Lagrangian Resolution of the Multilane Hybrid Traffic Flow Model

[Microscopic modeling of the relaxation phenomenon using a macroscopic lane-changing model. Trans. Res. B, forthcoming]

Jorge A. Laval[1] Ludovic Leclercq[2]

[1]School of Civil and Environmental Engineering
Georgia Institute of Technology

[2]Laboratoire Ingénierie Circulation Transport
LICIT (INRETS/ENTPE)

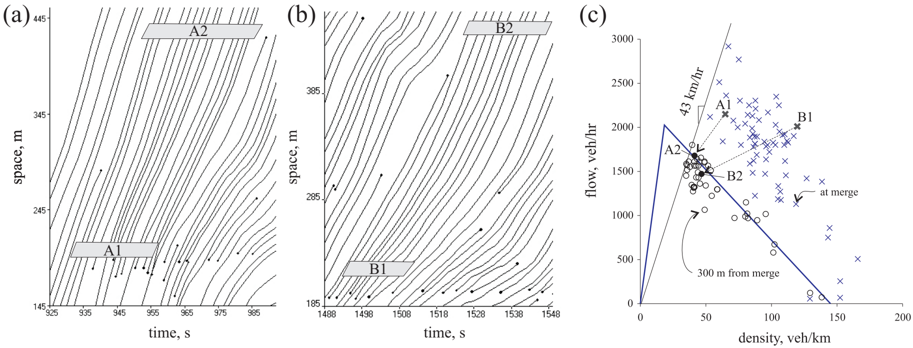
Mathematical Models of Traffic Flow

Motivation

Current Lane-Changing models

- tend to be inconsistent
- have too many parameters
  - curtesy
  - forced LCs
- do not address relaxation after the LC

Relaxation phenomenon



Outline

Outline

The multilane hybrid (MH) model

[Laval and Daganzo. Lane-changing in traffic streams. *Trans. Res. B*, 40 (3): 251-264, 2006]

Definition

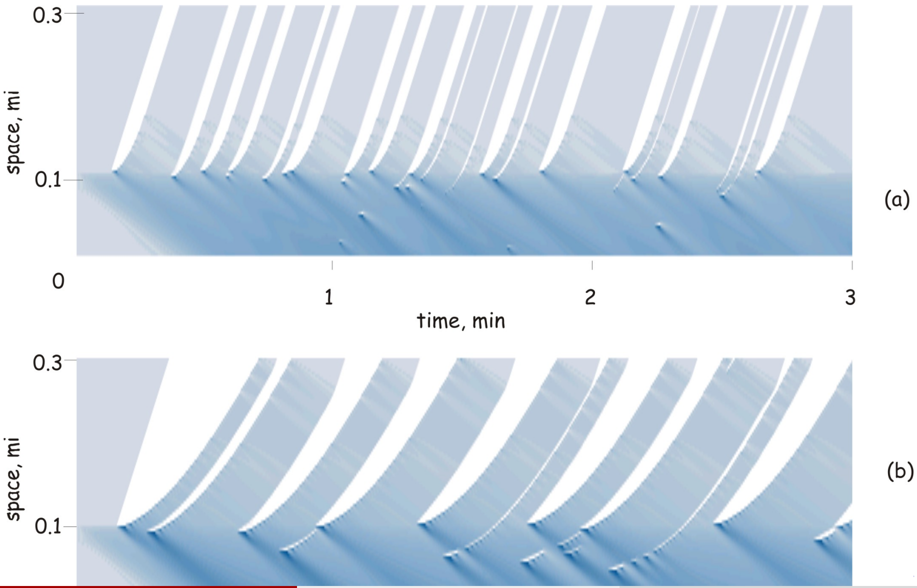
- Conservation law in each lane

$$\frac{\partial k_\ell}{\partial t} + \frac{\partial q_\ell}{\partial x} = \Phi_\ell \ , \qquad \ell = 1 \ldots n \tag{1}$$

  Where Φ_ℓ is the *net lane-changing rate onto* lane ℓ in units of veh/time-distance.

