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A permutation Unitary Events method

Mélisande ALBERT

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Multiple independence tests for point processes: a permutation Unitary Events approach based on delayed coincidence count

Mélisande ALBERT[1], Yann BOURET[2], Magalie FROMONT[3], Patricia REYNAUD-BOURET[1].

[1]Univ. Nice Sophia Antipolis, LJAD

Université Nice Sophia Antipolis

[2]Univ. Nice Sophia Antipolis, LPMC

[3]Univ. Européenne de Bretagne, IRMAR

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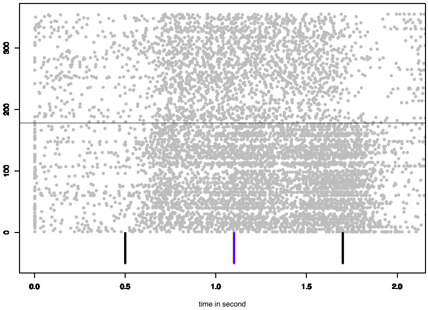

Global problematic

Synchrony detection?

Notion of coincidence

Neurons fire nearly at the same time

Model based methods:

- *Unitary Events methods* [Grün (1996), Grün, et al. (1999) or Tuleau-Malot, et al. (2014)].
- *Smoothed JPSTH methods* [Ventura et al. (2005)].

Surrogate data methods:

- *Across time* such as dithering approaches, [Louis et al. (2010)].
- *Across trials* such as the Trial Shuffling [Pipa et al. (2003)].

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First step

Test independence on a time window.

Statistical Modeling for Neuronal Activity

Spike trains modeled by point processes on an interval (say $[0,1]$).

Definition

Point process on $[0,1]$ = random countable set of points in $[0,1]$.
$\mathcal{X} :=$ the set of almost surely finite point processes on $[0,1]$.

Example : homogeneous Poisson process with intensity $\lambda > 0$.

$\hookrightarrow$ In the following, no model assumption for the point processes.

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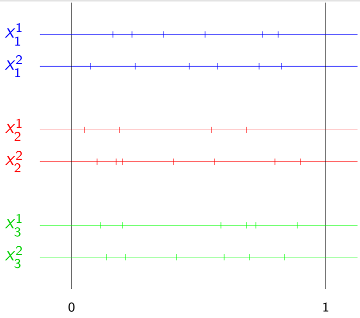
Observation: $\mathbb{X}_n = (X_1, \ldots, X_n)$,

where n = number of trials, and $X_i = (X_i^1, X_i^2)$ i.i.d. in $\mathcal{X}^2$.



Aim:

Test $(\mathcal{H}_0): \; X^1 \perp\!\!\!\perp X^2$ against $(\mathcal{H}_1): \; X^1 \not\perp\!\!\!\perp X^2$.

Denote $\mathbb{X}_n^{\perp\!\!\!\perp}$ an sample as above satisfying $(\mathcal{H}_0)$.

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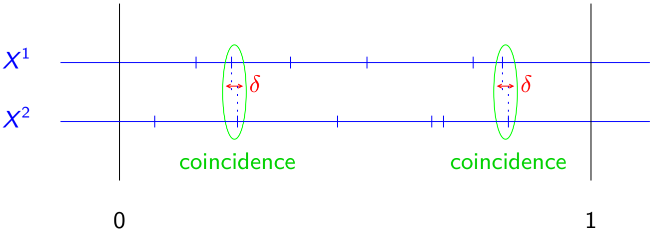
Notion of (delayed) coincidence for point processes

φ_δ^{coinc} counts the number of coincidences between two point processes:

$$\varphi_\delta^{coinc}\left(X^1, X^2\right) = \sum_{T \in X^1} \sum_{S \in X^2} \mathbb{1}_{\{|T-S| \leq \delta\}}.$$



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Test statistic based on the total number of delayed coincidences

$$C^{obs} = C(\mathbb{X}_n) = \sum_{i=1}^{n} \varphi_\delta^{coinc}\left(X_i^1, X_i^2\right).$$

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General idea

Reject independence when there are too many (resp. too few) coincidences compared to what is expected under independence.

How to recreate the distribution under independence?

Construct a new sample $\tilde{\mathbb{X}}_n$ from the original one, i.e. $\mathbb{X}_n$, such that

$$\mathcal{L}\left(C\left(\tilde{\mathbb{X}}_n\right)\middle|\mathbb{X}_n\right) \quad \approx \quad \mathcal{L}\left(C\left(\mathbb{X}_n^{\perp\!\!\!\perp}\right)\right),$$

whether $\mathbb{X}_n$ satisfies independence or not.

