Non destructive testing of cables using non linear vibrations:

simple o.d.e models, Duffing equation for one d.o.f.

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The strong linkage between damage and nonlinear elasticity of materials leads to many researches in the field of nonlinear response of mechanical structures . Due to severe mechanical environment conditions (Vibration, Shock...), damage may appear in structures. In a previous S & V symposium we have presented some interesting experiments who have shown that non linear acoustic spectroscopy is a very powerful technique in order to detect damage in structures The proposed method consists of the observation of nonlinear responses obtained from an array of sensors when the structure is excited by dual frequency sources : the low frequency source yields the power while the second acts as a probe . The nonlinear response is obtained by spectral analysis of signals coming from sensors , the response amplitude is proportional to the local nonlinearity (failure nonlinear elastic response) . Now we have developed the theoretical and mathematical approach of the problem and we show a good correlation between the modeling and the experiment. We explain also the ideas that are behind the concept of localization of damage. We also show that this technique based on non linear acoustic is a good concept in order to detect buried objects.

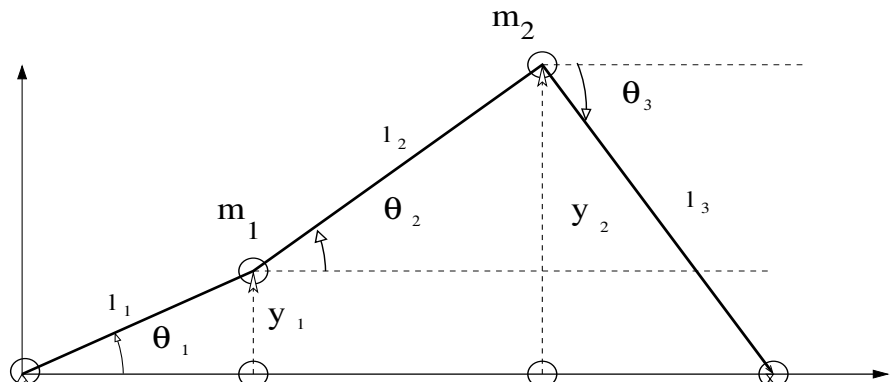
1 Introduction

1.1 Orientation

Although there is an active investigation of experiments for non destructive testing using non linear vibrations, there is a lack of models and numerical solutions to compare with the experimental results.

2 Transverse vibrations: vibrating masses on stretched cables in large displacement

As a very simplified model of the experiment described in [**?**], we consider n masses m_i attached to horizontal springs (or cables) which are submitted to a large tension T_0, at rest, see figure 1; the tension is positive when the cable is in traction which is assumed trough the vibration; the masses are moving (vertically) transversely to the position of the springs at rest; we denote by upper case letters quantities in the rest position and lower case in the current configuration, except for the tension T_i in the current configuration.



2.1 Masses in transverse displacement

2.1.1 Notations and assumptions

In a linear model of transverse vibration, it is usually assumed that the tension is so large at rest that it remains constant during the vibration. Here, due to large displacements the tension does depend on the change of length, we assume a linear elastic relation between tension and change of length. See the figure 1 with two masses and 3 cables.

- L_i length at rest; l_i length at time t; as the masses are moving perpendicularly to the position of the cables at rest:

$$l_i^2 = L_i^2 + (y_i - y_{i-1})^2$$

- **the change of tension of the linear elastic spring due to the change of of length**
  $T_i = T_0 + k_i[l_i(y) - L_i] = T_0 + k_i(\sqrt{L_i^2 + (y_i - y_{i-1})^2} - L_i)$;
  this tension is directed **along the axis of the spring** and positively oriented from m_i to m_{i+1}.
- Denote by θ_i, the angle of the i^{th} spring with the horizontal axis; at rest $\theta_i = 0$ we have

Using Newton law, the equations of dynamics are:

$$m_i y_i" = -T_i sin(\theta_i) + T_{i+1} sin(\theta_{i+1}) + \mathcal{F}_i \quad i = 1 \dots n \qquad (2.1)$$

where $-T_i sin(\theta_i) + T_{i+1} sin(\theta_{i+1})$ is the vertical component of the force due to cables acting on mass i; we assume no longitudinal movement so that the longitudinal component of the force does not work. The applied load on mass i is denoted by $\mathcal{F}_i$.

