Hidden context tree modeling of EEG data

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joint work with A. Duarte, R. Fraiman, G. Ost and C. Vargas

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Looking for experimental evidence that the brain is a statistician

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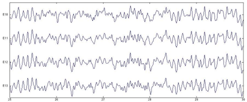
Hidden context tree models.

Neurobiological problem

A random source produces sequences of auditory stimuli.

Random Source



How to retrieve the structure of the source from EEG data?

Example of a random source: samba

- Auditory segments:
  - 2 - strong beat
  - 1 - weak beat
  - 0 - silent event
- Chain generation:
  - start with a deterministic sequence

$$\cdots \mathbf{2}\ 1\ \mathbf{0}\ 1\ \mathbf{2}\ 1\ \mathbf{0}\ 1\ \mathbf{2}\ 1\ \mathbf{0}\ 1\ \mathbf{2} \cdots$$

  - replace in a iid way each symbol 1 by 0 with probability ϵ.

A typical sample would be

$$\cdots \mathbf{2}\ 1\ \mathbf{0}\ {\color{red}1}\ \mathbf{2}\ 1\ \mathbf{0}\ 1\ \mathbf{2}\ {\color{red}1}\ \mathbf{0}\ {\color{red}1}\ \mathbf{2} \cdots$$
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How to define the structure of this source?

- By describing the algorithm producing each next symbol, given the shortest relevant sequence of past symbols.

The structure of the random source

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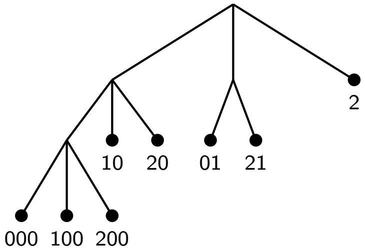

Contexts(w)	p(0|w)	p(1|w)	p(2|w)
2	ϵ	$1-\epsilon$	0
21	ϵ	$1-\epsilon$	0
01	0	0	1
20	ϵ	$1-\epsilon$	0
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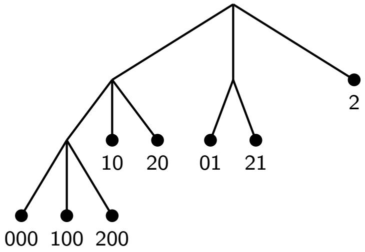

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The stochastic chain generated by the source samba

$$
\begin{array}{cccccccccc}
\cdots & \mathbf{2} & 1 & 0 & 0 & \mathbf{2} & 1 & 0 & 1 & \cdots \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\
 & X_{-5} & X_{-4} & X_{-3} & X_{-2} & X_{-1} & X_0 & X_1 & X_2 &
\end{array}
$$

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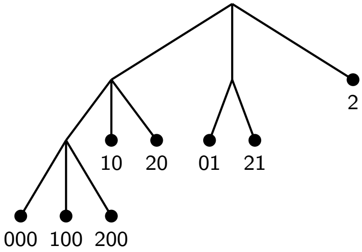

The neurobiological question

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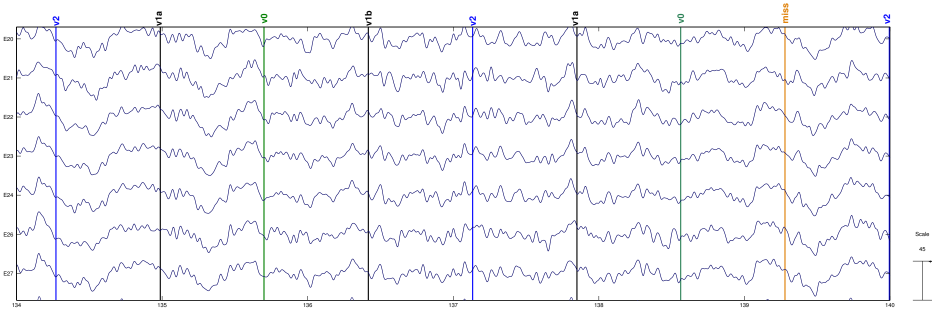
from the EEG data recorded during the exposure to the sequence of auditory stimuli generated by the samba source?

EEG data


v2
v1a
v0
v1b
v2
v1a
v0
miss
v2
E20
E21
E22
E23
E24
E26
E27
134
135
136
137
138
139
140
Scale
45

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- EEG data recorded with 18 electrodes
- for each electrode e and each step n
- call $Y_n^e = (Y_n^e(t), t \in [0, T])$ the EEG signal recorded at electrode e during the exposure to the auditory stimulus X_n
- $Y_n^e \in L^2([0, T])$, where $T = 450$ms is the time distance between the onsets of two consecutive auditory stimuli

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- family $\{Q_w : w \in \tau\}$ of probabilities on $(F, \mathcal{F})$
- stochastic chain $(X_n, Y_n) \in A \times F$.