The different resampling approaches

Trial Shuffling, Full Bootstrap and Permutation

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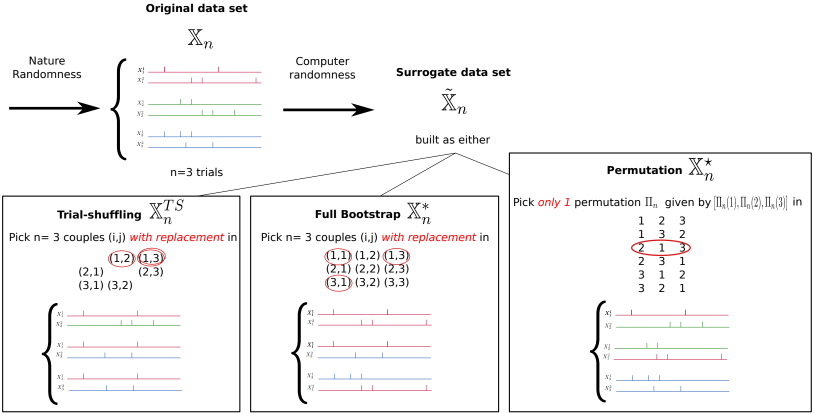

Unconditional distribution: *all* possible *choices* of both *Nature and Computer* randomness

Conditional distribution: *1 fixed* original data set (*Nature* randomness), *all* possible *choices* of *Computer* randomness

Conditional distributions of the number of coincidences

How do they perform?

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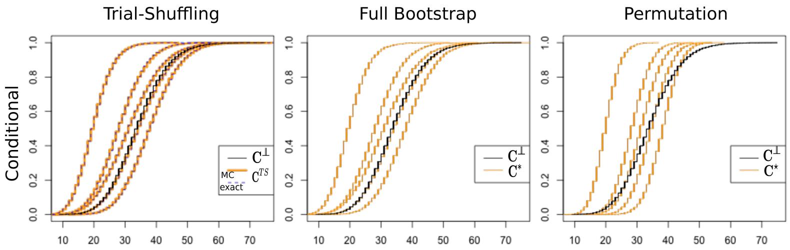
$$\mathcal{L}\left(C\left(\tilde{\mathbb{X}}_n\right)\middle|\mathbb{X}_n\right) \approx \mathcal{L}\left(C\left(\mathbb{X}_n^{\perp\!\!\perp}\right)\right)?$$



Centering issue !!!

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In view of the statistical literature, it is not possible to estimate $\mathcal{L}\left(C\left(\mathbb{X}_n^{\perp\!\!\perp}\right)\right)$ directly, BUT,

$$\mathcal{L}\Big(C\big(\tilde{\mathbb{X}}_n\big) - \mathbb{E}\Big[C\big(\tilde{\mathbb{X}}_n\big)\,|\mathbb{X}_n\Big]\Big|\mathbb{X}_n\Big) \quad \approx \quad \mathcal{L}\Big(C\big(\mathbb{X}_n^{\perp\!\!\perp}\big) - \mathbb{E}\Big[C\big(\mathbb{X}_n^{\perp\!\!\perp}\big)\Big]\Big).$$

YET, $\mathbb{E}\Big[C\big(\mathbb{X}_n^{\perp\!\!\perp}\big)\Big]$ is unknown...

Centering trick

Let $\hat{C}_0(\mathbb{X}_n) = \dfrac{1}{n-1}\sum_{i\neq j}\varphi_\delta^{coinc}(X_i^1, X_j^2)$, s.t. $\mathbb{E}\Big[\hat{C}_0(\mathbb{X}_n)\Big] = \mathbb{E}\Big[C\big(\mathbb{X}_n^{\perp\!\!\perp}\big)\Big]$,

and let

$$U(\mathbb{X}_n) = C(\mathbb{X}_n) - \hat{C}_0(\mathbb{X}_n).$$

Then

$$\mathcal{L}\Big(U\big(\tilde{\mathbb{X}}_n\big) - \mathbb{E}\Big[U\big(\tilde{\mathbb{X}}_n\big)\,|\mathbb{X}_n\Big]\Big|\mathbb{X}_n\Big) \quad \approx \quad \mathcal{L}\Big(U\big(\mathbb{X}_n^{\perp\!\!\perp}\big)\Big).$$

with

$$\mathbb{E}\Big[U\big(\mathbb{X}_n^{TS}\big)\Big|\mathbb{X}_n\Big] = -\frac{U(\mathbb{X}_n)}{n}, \text{ and } \begin{cases} \mathbb{E}[U(\mathbb{X}_n^*)|\mathbb{X}_n] = 0, \\ \mathbb{E}[U(\mathbb{X}_n^\star)|\mathbb{X}_n] = 0. \end{cases}$$

Conditional distributions of the centered number of coincidences

How do they perform?