3 Transverse vibration: approximate non dimensional equation for a small load

3.1 Introduction

It is not obvious to prescribe the right data to obtain clear inter-modulation peaks; this is also a real trouble of the experiments!.

So it is very useful to obtain an asymptotic expansion of the solution in order to determine rough windows of the data for which inter-modulation lobes are non negligible *and to relate the level of the inter-modulation lobes to the level of the damage*; see subsection 4.2

For one d.o.f., the linearized equation is:

$$m_1 \ddot{y_1} = -T_0(\frac{1}{L_1} + \frac{1}{L_2})y_1 + \tilde{F}_1 cos(\tilde{\alpha}\tilde{t}) \qquad (3.1)$$

Approximate equation

Normalization We normalize time $\tilde{t}$

$$\omega^* = \left(\frac{T_0(\frac{1}{L_1} + \frac{1}{L_2})}{m} \right)^{\frac{1}{2}}, \quad \tilde{t} = \frac{t}{\omega^*}, \quad \alpha = \frac{\tilde{\alpha}}{\omega^*} \tag{3.3}$$

and displacement with the total lenth at rest:

And the differential equation may be approximated by a so-called Duffing equation:

$$\frac{d^2u}{dt^2} = -u + \epsilon u^3 + Fcos(\alpha t) + \dots \tag{3.4}$$

with the small parameter:

$$\epsilon = \frac{\tilde{\lambda}^2 L_{tot}^2}{2m\omega^{*2}} \left(\frac{T_0 - k_1 L_1}{L_1^3} + \frac{T_0 - k_2 L_2}{L_2^3} \right)$$

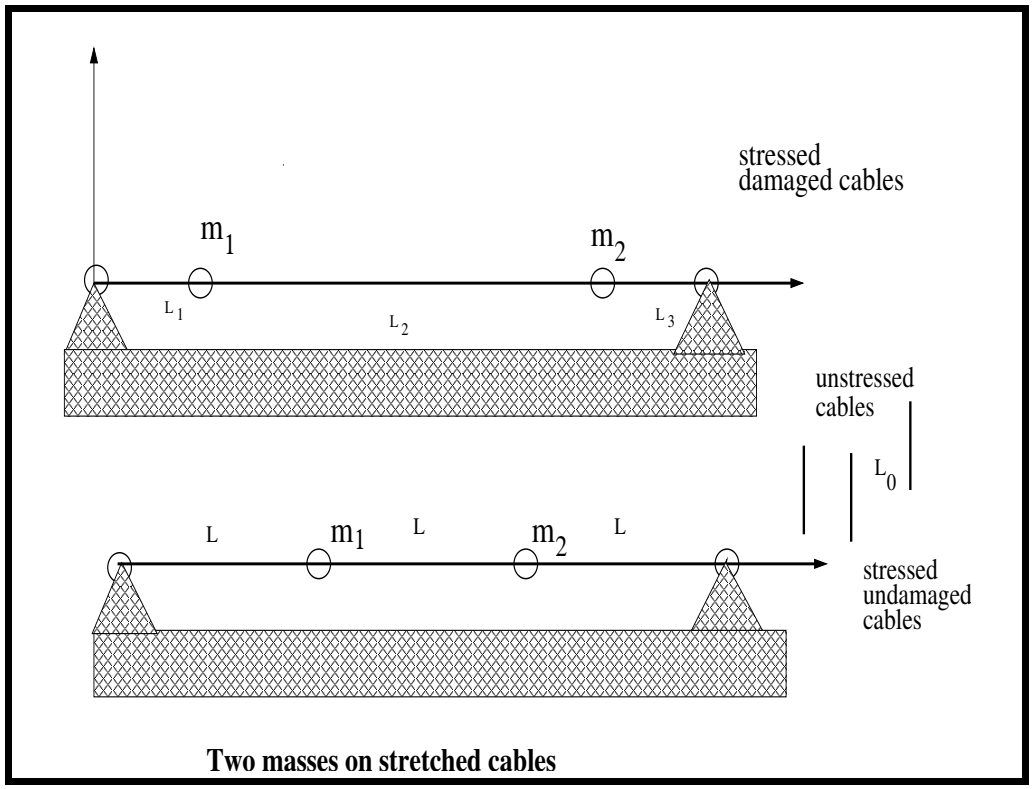


Figure 2: Up: one damaged cable and 2 undamaged cables ; down: 3 undamaged cables

3.1.1 A possible damage of a cable

Here a simple breakage model of several fibers of a cable is addressed, this causes decrease of rigidity k_1 say for cable 1; internal contacts are *not*

4 Numerical and asymptotic solution of Duffing equation in the frequency domain

4.1 Introduction

One of the main issues of the use of the inter-modulation lobes for *non destructive testing* is their level relative to the main lobe located at the load frequency $\alpha/2\pi$.

