Traffic Flow on Single Links with Bottlenecks:
Variational Theory, Analysis, Application and Empirical Evidence

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Luminy, October 2007

References

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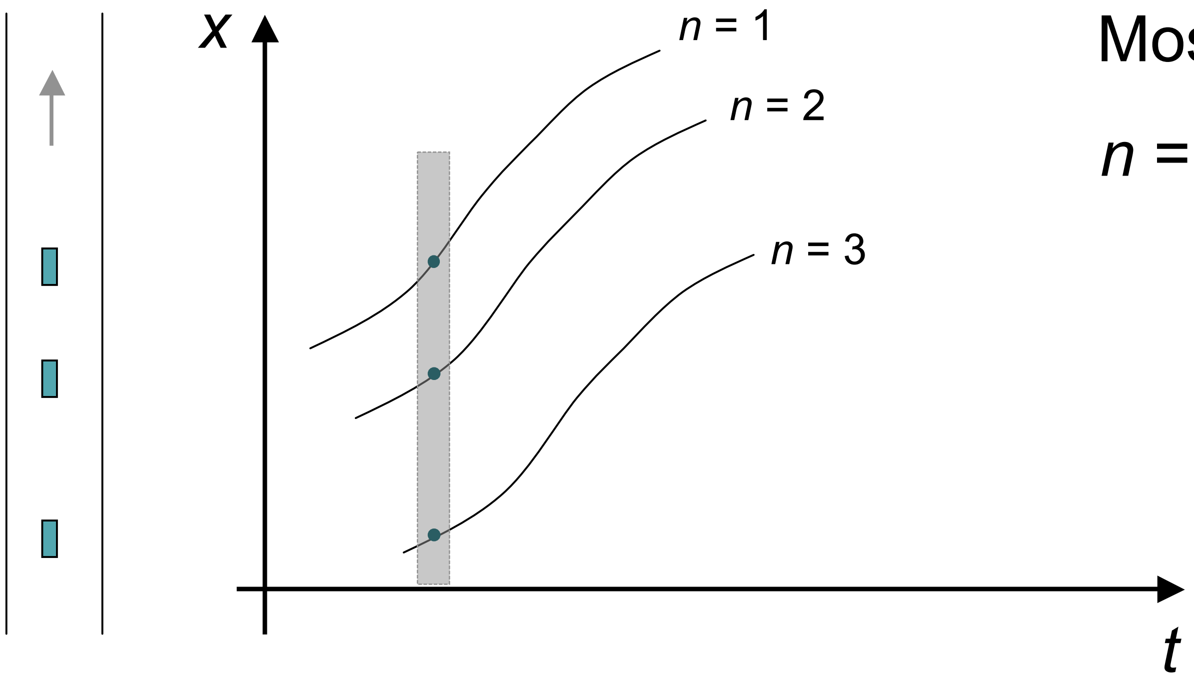
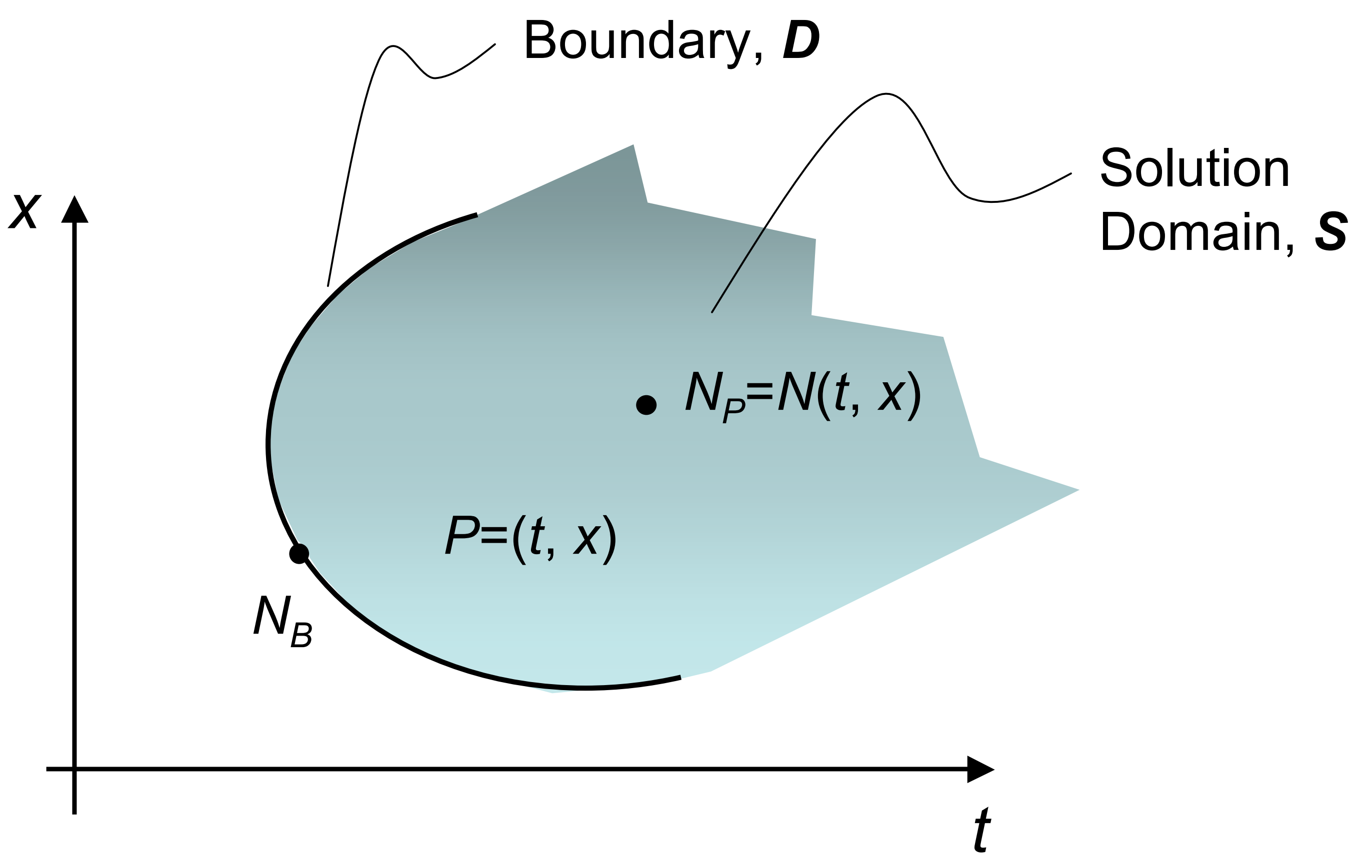
A Single Stream: Description



