \right\rangle$$

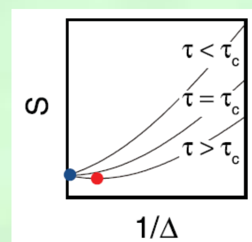
K: spike count

Δ: binsize

Poisson: $E[K^2] = E[K]^2 + E[K]$

$$S(\Delta) = \phi(0) + \frac{\langle \lambda \rangle}{\Delta} - \frac{1}{\Delta^2}\int_0^\Delta dt \int_{-t}^{t} \phi(s)\, ds$$

$$\phi(s) \equiv \langle \delta\lambda(t+s)\delta\lambda(t) \rangle$$



Finite binsize:

$$\left.\frac{dS}{d(1/\Delta)}\right|_{\Delta=\infty} < 0$$

>>> Rate fluctuation is detectable, if

$$\frac{1}{\langle \lambda \rangle}\int_{-\infty}^{\infty} \langle \delta\lambda(t+s)\delta\lambda(t) \rangle ds > 1$$

Koyama Shimokawa & Shinomoto, J Phys A 2004; 2007; Shintani & Shinomoto, Phys Rev E 2012

Hawkes process

Onaga

$$\lambda(t) = \rho + \alpha \sum_k f(t - t_k)$$ >>> correlation: $$\phi(s) \equiv \langle \delta\lambda(t+s)\delta\lambda(t)\rangle$$

Hawkes, Biometrica 1971

>>>>>> $$\tilde{\phi}_\omega = \frac{\alpha \tilde{f}_\omega + \alpha \tilde{f}_{-\omega} - \alpha^2 \tilde{f}_\omega \tilde{f}_{-\omega}}{(1 - \alpha \tilde{f}_\omega)(1 - \alpha \tilde{f}_{-\omega})} \langle \lambda \rangle$$

Our detectability condition:

$$\frac{1}{\langle\lambda\rangle} \int_{-\infty}^{\infty} \langle \delta\lambda(t+s)\delta\lambda(t)\rangle \, ds = \frac{2\alpha - \alpha^2}{(\alpha - 1)^2} = 1$$ <<< $$\frac{1}{\langle\lambda\rangle} \int_{-\infty}^{\infty} \langle \delta\lambda(t+s)\delta\lambda(t)\rangle ds > 1$$

$$\alpha_c = 1 - 1/\sqrt{2} \approx 0.2929.$$ >>> independent of temporal excitation profile

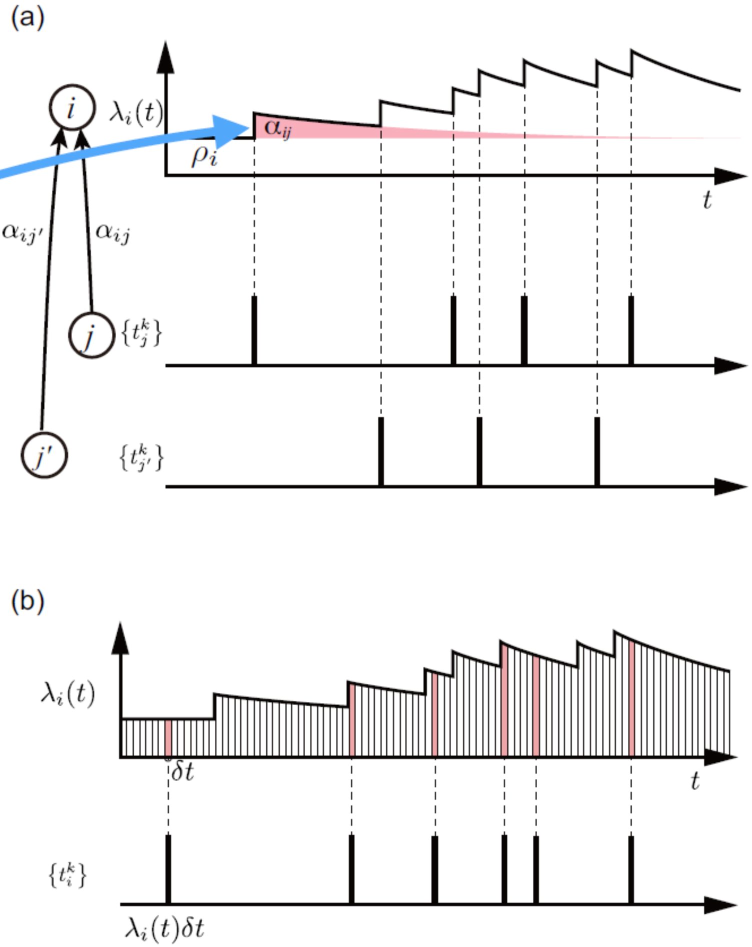
Onaga & Shinomoto, Phys Rev E 2014

Multivariate Hawkes process

Occurrence rate:

$$\lambda_i(t) = \rho_i + \sum_{j=1}^{N} \alpha_{ij} \sum_k f(t - t_j^k)$$

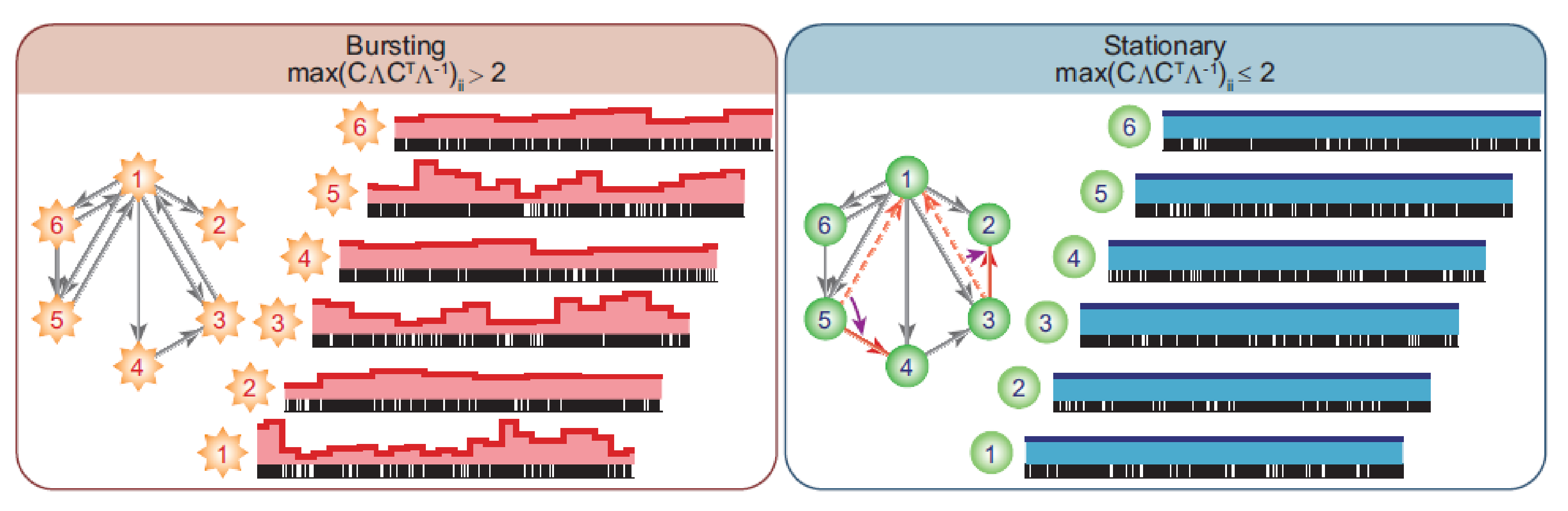
$$\int_0^\infty f(t)dt = 1$$



Multivariate Hawkes process


Onaga

multi-dimensional Hawkes process: $\lambda_i(t) = \rho_i + \sum_{j=1}^{N} \alpha_{ij} \int_{-\infty}^{\infty} f(t-s)\nu_j(s)ds$



bursts of event occurrences take place, if

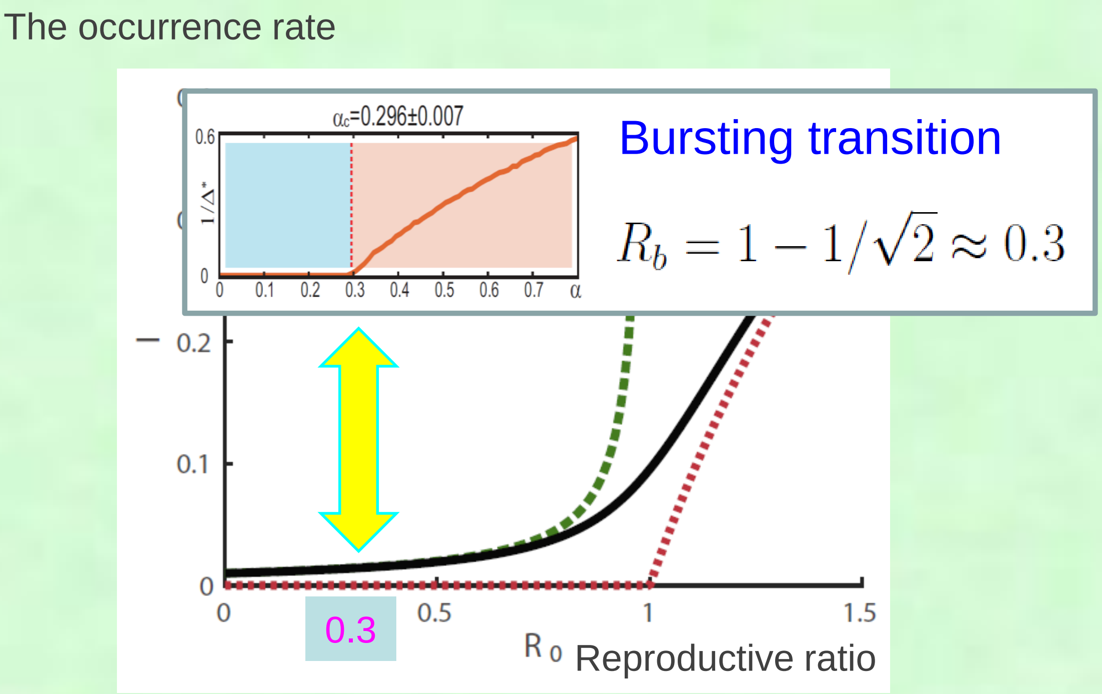
$\max_i \left(C\Lambda C^T \Lambda^{-1}\right)_{ii} > 2$, where $C \equiv \sum_{n=0}^{\infty} \alpha^n = (I-\alpha)^{-1}$ and $\Lambda = \mathrm{diag}(\langle\lambda\rangle)$

Onaga & Shinomoto, Phys Rev E 2014

Conclusion

1. Epidemic transition disappears if there is spontaneous activity.

2. Bursting transition occurs at $R_b = 1 - 1/\sqrt{2} \approx 0.3$ (<<1).



Ongoing project: Construct a method for taming bursts in tweets.

Thank you

You are (individually) welcome to Kyoto (though I cannot organize workshops).