- disruptive lane-changes $\sim$Poisson(Φ)
- disruptive lane-changes act as moving bottleneck in target lane

Moving Bns = key to capacity drop



The multilane hybrid (MH) model I

- the sending (demand) function

$$\lambda_\ell = \min\{uk_\ell, Q\} \tag{2}$$

- the receiving (supply) function

$$\mu_\ell = \min\{w(\kappa - k_\ell), Q\} \tag{3}$$

- for triangular FD with
  - free-flow speed u
  - wave speed w, and
  - jam density/lane κ

  $\Rightarrow Q = uw\kappa/(u+w)$ is the one-lane capacity.

The multilane hybrid (MH) model II

- The desired number of lane-changing moves

$$L_{\ell\ell'}\Delta x = \lambda_\ell \pi_{\ell\ell'}\Delta t \qquad \forall \ell, \forall \ell' \neq \ell \tag{4}$$

where

$$\pi_{\ell\ell'} = \frac{\Delta v_{\ell\ell'}}{u\tau} \qquad \text{(discretionary LCs)} \tag{5}$$

- the parameter τ has units of time.
- the desired number of through moves is

$$T_{\ell'} = (1 - \sum_{\ell \neq \ell'} \pi_{\ell'\ell}\Delta t)\lambda_{\ell'} \qquad \forall \ell. \tag{6}$$

The multilane hybrid (MH) model III

- available capacity is prorated according to demands $\Rightarrow$ fraction of the demand able to advance to ℓ' is

$$\gamma_{\ell'} = \min\{1, \frac{\mu_{\ell'}}{T_{\ell'} + \sum_{\ell \neq \ell'} \Delta x L_{\ell\ell'}}\} \quad (7)$$

- and the transfers to the target lane ℓ are

$$\Phi_{\ell\ell'} = \gamma_{\ell'} L_{\ell\ell'} \quad , \forall \ell \neq \ell' \quad (8)$$

$$q_{\ell'} = \gamma_{\ell'} T_{\ell'} \quad (9)$$

Outline

The car-following linear (CFL) model

- exact car-following formulation of kinematic wave theory with triangular FD for a single lane

$$x^i_{j+1,\ell} = \min\{x^i_{j\ell} + u\Delta t, x^{i-1}_{j\ell} - 1/\kappa\} \tag{10}$$

  where x^i_j is the position of vehicle i along lane ℓ at time-step j.
- this scheme is free of numerical errors provided that

$$\Delta t = \frac{1}{w\kappa} \tag{11}$$

Outline

Quantizing Φ

- Recall

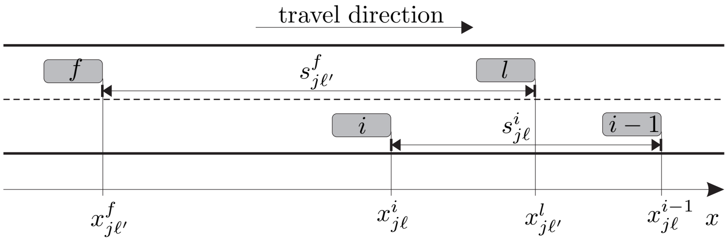
$$\gamma_{\ell'} = \min\{1, \frac{\mu_{\ell'}}{(1 - \sum_{\ell \neq \ell'} \pi_{\ell'\ell} \Delta t)\lambda_{\ell'} + \sum_{\ell \neq \ell'} \Delta x L_{\ell\ell'}}\} \tag{12}$$

- taking the limit $\Delta t, \Delta x \to 0$

$$\Phi_{\ell\ell'} = \min\{1, \frac{\mu_{\ell'}}{\lambda_{\ell'}}\} \frac{\pi_{\ell\ell'} \lambda_{\ell}}{u} \tag{13}$$

- this continuum process needs to be quantized into vehicle units.

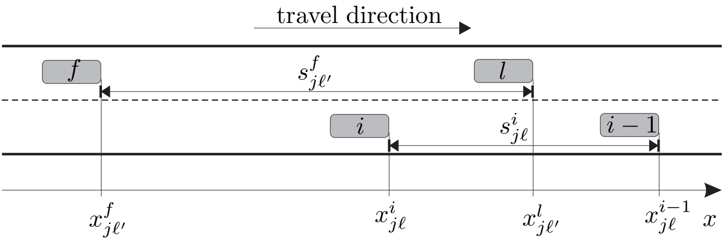
Quantizing Φ



- let $p^{ij}_{\ell\ell'}$ be the probability of vehicle i in lane ℓ of switching to lane ℓ' during time-step j.