Moskowitz's Function

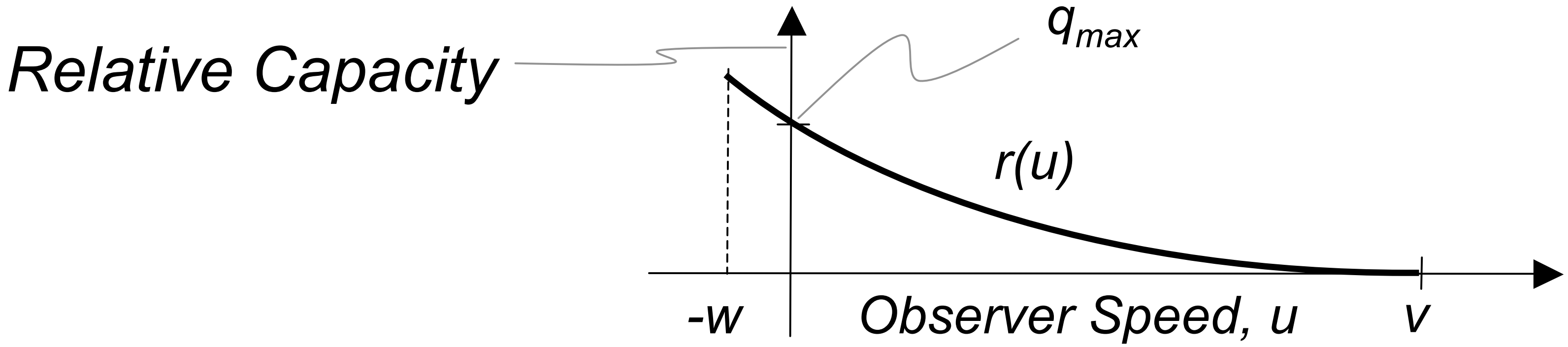
$n = N(t,x)$

A Single Stream: Prediction



Variational Theory (VT): Basic Ideas

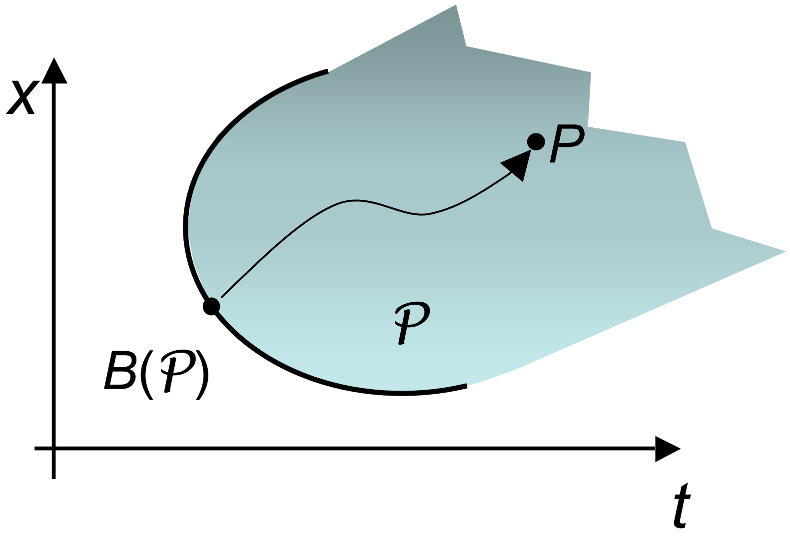
N(*t*,*x*) is L-Continuous $-\frac{\Delta N}{\Delta x} \in [0, \kappa]$ $\frac{\Delta N}{\Delta t} \in [0, q_{max}]$



N_P is largest possible subject to capacity constraints

(Daganzo, 2005;2006)

Variational Theory (VT): Mathematical Expression



- Observer ≡ Valid Path: $\mathcal{P}$, $x(t)$
- Observer Speed: $x'(t)$
- Observer Bound:

$$\int_{\mathcal{P}} r(x')dt \equiv \Delta(\mathcal{P})$$

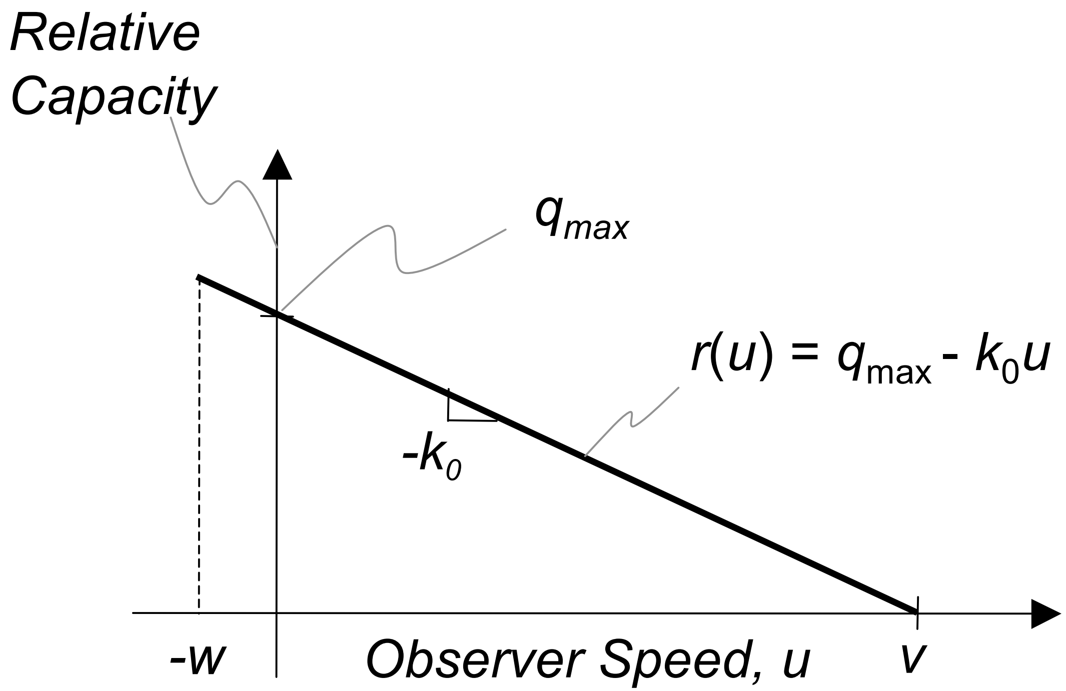
- Observer Constraint:

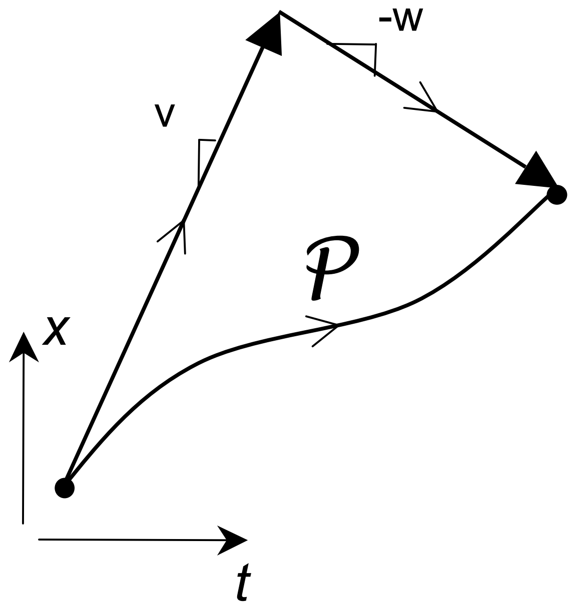
$$N_P \leq N_{B(\mathcal{P})} + \Delta(\mathcal{P})$$

VT Expression: $N_P^{VT} = \inf \{N_{B(\mathcal{P})} + \Delta(\mathcal{P}) : \forall \mathcal{P}\}$

(Daganzo, 2005;2006)

Simple Case: Linear



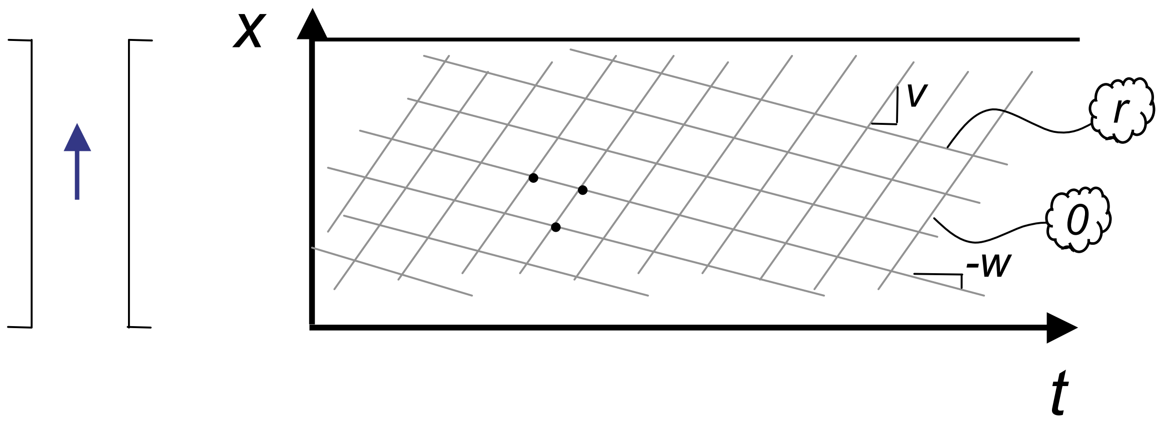


Bound is Path-Independent: $D(\mathcal{P}) = q_{max}Dt - k_0Dx$

(Daganzo, 2005a)

Network Solution with Dynamic Programming (DP)