The choice of the data of the system in order to be able to observe the inter-modulation lobes is not obvious numerically. The use of the double scale approximation is a valuable tool in order to design a system. However, in the design process, it should be used cautiously because the generated data fall easily out of its domain of validity.

4.2 Peaks of the exact Fourier transform of the approximate solution $u_0 + \epsilon u_1$

As

$$\mathcal{F}(cos(2\pi iat)\chi_{[0,A]}) = \frac{A}{2}(\tau_a e^{-i\pi A\nu}\mathbf{sinc}(\pi\nu\mathbf{A}) + \tau_{-a} e^{-i\pi A\nu}\mathbf{sinc}(\pi\nu\mathbf{A}))$$

with sinus cardinal (or sampling function)

$$\text{angular frequency} \mapsto \text{ level of the peak} \tag{4.1}$$

$$1 + \epsilon\beta_1 \mapsto \frac{t_{max}}{2} a \tag{4.2}$$

$$\alpha \mapsto \frac{t_{max}}{2} \left(-\phi + \epsilon \frac{3.\phi.(2.a^2 + \phi^2)}{4(1-\alpha^2)} \right) \tag{4.3}$$

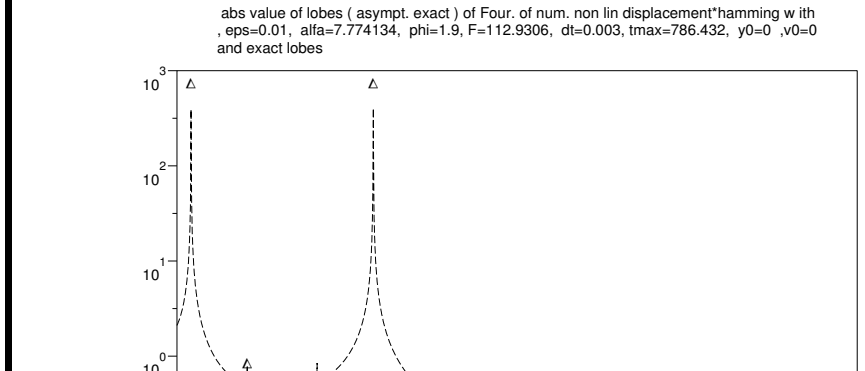
$$3.(1 + \epsilon.\beta_1) \mapsto \epsilon \frac{t_{max}}{2} \frac{a^3}{32} \tag{4.4}$$

$$(-2 + \alpha - 2.\epsilon.\beta_1) \mapsto \epsilon \frac{t_{max}}{2} \frac{(3.a^2.\phi/4)}{(1-(-2+\alpha)^2)} \tag{4.5}$$

$$(2 + \alpha + 2.\epsilon.\beta_1) \mapsto \epsilon \frac{t_{max}}{2} \frac{(3.a^2.\phi/4)}{(1-(2+\alpha)^2)} \tag{4.6}$$

$$3.\alpha \mapsto \epsilon \frac{t_{max}}{2} \frac{\phi^3}{(4.(1-9.\alpha^2))} \tag{4.7}$$

Proposition 4.1. *From this table, it is clear that the level of the intermodulation lobes depends linearly on ϵ; the slope itself depends on the value t_{max} of the final time, on the level of $\phi = \frac{-F}{1-\alpha^2}$ related to the applied load , and on the level of the*


abs value of lobes (asympt. exact) of Four. of num. non lin displacement*hamming w ith
, eps=0.01, alfa=7.774134, phi=1.9, F=112.9306, dt=0.003, tmax=786.432, y0=0 ,v0=0
and exact lobes
10^3
10^2
10^1
10^0

4.3 Data of the bridge

The following data are those used in the experiments reported in [**?**].