$$p^{ij}_{\ell\ell'} = \int_{t=t_j}^{t_{j+1}} \int_{x=x^i_{j\ell}}^{x^{i-1}_{j\ell}} \Phi(k_\ell(t,x), k_{\ell'}(t,x))dtdx \approx \Phi(k_\ell(t_j, x^i_{j\ell}), k_{\ell'}(t_j, x^i_{j\ell}))\Delta t s^i_{j\ell} \quad (14)$$

Quantizing Φ



- Densities and spacings are related by

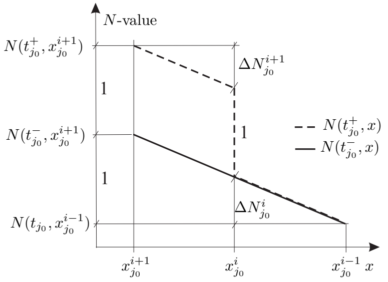
$$k_\ell(t_j, x^i_{j\ell}) = 1/s^i_{j\ell} \quad (15)$$

$$k_{\ell'}(t_j, x^i_{j\ell}) = 1/s^f_{j\ell'} \quad (16)$$

where f gives i's new follower in target lane ℓ'

- lane changes $\sim$Bernoulli($p^{ij}_{\ell\ell'}$)

Outline



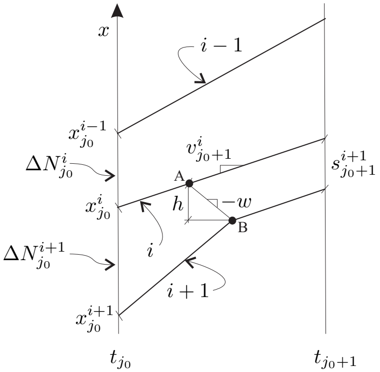
Definition

Let $N(t, x)$ be the vehicle number at (t, x) on the target lane under consideration, and

$$\Delta N^i_j = N(t_j, x^i_j) - N(t_j, x^{i-1}_j)$$

the difference in N between vehicle i and its leader $i - 1$

$$\Delta N^i_j = \begin{cases} 1, & i \text{ in "equilibrium"}; \\ < 1, & \text{o/w.} \end{cases}$$



- through line A-B pass $h\kappa$ vehicles, thus

$$\Delta N^{i+1}_{j_0} = \Delta N^{i+1}_{j_0+1} = h\kappa \tag{17}$$

- from the figure

$$h = s^{i+1}_{j_0+1} \frac{w}{v^i_{j_0+1} + w} \tag{18}$$

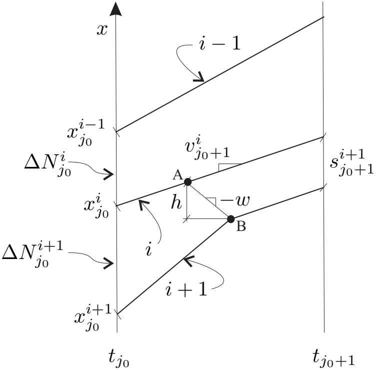
and therefore

$$s^{i+1}_{j_0+1} = \frac{\Delta N^{i+1}_{j_0+1}}{K(v^i_{j_0+1})} \tag{19}$$

where

$$K(v) = \frac{w\kappa}{v+w} \tag{20}$$

gives the density in congestion associated with speed v.



- also from the figure:

$$x^{i+1}_{j_0+1} = x^i_{j_0} + v^i_{j_0+1}\Delta t - s^{i+1}_{j_0+1} \qquad (21)$$

Hence,

$$x^{i+1}_{j+1} = x^i_j + v^i_{j+1}\Delta t - \frac{\Delta N^{i+1}_{j+1}}{K(v^i_{j+1})} \qquad (22)$$

Outline

The continuum model

- Conservation law in Lagrangian coordinates

$$\frac{\partial s}{\partial t} + \frac{\partial v}{\partial N} = 0 \tag{23}$$

- for non-equilibrium drivers we propose

$$\frac{\partial v}{\partial N} = -\varepsilon \tag{24}$$

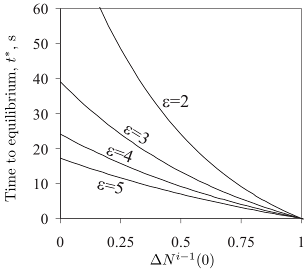
$\Rightarrow \varepsilon$ can be interpreted as the speed difference the follower is willing to accept with respect to its leader in order to attain an equilibrium spacing.