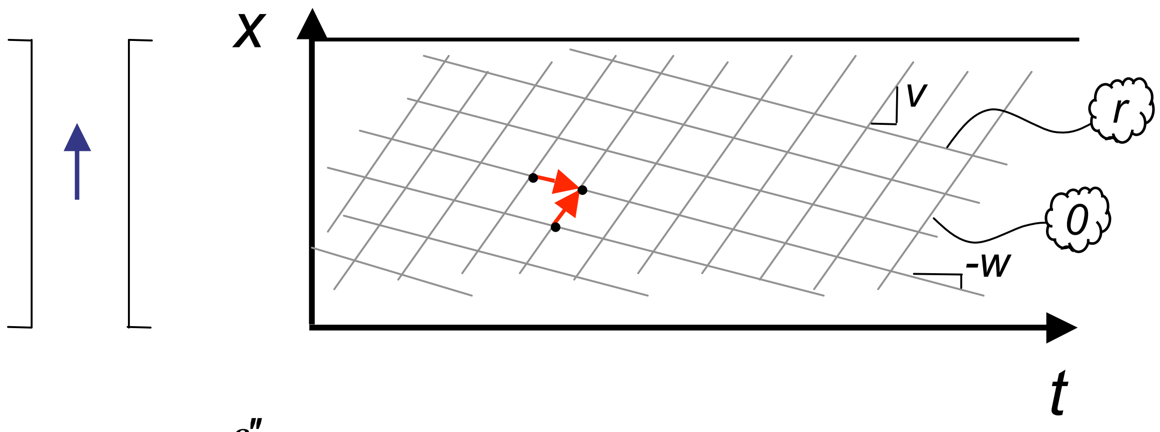
Sufficient network:



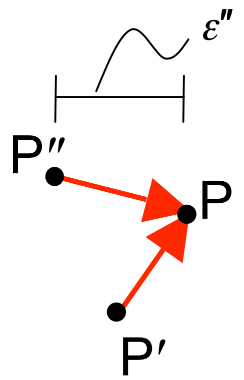
(Daganzo, 2005a)

Network Solution with Dynamic Programming (DP)

Sufficient network:



Stencil:

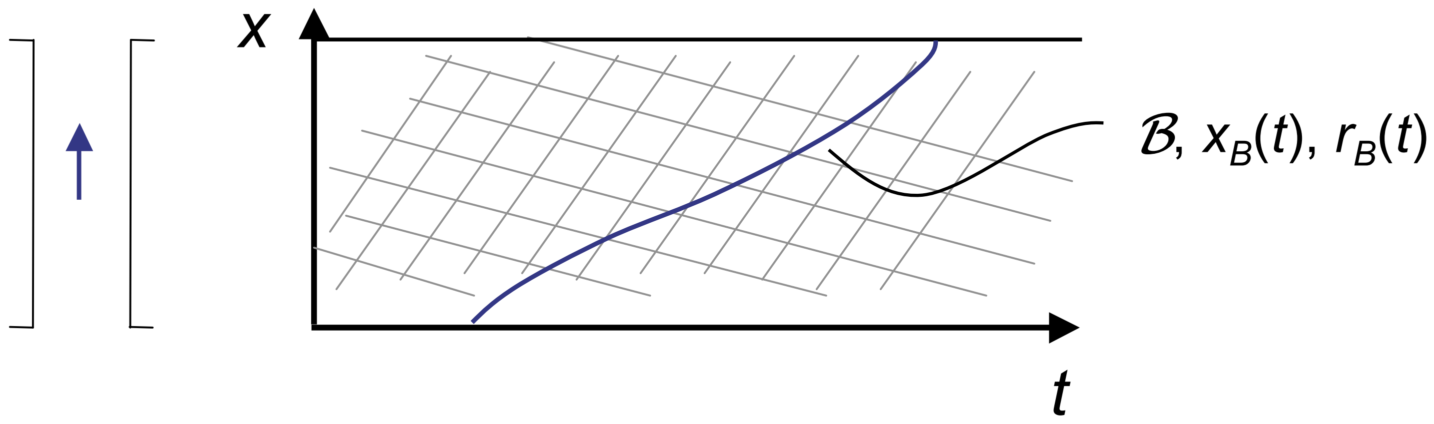


DP: $N_P = min\ \{N_{P'}, N_{P''} + r\varepsilon''\}$

(Daganzo, 2005a)

Bottlenecks in VT

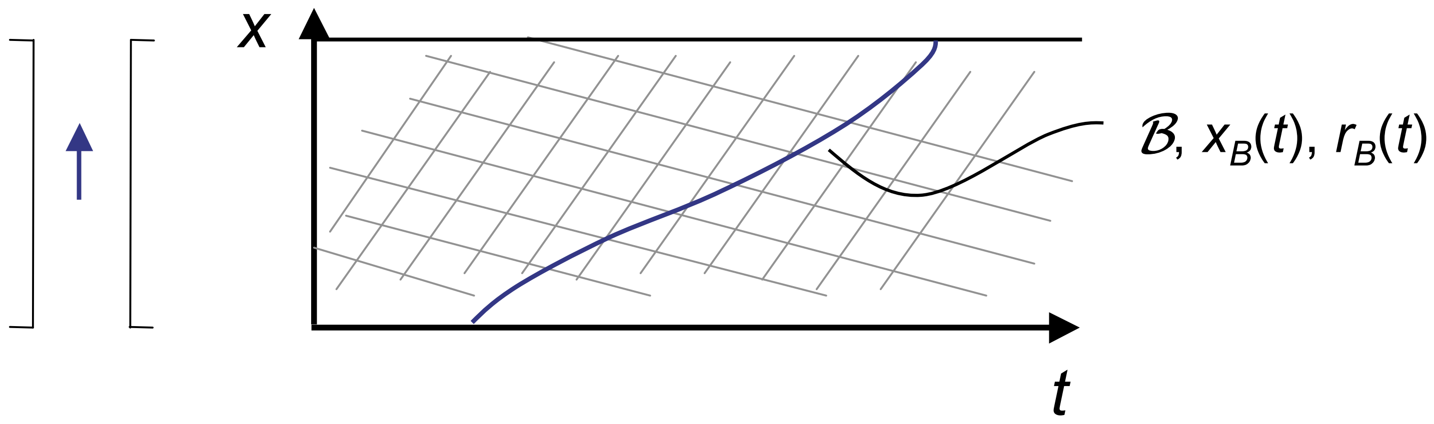
Sufficient network with shortcut:



(Daganzo, 2005a); (Daganzo & Menendez, 2005b)

Bottlenecks in VT

Sufficient network with shortcut:



Still a DP Problem

(Daganzo, 2005a); (Daganzo & Menendez, 2005b)

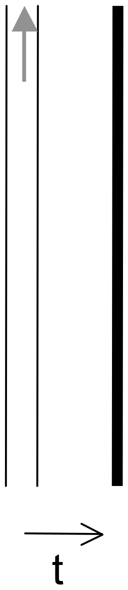
Questions about $N^{VT}(t,x)$

1. Well-posed?

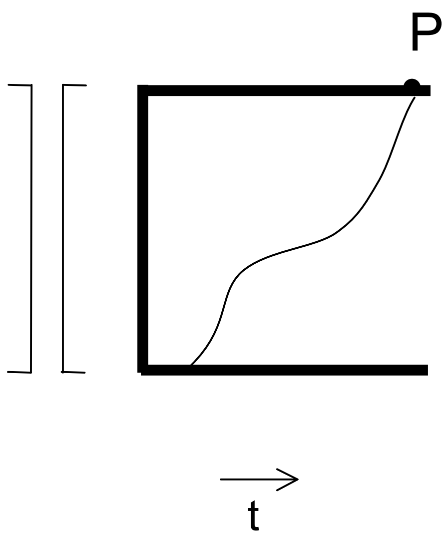
2. Related to known models: KW ; CF ?

3. Realistic?

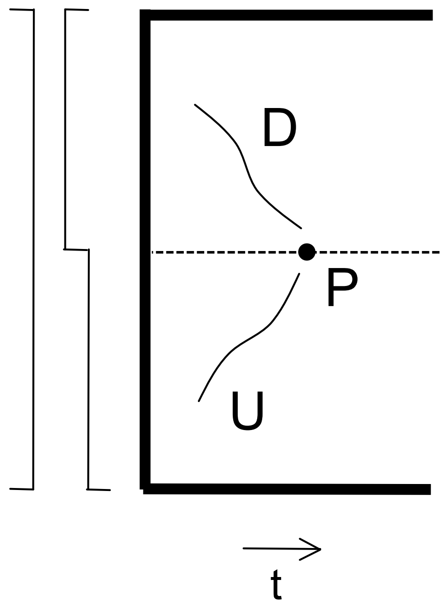
(Q1) Well-posed?