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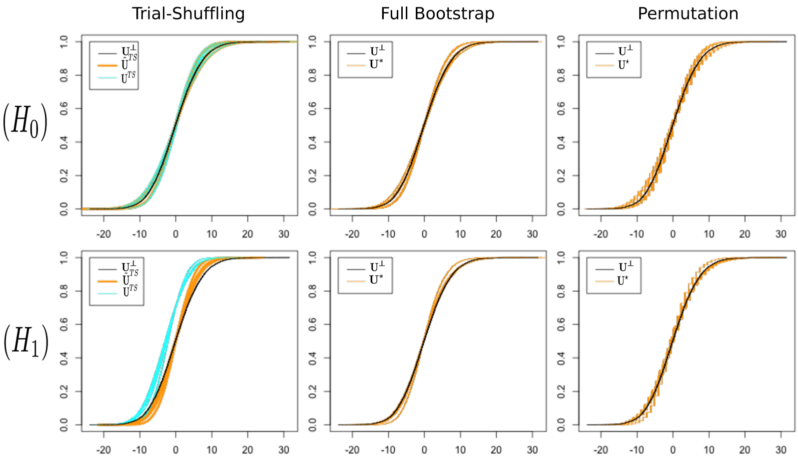
$$\mathcal{L}\left(U\left(\tilde{\mathbb{X}}_n\right) - \mathbb{E}\left[U\left(\tilde{\mathbb{X}}_n\right)\,|\mathbb{X}_n\right]\middle|\mathbb{X}_n\right) \quad \approx \quad \mathcal{L}\left(U\left(\mathbb{X}_n^{\perp\!\!\!\perp}\right)\right)?$$



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$$\mathcal{L}\left(U\left(\tilde{\mathbb{X}}_n\right) - \mathbb{E}\left[U\left(\tilde{\mathbb{X}}_n\right)|\mathbb{X}_n\right]\Big|\mathbb{X}_n\right) \quad \approx \quad \mathcal{L}\left(U\left(\mathbb{X}_n^{\perp\!\!\perp}\right)\right)?$$

Critical value:

$(1-\alpha)$-quantile of $\mathcal{L}\left(U\left(\tilde{\mathbb{X}}_n\right) - \mathbb{E}\left[U\left(\tilde{\mathbb{X}}_n\right)|\mathbb{X}_n\right]\Big|\mathbb{X}_n\right)$,

$\Rightarrow$ with Monte Carlo approximation.

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Trial Shuffling *p*-values with Monte Carlo approximation:

- Simulate B *Trial Shuffling samples* $\mathbb{X}_n^{TS,1}, \dots, \mathbb{X}_n^{TS,B}$.
- Compute the centered test statistics:

$$\tilde{U}_b^{TS} = U\left(\mathbb{X}_n^{TS,b}\right) + \frac{U(\mathbb{X}_n)}{n}.$$

- Define the p-value by

$$\hat{\alpha}^{TS}(\mathbb{X}_n) = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}_{\left\{\tilde{U}_b^{TS} \geq U(\mathbb{X}_n)\right\}},$$

and reject independence if it is smaller than α.

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Full Bootstrap p-values with Monte Carlo approximation:

- Simulate B *Full Bootstrap samples* $\mathbb{X}_n^{*,1}, \ldots, \mathbb{X}_n^{*,B}$.
- Compute the centered test statistics:

$$U_b^* = U\left(\mathbb{X}_n^{*,b}\right).$$

- Define the p-value by

$$\hat{\alpha}^*(\mathbb{X}_n) = \frac{1}{B} \sum_{b=1}^{B} \mathbb{1}_{\left\{U_b^* \geq U(\mathbb{X}_n)\right\}},$$

and reject independence if it is smaller than α.

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Permutation p-values with Monte Carlo approximation:

- Simulate B *permuted samples* $\mathbb{X}_n^{\star,1}, \ldots, \mathbb{X}_n^{\star,B}$.
- Compute the centered test statistics:

$$U_b^\star = U\left(\mathbb{X}_n^{\star,b}\right), \quad \text{and} \quad U_{B+1}^\star = U(\mathbb{X}_n)\,.$$

- Define the p-value by [Romano and Wolf (2005)]

$$\hat{\alpha}^\star(\mathbb{X}_n) = \frac{1}{B+1}\sum_{b=1}^{B+1} \mathbb{1}_{\left\{U_b^\star \geq U(\mathbb{X}_n)\right\}},$$

and reject independence if it is smaller than α.

Then, thanks to [Romano and Wolf (2005)],

$$\mathbb{P}_{(\mathcal{H}_0)}\left(\hat{\alpha}^\star(\mathbb{X}_n) \leq \alpha\right) \leq \alpha,$$

(only for the permutation approach).

Test of independence
How do they perform?