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- and any sequence $I_m^n = (I_m, \ldots, I_n)$ of $\mathcal{F}$-measurable sets,

$$\mathbb{P}\Big(Y_m^n \in I_m^n | X_{m-\ell(\tau)+1}^{n} = x_{m-\ell(\tau)+1}^{n}\Big) = \prod_{k=m}^{n} Q_{c_\tau(x_{k-\ell(\tau)+1}^{k})}(I_k)$$

- $\ell(\tau) =$ height of τ
- $c_\tau(x_{k-\ell(\tau)+1}^{k}) =$ context assigned to $x_{k-\ell(\tau)+1}^{k}$ by τ

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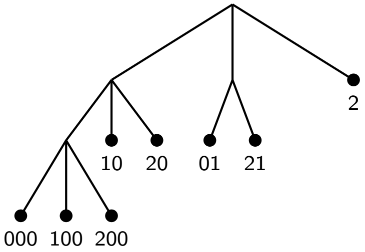
Question: Is $(X_n, Y_n^e)_{n\in\mathbb{Z}}$ a HCTM compatible with τ?



is $(X_n, Y_n^e)_{n\in\mathbb{Z}}$ a HCTM compatible with τ?

In other terms, for any $w \in \tau$, is it true that

$$\mathcal{L}(Y_n^e | X_{n-\ell(w)+1}^n = w, X_{-\infty}^{-\ell(\tau)} = u) = \mathcal{L}(Y_n^e | X_{n-\ell(w)+1}^n = w, X_{-\infty}^{-\ell(\tau)} = v)$$

for any pair of strings u and v?

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- Start with a maximal admissible candidate tree
- For any string w and pair of symbols $a, b \in A$ with aw and bw belonging to the candidate tree
- test the equality

$$\mathcal{L}(Y_n^e | X_{n-\ell(w)}^n = aw) = \mathcal{L}(Y_n | X_{n-\ell(w)}^n = bw)$$

Pruning the tree

- If for all pairs of symbols (a, b) the equality is rejected then prune all the leaves aw
- Repeat the pruning procedure until no more pruning is required

How to test the equality

$$\mathcal{L}(Y_n|X_{n-\ell(w)}^n = aw) = \mathcal{L}(Y_n|X_{n-\ell(w)}^n = bw)\,?$$

Apply the projective method introduced by Cuestas-Albertos, Fraiman and Ransford (2006).

Experimental results

- Context tree selection procedure for the EEG data recorded during the exposure to the sequence of auditory stimuli generated by the samba source
- Sample composed by 20 subjects
- For each subject EEG data from 18 electrodes was recorded

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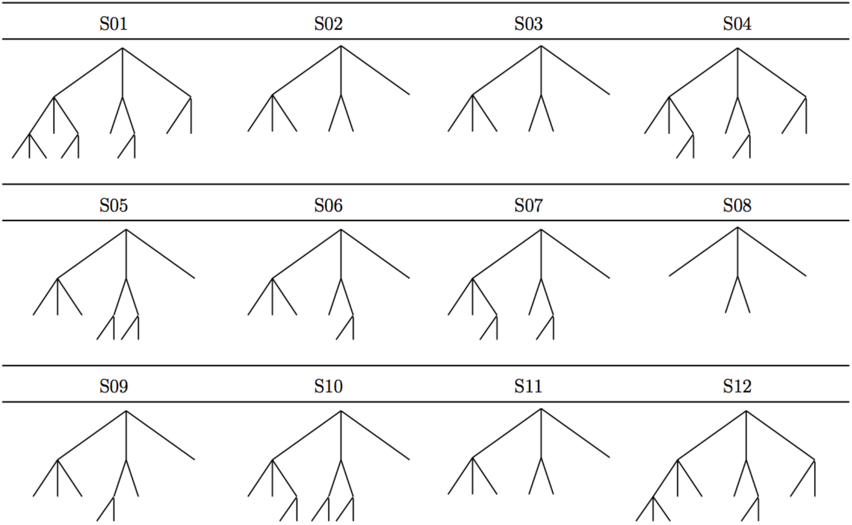

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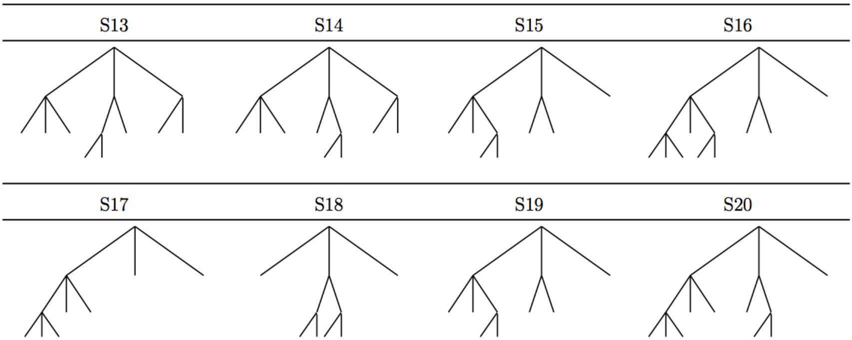
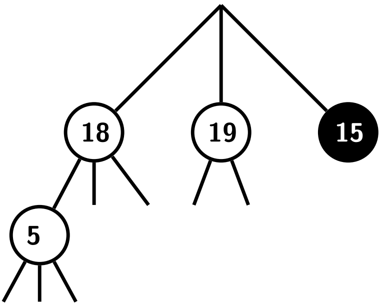


Summary



White nodes indicate the number of subjects which correctly identify the node as not being a context. **Black nodes** indicate the number of subjects which correctly identify the node as a **context**. For instance, 18 subjects correctly identify that the symbol 0 alone is not enough to predict the next symbol. And 15 subjects correctly identify the symbol 2 as **a context**.