- we obtain:

$$\frac{d}{dt}\Delta N^{i+1}(t) = \varepsilon K(v^i(t)) \tag{25}$$

The continuum model

Example



bottleneck with constant acceleration rate $\beta > 0$

$$\Delta N^{i+1}(t) = \Delta N^{i+1}(0) + \frac{\varepsilon w \kappa}{\beta} \ln[1 + \frac{\beta t}{v^i(0) + w}]$$

$\Rightarrow$ the time to convergence to equilibrium, t^*, equals

$$t^* = \frac{v^i(0) + w}{\beta} (\exp[\beta \frac{1 - \Delta N^{i+1}(0)}{\varepsilon w \kappa}] - 1)$$

The discrete model I

- once v_{j+1}^{i+1} is known the follower spacing obeys

$$s_{j+1}^{i+1} = s_j^{i+1} + (v_{j+1}^i - v_{j+1}^{i+1})\Delta t. \tag{26}$$

- for $j > j_0$: differences in vehicle number and spacings are related by (19), ie

$$\Delta N_j^{i+1} = s_j^{i+1} K(v_j^i) \qquad \text{with} \qquad j > j_0. \tag{27}$$

- Combining (26)-(27) gives

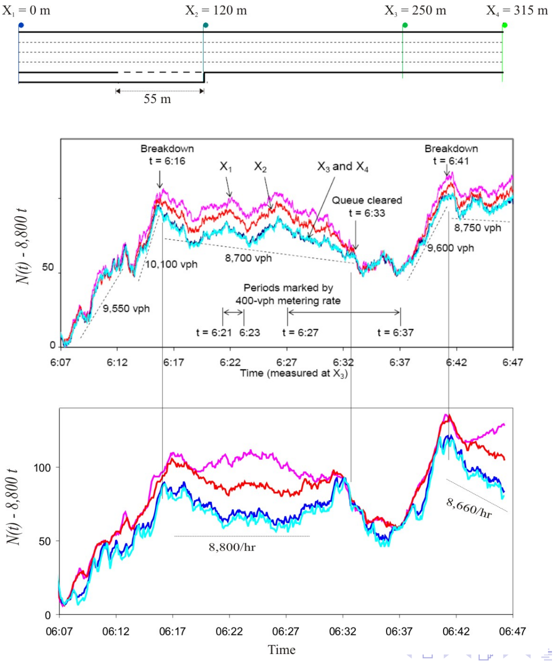
$$\Delta N_{j+1}^{i+1} = [\frac{\Delta N_j^{i+1}}{K(v_j^i)} + (v_{j+1}^i - \tilde{v}_{j+1}^{i+1})\Delta t] K(v_{j+1}^i) \tag{28}$$

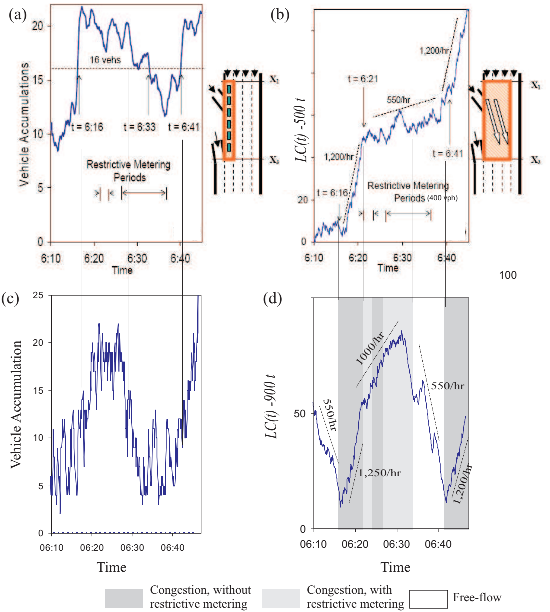
where we have introduced the follower's desired speed, $\tilde{v}_{j+1}^{i+1}$, since its actual speed v_{j+1}^{i+1} is not known at this stage.

