Adaptive Waveform Learning

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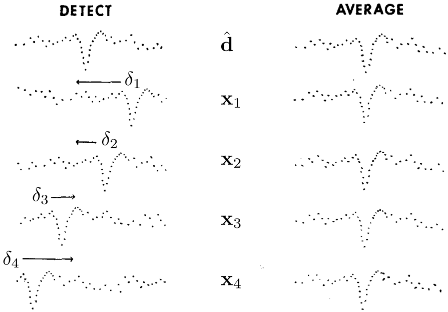
Latency (jitter) compensation

Neuroscience experiments: measuring response to stimulus
Low SNR $\rightarrow$ use of multitrial data
Variability in latency must be compensated.

Woody's method (1967)



$$\mathbf{x}_m = \mathbf{d}(\cdot - \delta_m) + \varepsilon_m$$

$\mathbf{x}_m$	trial, epoch (or channel)
$\mathbf{d}$	signal of interest
δ_m	latency
ε_m	noise

Drawback: only one constant signal component

Linear Decompositions

$$\mathbf{x}_m = \sum_k a_{km} \mathbf{d}_k + \varepsilon_m$$

Principal Components Analysis (PCA):

- Maximizes variance
- Orthogonality between components

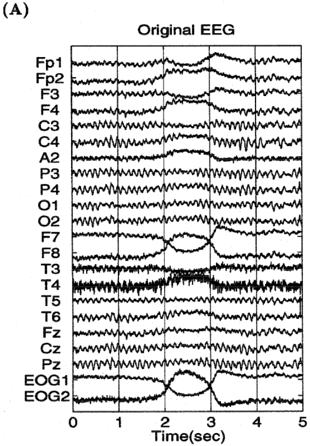
Independent Components Analysis (ICA):

- Statistical independence between components

Dictionary Learning:

- Sparsity of components

Drawback: no explicit modeling of temporal variability

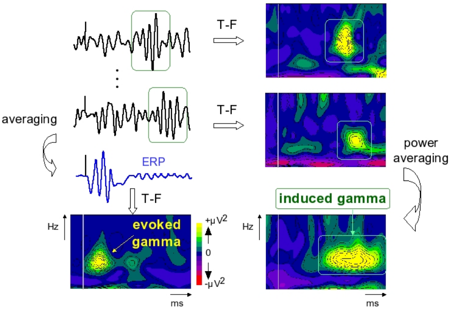


[Jung et al 2000]

Time-Frequency analysis

- Representations in short-time Fourier or wavelet bases
- Time-frequency: meaningful
- Complex coeffs: phase / amplitude separation

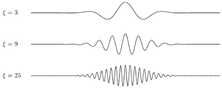
Drawback: smearing of activity in average map due to latency jitter



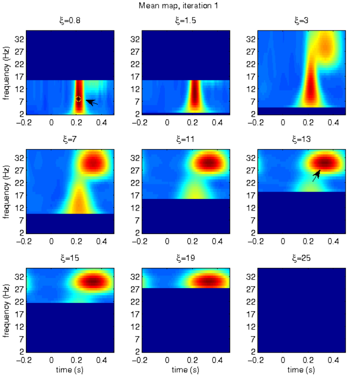
Bertrand et al. 1996

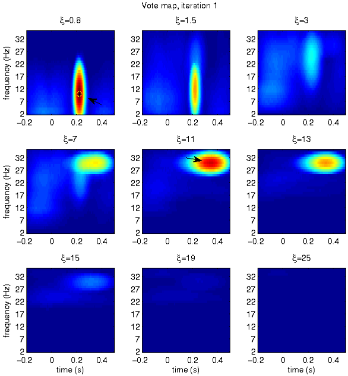
Average map vs. vote map

Large dictionary with 3 dimensions:
time - frequency - number of periods ξ



Example - true signal has $\xi = 11$.





Average map
Global max at $\xi = 13$.

Vote map
Global max at $\xi = 11$: Consensus Atom

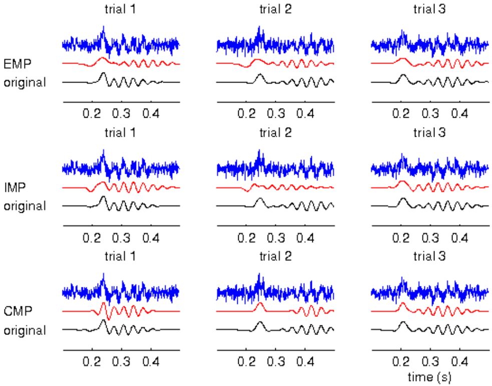
Consensus Matching Pursuit (CMP)

$$\mathbf{x}_m = \sum_k a_{km} \mathbf{d}_{km} + \varepsilon_m$$

At iteration k
for each trial m
find d_{km}

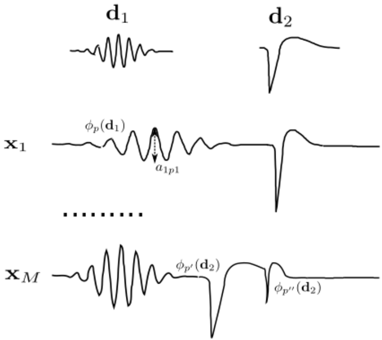
- local maximum of t-f-ξ map
- closest to Consensus Atom

Drawback: atoms not physiologically realistic



[Bénar, Papadopoulo, Torrésani, Clerc, 2009]

Towards physiologically realistic atoms



Discrete set of deformations: ϕ_p
including combinations of:

- translations
- dilations ...