Initial
Value
Problem



Finite Highway
Problem



Composite
Highway
Problem

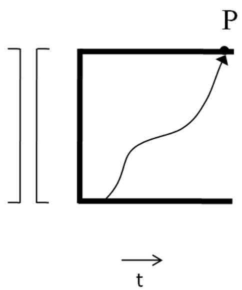
(Daganzo, 2006)

(Q1) Well-posed?

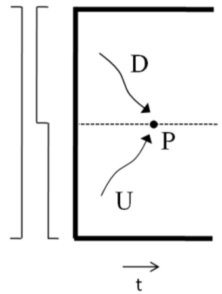
Assume: Data are L- Continuous
Bottlenecks have non-negative speeds & rel. capacities

Then:

Initial Value Problem



Finite Highway Problem



Composite Highway Problem

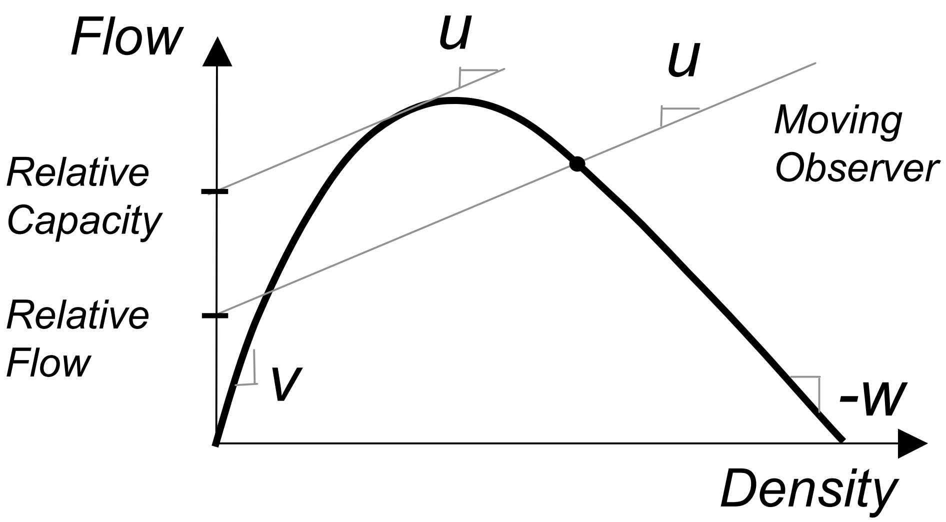
Well-posed

Well-posed if: $N_P \geq N_P^{VT}$

Well-posed if: $N_P = min\ \{ N_P^U, N_P^D \}$

(Daganzo, 2006)

(Q2) Related to KW?

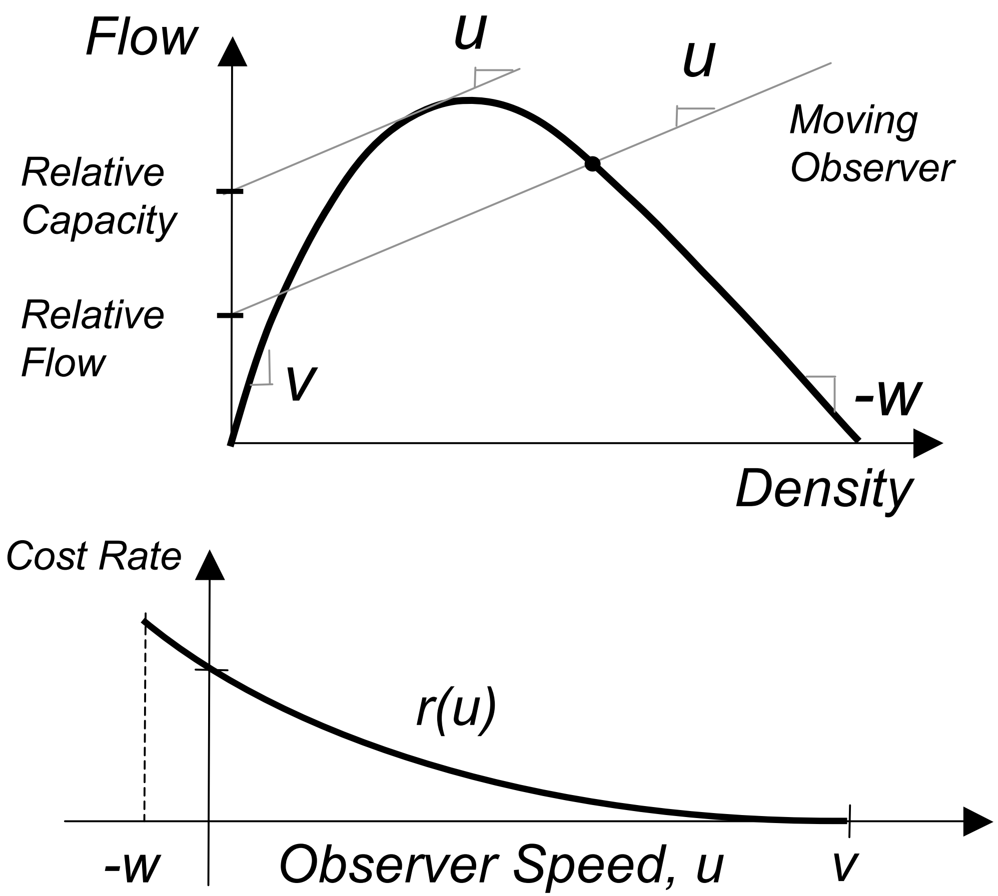


$$\frac{\partial N}{\partial t} = Q\left(-\frac{\partial N}{\partial x}\right)$$

Flow Density

(Daganzo, 2005; 2006)

(Q2) Related to KW?



$$\frac{\partial N}{\partial t} = Q\left(-\frac{\partial N}{\partial x}\right)$$

Flow Density

KW has a relative capacity function

(Daganzo, 2005; 2006)

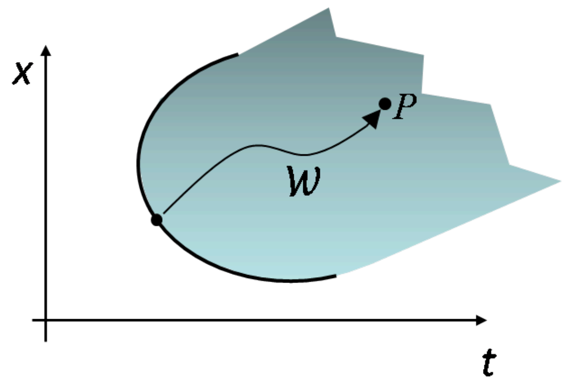
(Q2) Related to KW?

Solution must satisfy the relative capacity constraint:

$$N_P^{KW} \le inf\{N_{B(\mathcal{P})} + \Delta(\mathcal{P}) : \forall \mathcal{P}\} = N_P^{VT}$$

Solution defined by waves that satisfy:

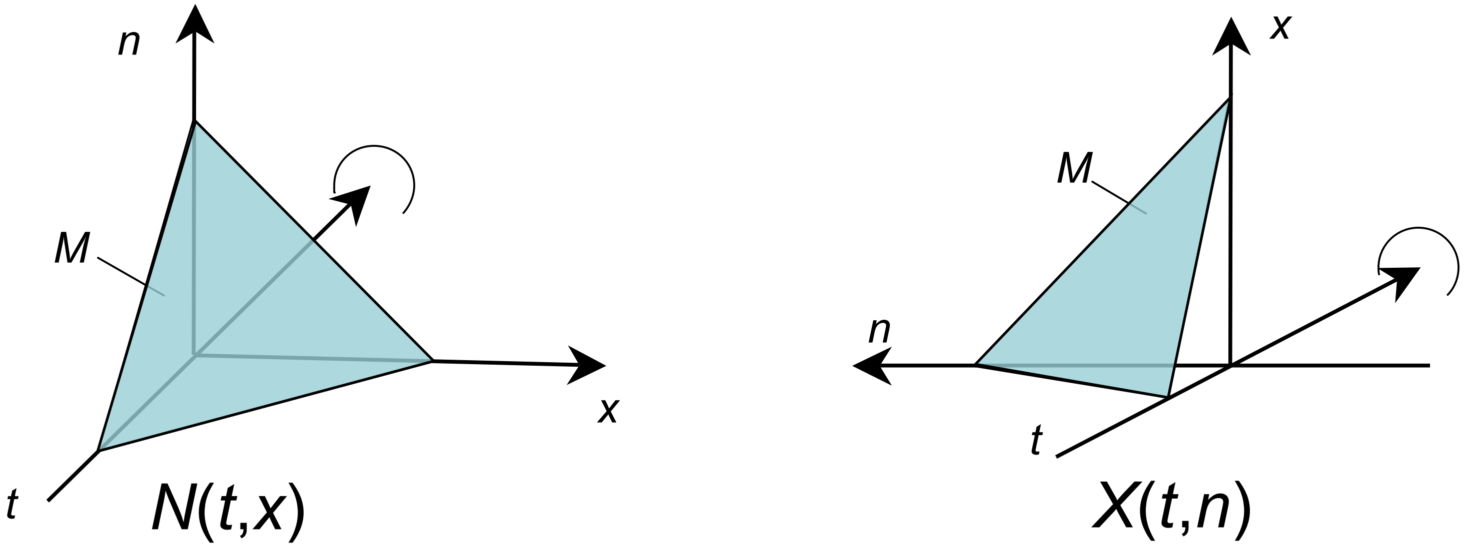
$$N_P^{KW} = N_{B(w)} + \Delta(\mathcal{W}) \ge inf\{N_{B(\mathcal{P})} + \Delta(\mathcal{P}) : \forall \mathcal{P}\} = N_P^{VT}$$



Hence: $N_P^{KW} = N_P^{VT}$

(Daganzo, 2005; 2006)

(Q2) Related to CF?

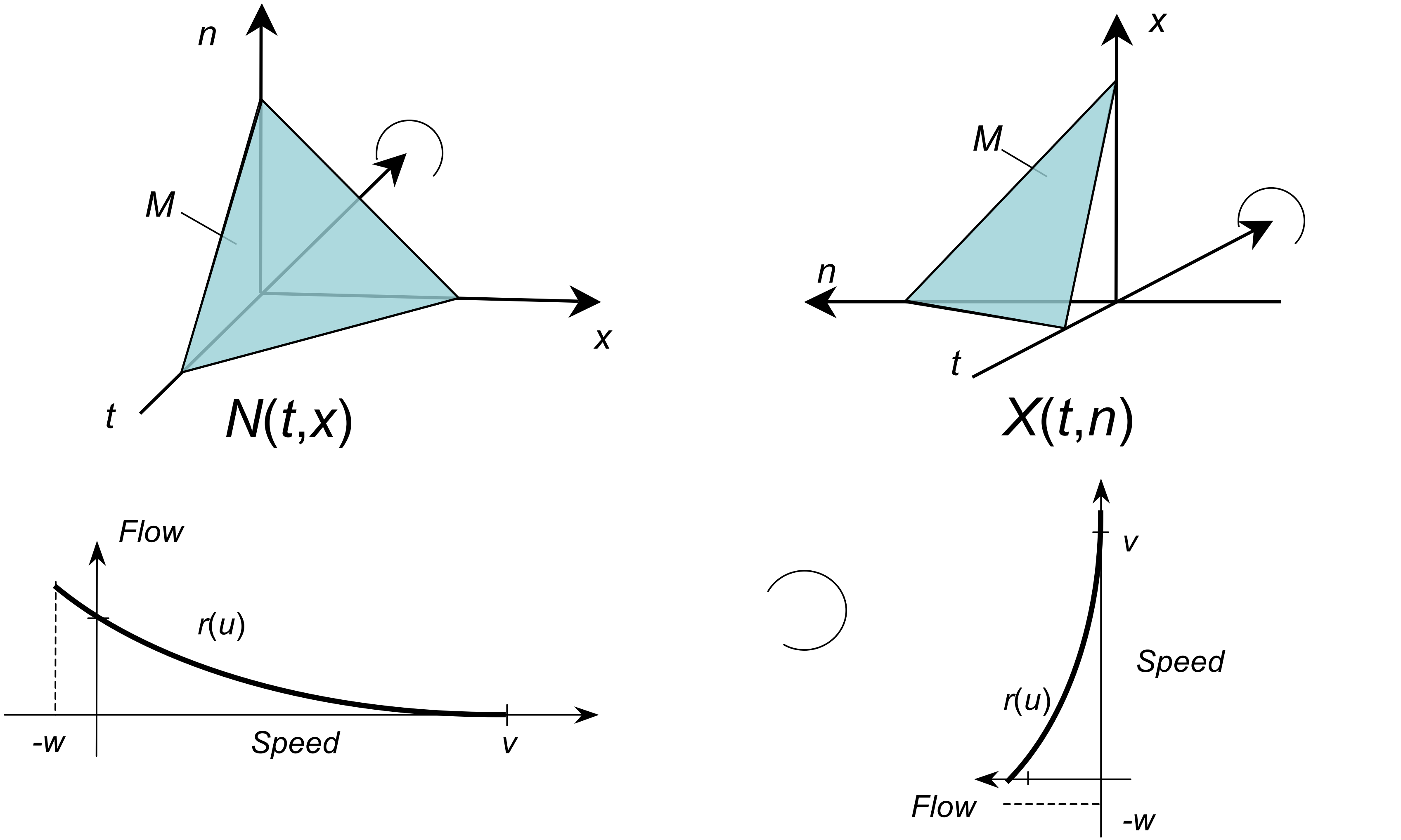


Expressions of the same *M*
to be found

Vehicles at given positions   Positions of given vehicles

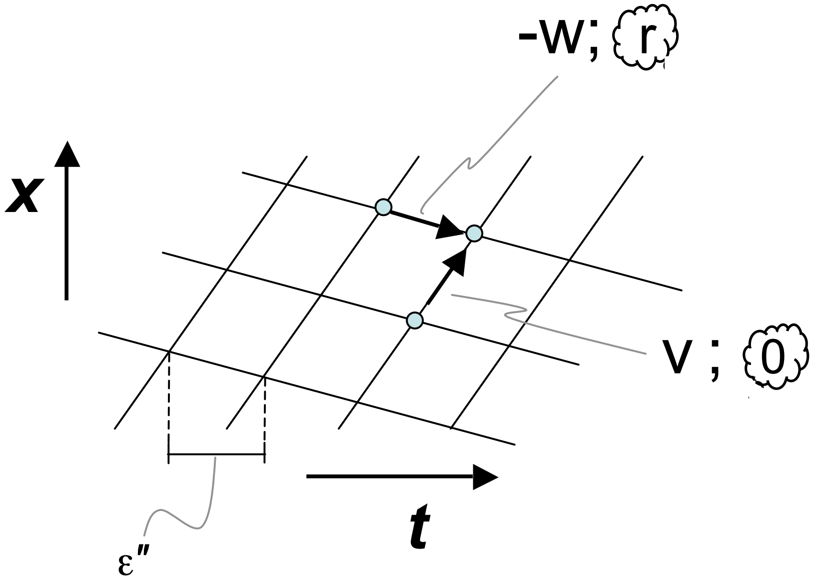
(Daganzo, 2006)

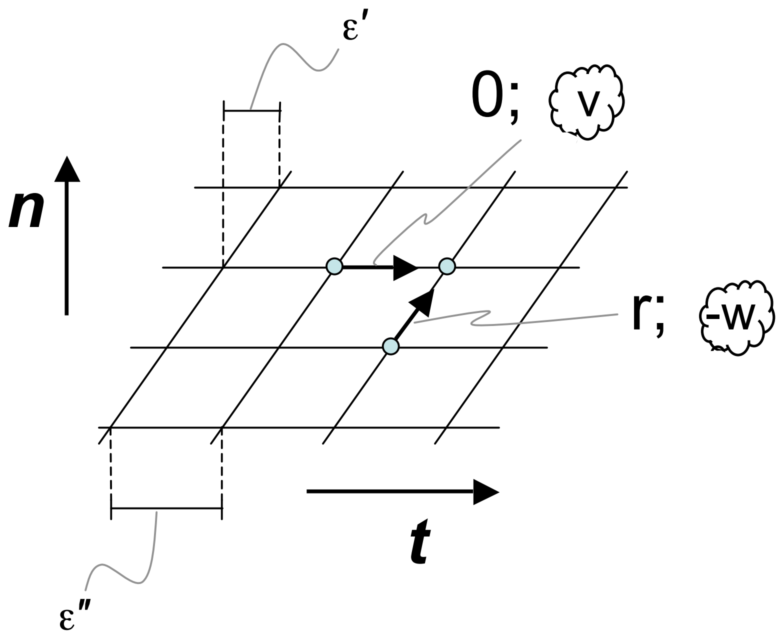
(Q2) Related to CF?