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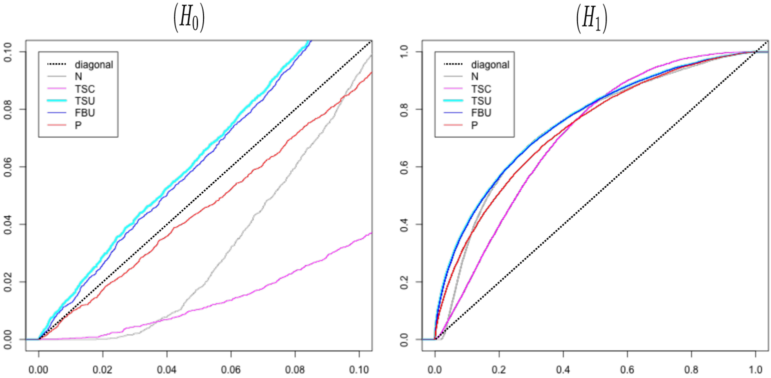
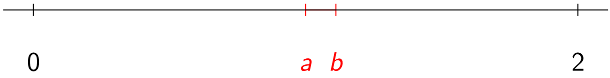


Initial problematic

Detect the synchronizations.

Idea : simultaneously test independence on sliding time windows $[a_k, b_k]$,



$$(\mathcal{H}_{0,k}) : X^1 \perp\!\!\!\perp X^2 \text{ on } [a_k, b_k] \quad (\mathcal{H}_{0,k}) : X^1 \not\perp\!\!\!\perp X^2 \text{ on } [a_k, b_k].$$

Aim:

Control the m tests at a global level α.

⚠ The errors accumulate !

↪ Benjamini and Hochberg multiple testing procedure to control the False Discovery Rate (1995).

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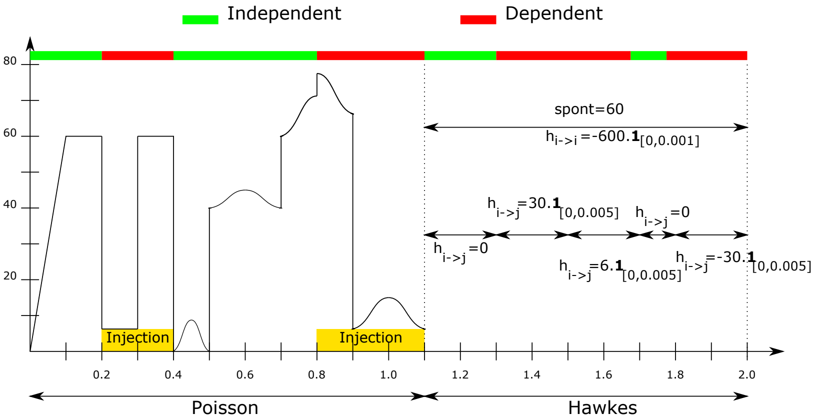

A: Description of Experiment 1

- $n = 50$,
- δ varies in $\{0.001, 0.002, \ldots, 0.04\}$,
- $B = 10000$ steps in the Monte Carlo approximation of the quantiles,
- $m = 191$ simultaneous tests.

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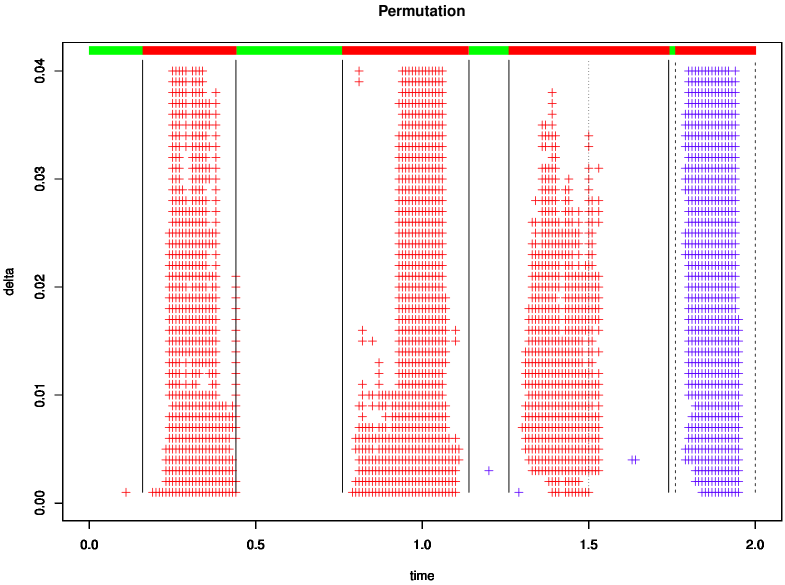

Work still in progress and Perspectives

Conclusions and perspectives

- Centering when applying bootstrap-based methods.
- The permutation approach is more reliable.
- Asymptotic theoretical results for full bootstrap and permutation approaches.
- Non-asymptotic results for the permutation tests?
- Choice of δ for the notion of coincidences?
- More than two neurons?

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