Goal: learn the templates $\mathbf{d}_k$ and which of their deformations are active
$\rightarrow$ Adaptive Waveform Learning

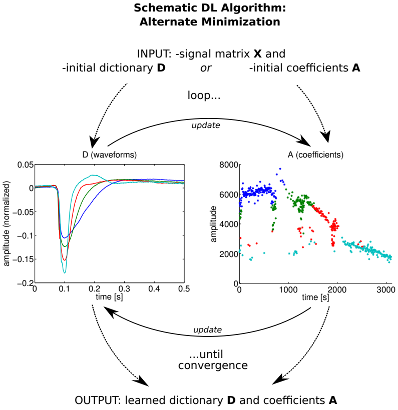
$$\min_{a_{kpm},\, \mathbf{d}_k} \sum_{m=1}^{M} \left(\left\| \mathbf{x}_m - \sum_{k=1}^{K} \sum_{p=-P}^{P} a_{kpm} \phi_p(\mathbf{d}_k) \right\|_2^2 + \lambda \sum_{k=1}^{K} \sum_{p=-P}^{P} |a_{kpm}| \right)$$

[Hitziger et al, Int Conf Learning Repr 2013]

Adaptive Waveform Learning

$$\mathbf{D} = \underset{\mathbf{D}}{\operatorname{argmin}} \min_{\{\mathbf{a}_m\}} \sum_{m=1}^{M} \left(\frac{1}{2} \|\mathbf{x}_m - \mathbf{D}\mathbf{a}_m\|_2^2 + \lambda \|\mathbf{a}_m\|_1 \right)$$

Dictionary $\mathbf{D}$:
templates with all their deformations
Coefficients $\mathbf{a}_m$: atom selection



Adaptive Waveform Learning algorithm

Algorithm 1 Generic AWL

Input: $\{\mathbf{x}_m\}$, $\{\phi_p\}$, $\lambda \in \mathbb{R}_0^+$, constraints $\mathcal{C}(\mathbf{a})$.

Initialize waveforms $\mathbf{d}_1, \ldots, \mathbf{d}_K$.
loop
    $\mathbf{a} \leftarrow \text{COEFF\_UPDATE}(\{\mathbf{x}_m\}, \{\phi_p\}, \{\mathbf{d}_k\}, \lambda, \mathcal{C})$.
    if stopping criterion reached: **break**.
    $\{\mathbf{d}_k\} \leftarrow \text{WF\_UPDATE}(\{\mathbf{x}_m\}, \{a_{klm}\}, \{\phi_{klm}\}, \{\mathbf{d}_k\})$.
end loop

Output: $\{a_{klm}\}, \{\phi_{klm}\}, \{\mathbf{d}_k\}$.

Coefficient update

procedure COEFF_UPDATE$(\{\mathbf{x}_m\}, \{\phi_p\}, \{\mathbf{d}_k\}, \lambda)$

  Create a dictionary $\mathbf{D} = \{\mathbf{d}_k^p\}$ with $\mathbf{d}_k^p = \phi_p(\mathbf{d}_k)$.

  for $m = 1$ to M **do**

    Solve through sparse coding:

$$a_{\cdot\cdot m} \leftarrow \underset{\{a_{kpm}\}}{\text{argmin}} \left\| \mathbf{x}_m - \sum_{k,p} a_{kpm} \mathbf{d}_k^p \right\|_2^2 + \lambda \sum_{k,p} |a_{kpm}| ,$$

    s.t. $\mathbf{a} \geq 0$ and

      $\mathcal{C}(\mathbf{a})$.

  end for

end procedure, **return a**

Waveform update

procedure $\text{WF\_UPDATE}\left(\{\mathbf{x}_m\}, \{a_{klm}\}, \{\phi_{klm}\}, \{\mathbf{d}_k\}\right)$

  for $k \in \{1, \ldots, K\}$ **do**

    $r_{km} = \mathbf{x}_m - \sum_{k' \neq k} a_{k'm}\phi_{k'm}(\mathbf{d}_{k'})$.

    $\psi_{klm} = a_{klm}\,\phi_{klm}$.

    $\mathbf{d}_k = \left(\sum_m \psi_{km}^t \psi_{km}\right)^+ \sum_m \psi_{km}^t(r_{km})$

  end for

  for $k = 1$ to K **do**

    Center waveform $\mathbf{d}_k$.