(Daganzo, 2006)

(Q2) Related to CF?



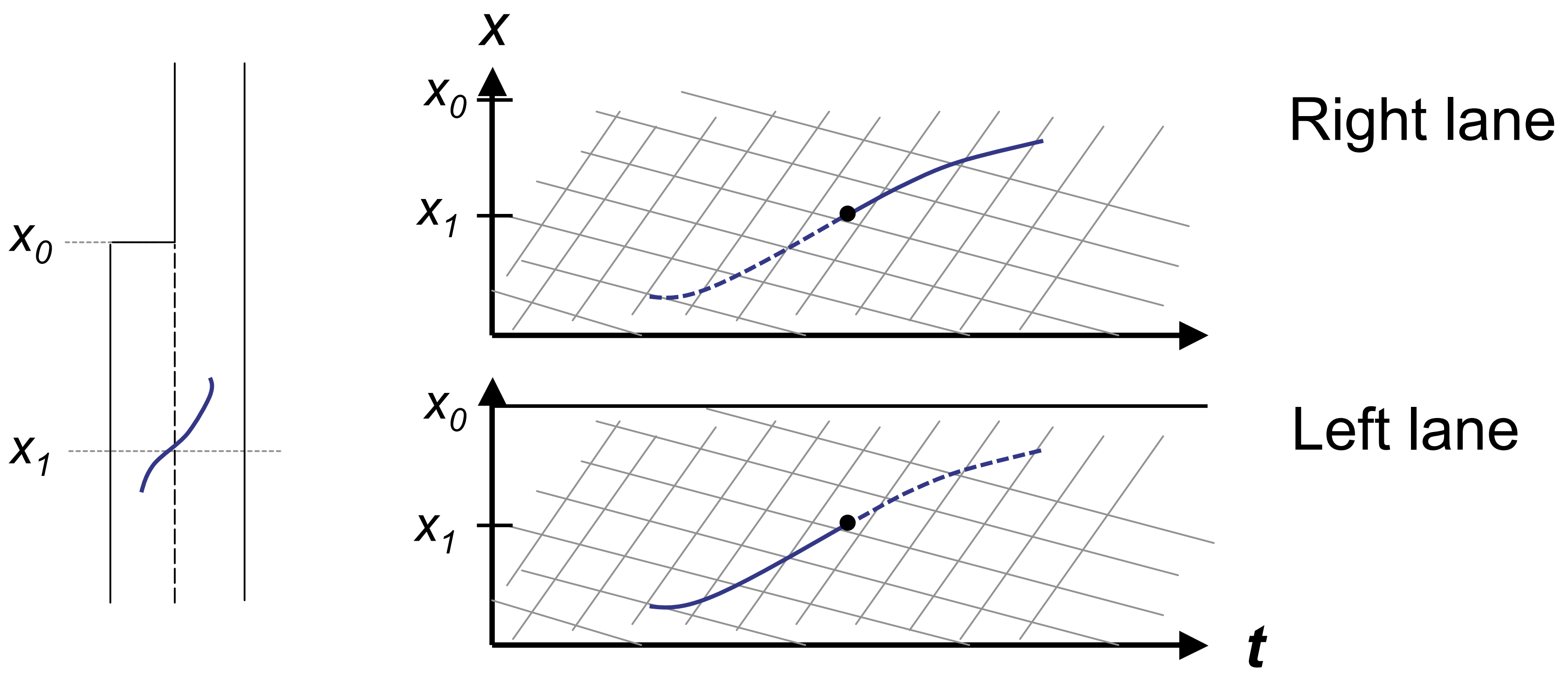


$$N_P = min\{N_{P'}; N_{P''} + r\varepsilon''\}$$

$$X_P = min\{X_{P'} + v\varepsilon'; X_{P''} - w\varepsilon''\}$$

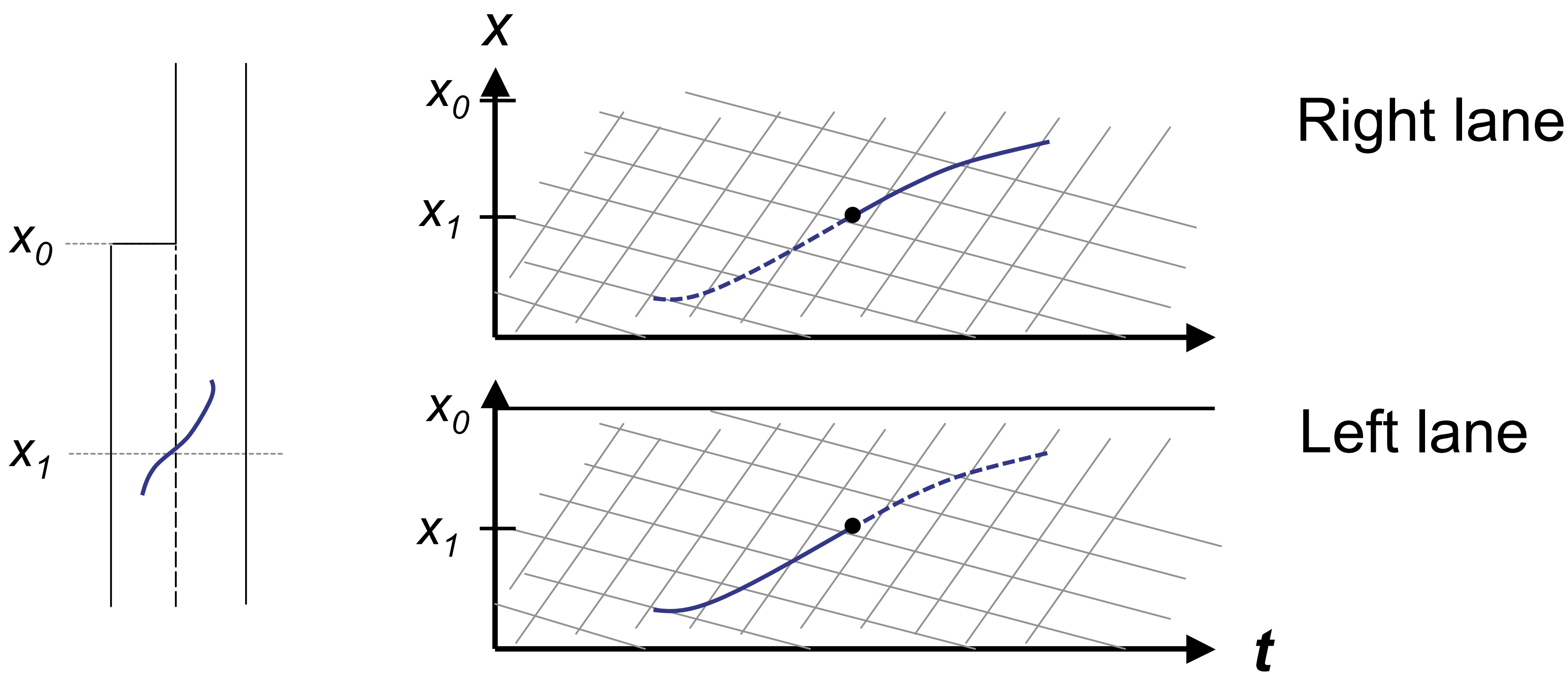
(Daganzo, 2006; 2006a)

(Q3) Realistic?