The discrete model II

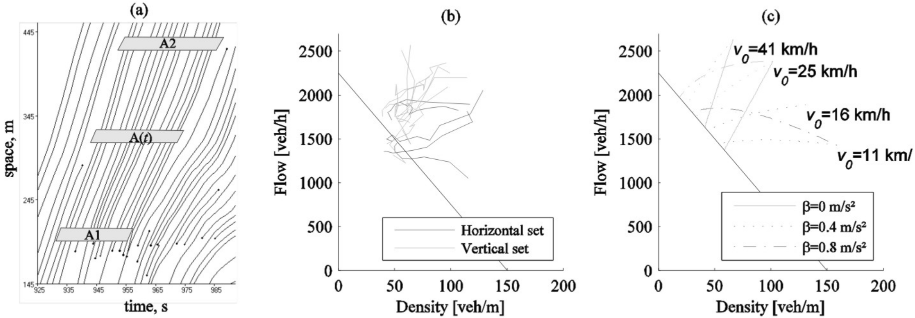
- we propose

$$\tilde{v}_{j+1}^{i+1} = \underbrace{v_{j+1}^{i}(1 - \Delta N_j^{i+1}) + v_j^i \Delta N_j^{i+1}}_{\text{MB-speed}} - \varepsilon. \tag{29}$$

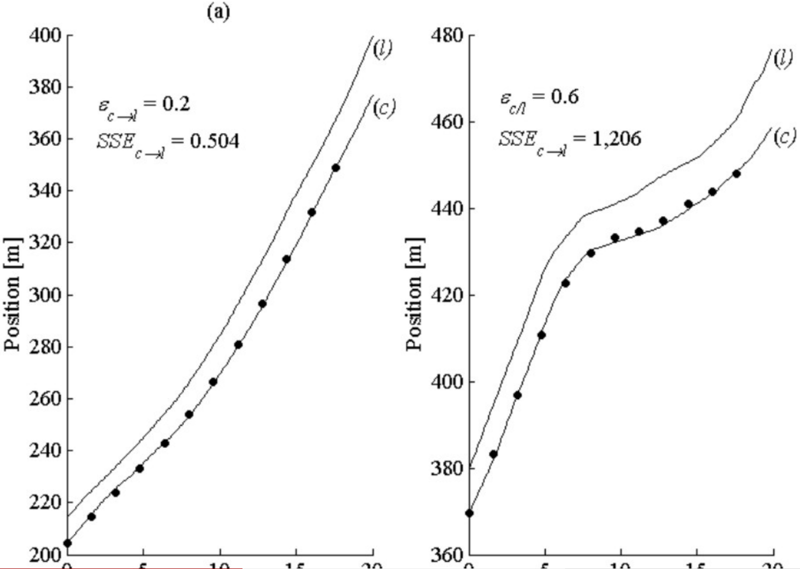

X1 = 0 m
X2 = 120 m
X3 = 250 m
X4 = 315 m
55 m
Breakdown
t = 6:16
X1
X2
X3 and X4
Breakdown
t = 6:41
Queue cleared
t = 6:33
8,750 vph
9,600 vph
10,100 vph
8,700 vph
9,550 vph
Periods marked by
400-vph metering rate
t = 6:21
6:23
t = 6:27
t = 6:37
N(t) - 8,800 t
Time (measured at X3)
8,800/hr
8,660/hr
Time


(a)
Vehicle Accumulations
16 vehs
t = 6:16
t = 6:33
t = 6:41
Restrictive Metering Periods
Time
X1
X3
(b)
LC(t) -500 t
1,200/hr
t = 6:21
550/hr
1,200/hr
t = 6:41
t = 6:16
Restrictive Metering Periods (400 vph)
Time
100
(c)
Vehicle Accumulation
Time
(d)
LC(t) -900 t
1000/hr
550/hr
550/hr
1,250/hr
1,200/hr
Time
Congestion, without restrictive metering
Congestion, with restrictive metering
Free-flow

Relaxation paths



Using trajectories



Conclusions

- A complete multilane microscopic model with only two additional parameters:
  - τ: mean time to execute a lane change
  - ε: speed-gap
- good agreement with field data
- continuum expression for Φ can be used in many applications
- analytical solutions for KW with merges
- variational theory with LCs