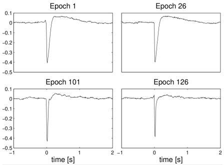
    $\mathbf{d}_k = \frac{\mathbf{d}_k}{\|\mathbf{d}_k\|_2}$.

  end for

end procedure, **return** $\{\mathbf{d}_k\}$

Adaptive Waveform Learning (AWL): epoched

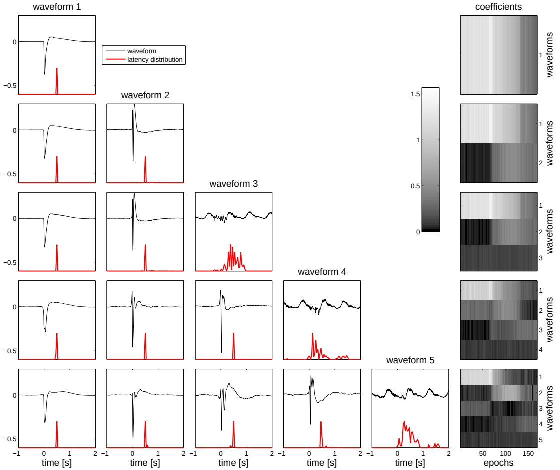
Manual epoching of 169 isolated spikes in a Local Field Potential recording



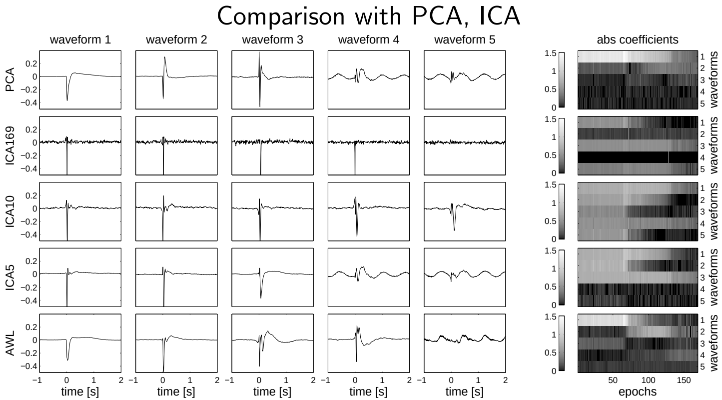
Hierarchical AWL

Results:

- *Oscillatory component*
- *Energy profile of coefficients*

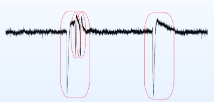


Adaptive Waveform Learning (AWL): epoched

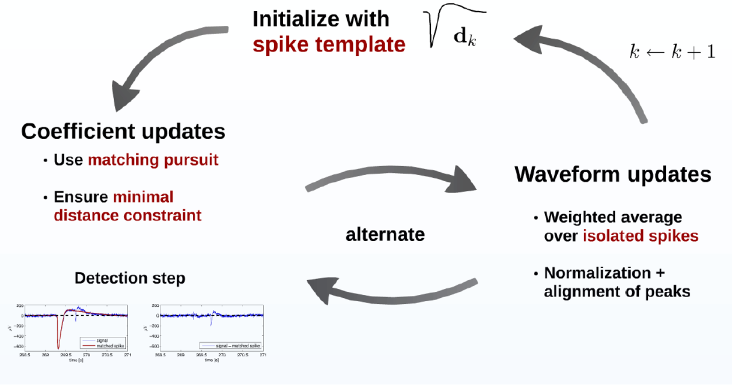


- *PCA/ICA: sinusoidal oscillatory component, separation not proper*
- *PCA: first component overly dominant*

Adaptive Waveform Learning (AWL): continuous



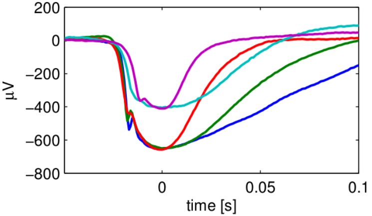
Hierarchical (start $k = 1$)



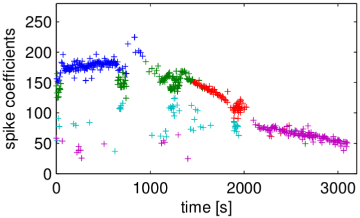
Adaptive Waveform Learning (AWL): continuous

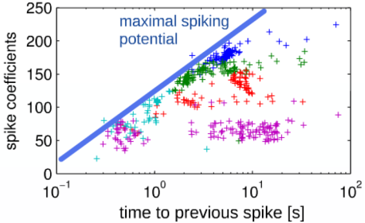
Processing full Local Field Potential recording

Learning 5 waveforms



Spike shapes vary in duration and amplitude





Coefficients are clustered

Inter-spike interval correlates with energy

Adaptive Waveform Learning (AWL): continuous

Neurovascular coupling

- *CBF around 1 Hz (respiration)*
- *Spiking rate matches CBF component*
- *Spikes synchronize, phase-locked to CBF*

Reversed causality: haemodynamics

drives spikes