General data The total lenth of the stressed cable, young modulus and the area of the cross section:

$$L_{tot} = 165m, E = 2 * 10^{11} N/m^2, A = 4 * 10^{-3} m^2$$

linear mass is $\rho = 43.8kg/m$; the total mass is $3615.5kg$; it is taken as the same for the damaged cable.

For the safe (undamaged) cable: The *measured* tension in the safe cable:

$$T_0 = 2.794 * 10^6 N,$$

from the usual stress-strain relation for bars, the relative elongation or strain between the unstressed ans prestressed configurations: $S_{stot} = T_0/(E * A) = 0.0035$;

The 1 d.o.f. model provides a good approximation of the fundamental frequency .

[The damaged cable] The *measured* tension is

$$T_{0dam} = 1.919 * 10^6 N;$$

The coefficients ϵ of formula (12) involved in Duffing equation:

$\epsilon_{safe} = -572.66$ $\epsilon_{dam} = -635.58.$

We notice a substantial increase of ϵ due to decrease of rigidity

of one cable; this increase causes an increase of the

intermodulation lobes as noted in subsection 4.2

For a much larger load $F = 0.0378982$, the intermodulations are quite visible. However, it is acknowledge that it corresponds to an experimental load of

$$\mathcal{F}_{exp} = mL_{tot}\omega^{*2} = 7676009N$$

To found a shaker wchich can produce such a large load is quite

Finally, we show numerical evidence that if we apply the load *at a frequency close to the eigenfrequency of the structure,* the experimental load of $489N$ is high enough in order to show intermodulations; the asymptotic expansions provided in this article are not valid with this type of frequency ; it will be considered in a forthcoming paper.

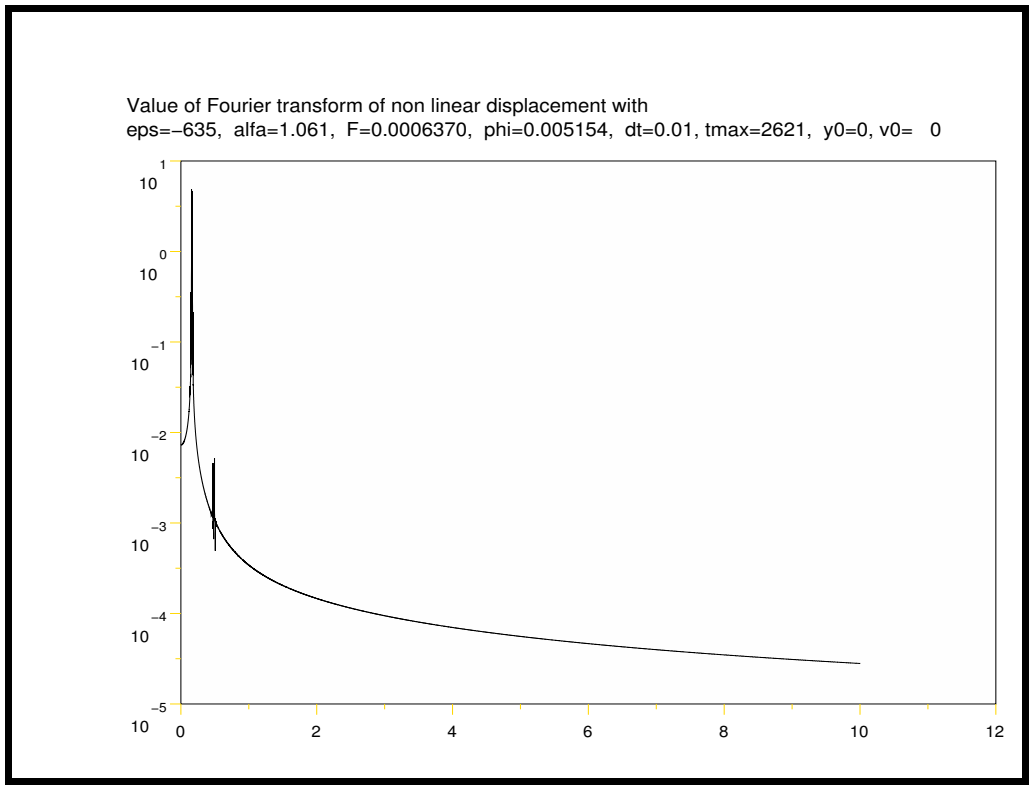


Figure 4: tm=2621, alpha=1.06, F=0.00006 eps=-635,