First test on finite elements

No document allowed. Questions marked with a * were treated in class.

Exercise 1 Give the variational formulation of the following problems. Specify the functional space on which the variational formulation holds.

1.1)* $-u'' = f$ in $]0,1[$, $u(0) = 0$, $u(1) = 0$.

1.2)* $-u'' = f$ in $]0,1[$, $u'(0) = 0$, $u'(1) = 0$.

1.3)* $-u'' = f$ in $]0,1[$, $u(0) + 3u'(0) = 0$, $u'(1) = 2$.

1.4)* $-u'' = f$ in $]0,1[$, $u'(0) = 1$, $u(1) = 5$.

Exercise 2 We wish to solve numerically

$$-u'' = f \text{ in }]0,1[, \qquad u(0) + 3u'(0) = 0, \qquad u'(1) = 2.$$

We define V_h as the space of continuous functions v which are polynomials of degree one on each interval $I_i = [x_i, x_{i+1}]$ where $x_i = (i-1)/n$, $i = 1, n+1$. Let us set $n = 3$.

2.1)* Construct the matrix of the linear system associated to the discrete problem.

2.2)* For $f(x) = x$, construct the right-hand side of the linear system associated to the discrete problem.

2.3)* Comment on the structure and inversibility of the system matrix constructed in 2.1.

Exercise 3 Quadratic interpolation.

3.1) Recalling that the two barycentric coordinates on the reference interval $[0,1]$ are $\lambda_0(x) = 1 - x$ and $\lambda_1(x) = x$, write the three basis functions on $[0,1]$ for Lagrange piece-wise quadratic polynomial interpolation in terms of λ_0 and λ_1.

3.2) Calculate the "quadratic" stiffness matrix $A_{ij} = \int_0^1 \phi_i' \phi_j'$ on the reference element $[0,1]$ by using the basis functions defined in 3.1.

3.3) Extend the question 3.1 to the reference triangle T with vertices $(0,0)$, $(1,0)$ and $(0,1)$.