(Laval & Daganzo, 2006)

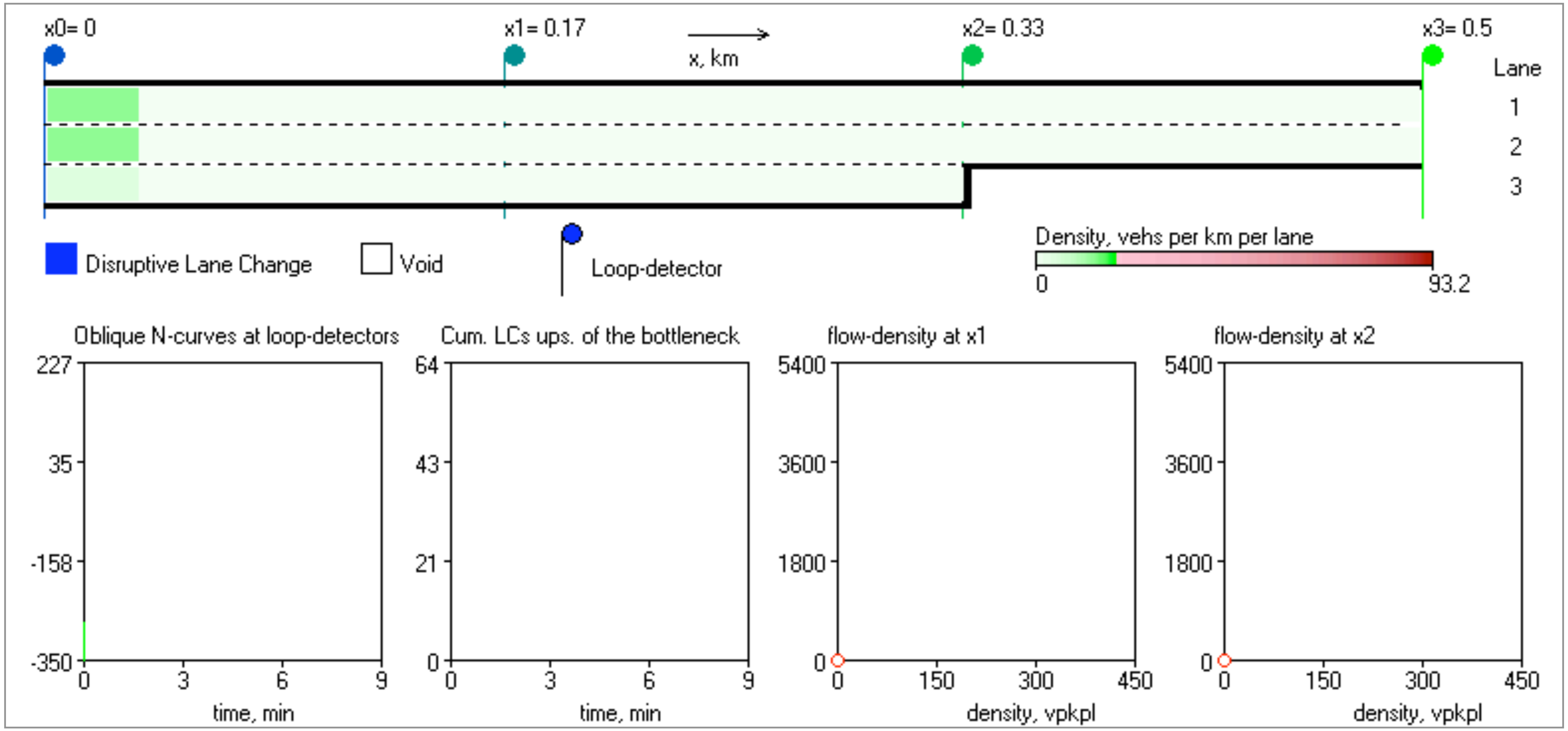
(Q3) Realistic?



Method is parsimonious:
- Optional moves → 1 behavior parameter (when)
- Mandatory moves → 1 behavior parameter (where)

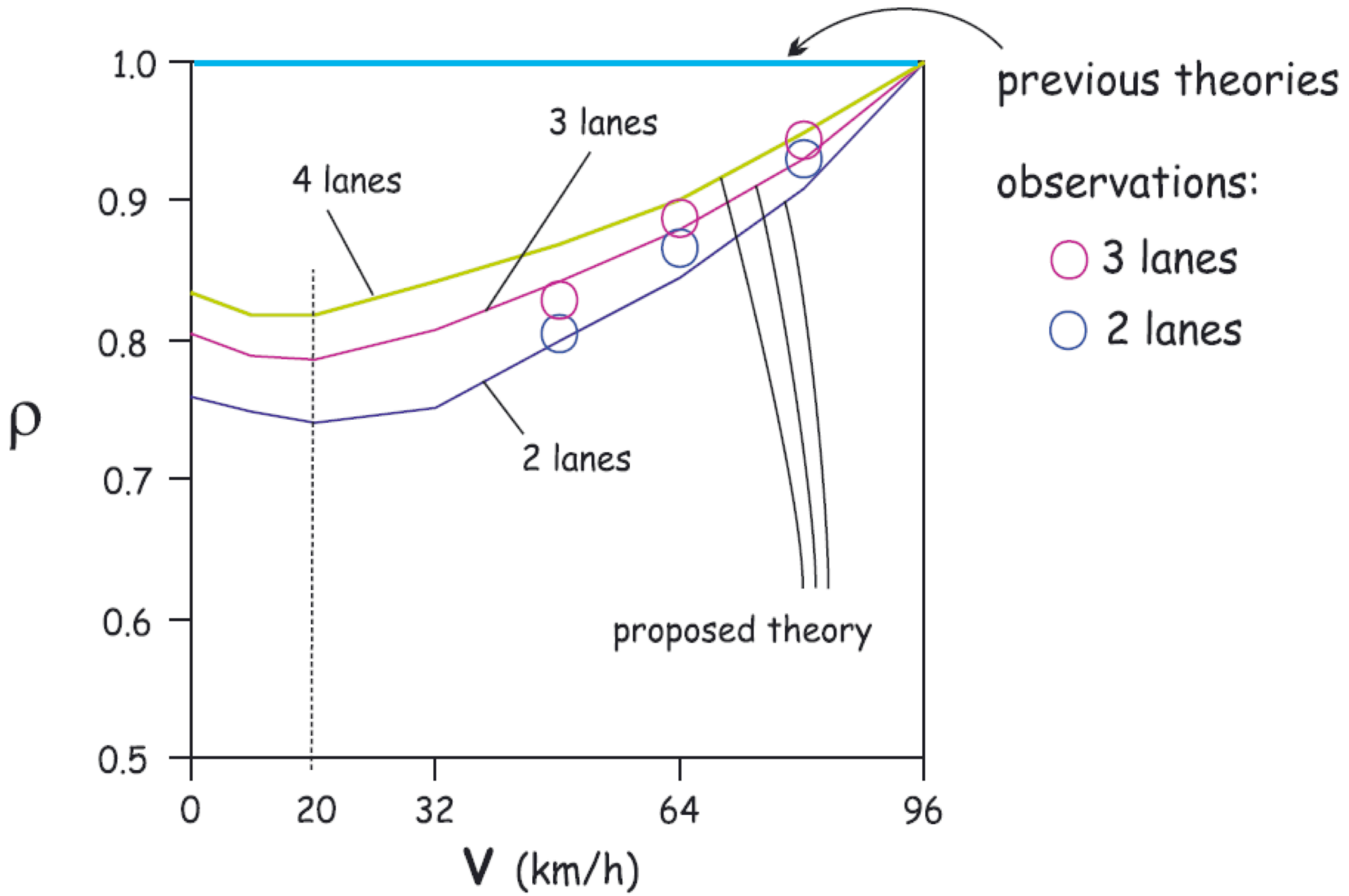
(Laval & Daganzo, 2006)

A Lane Drop Bottleneck



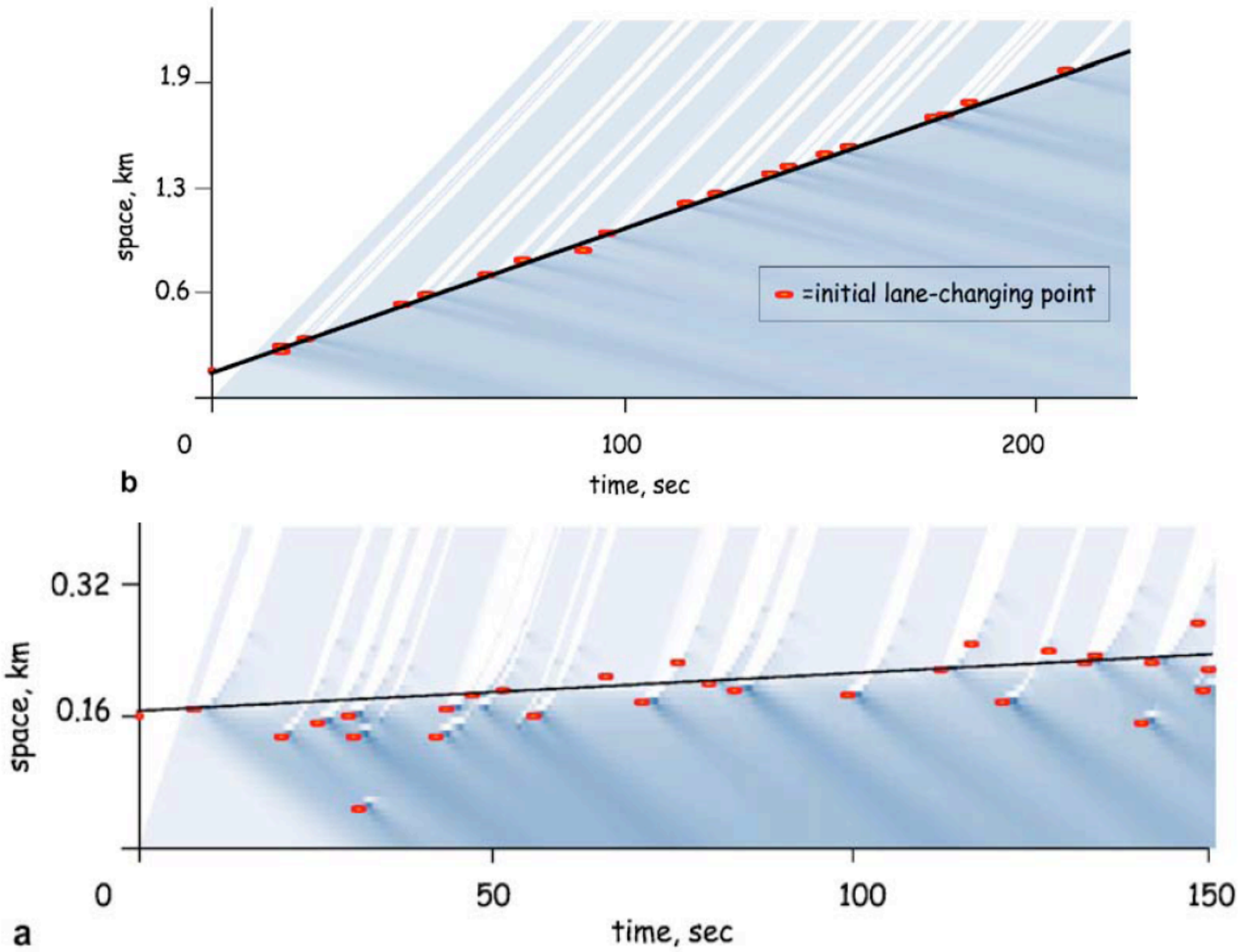
(Laval & Daganzo, 2006)

Capacity of Moving Bottlenecks



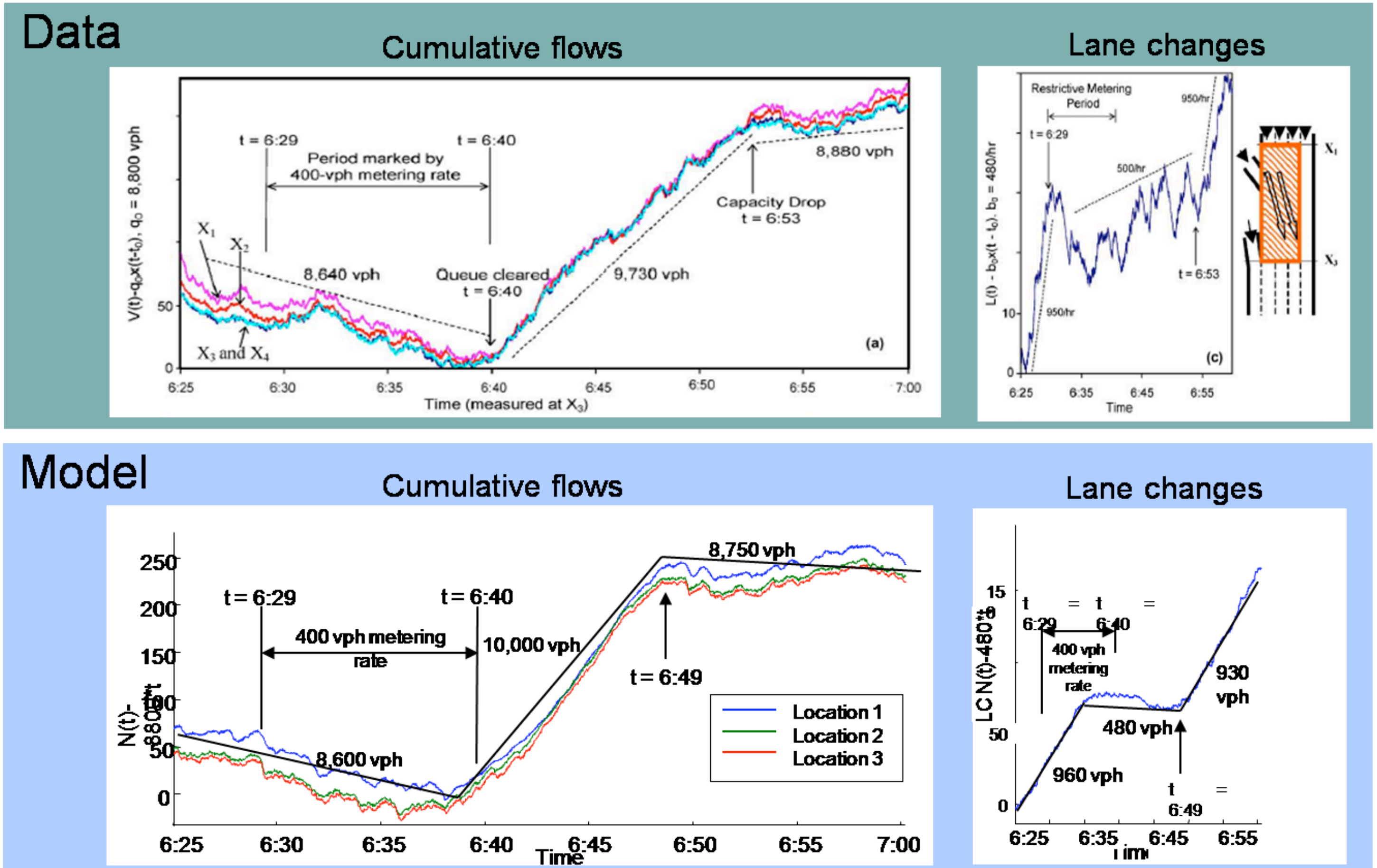
(Munoz & Daganzo, 2002); (Laval & Daganzo, 2006)

Causes of Moving Bottlenecks



(Laval & Daganzo, 2002)

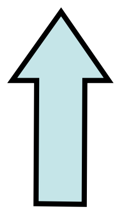
HOV and On-ramps: Mandatory and Optional Moves



(Laval & Cassidy and Daganzo, 2006)
(Menendez & Daganzo, 2007)

HOV with Mandatory Moves

Key: • HOV • LOV ◊ HOV Lane Designation

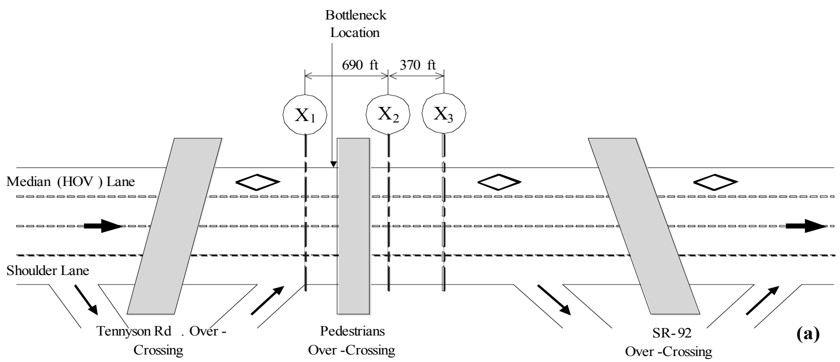


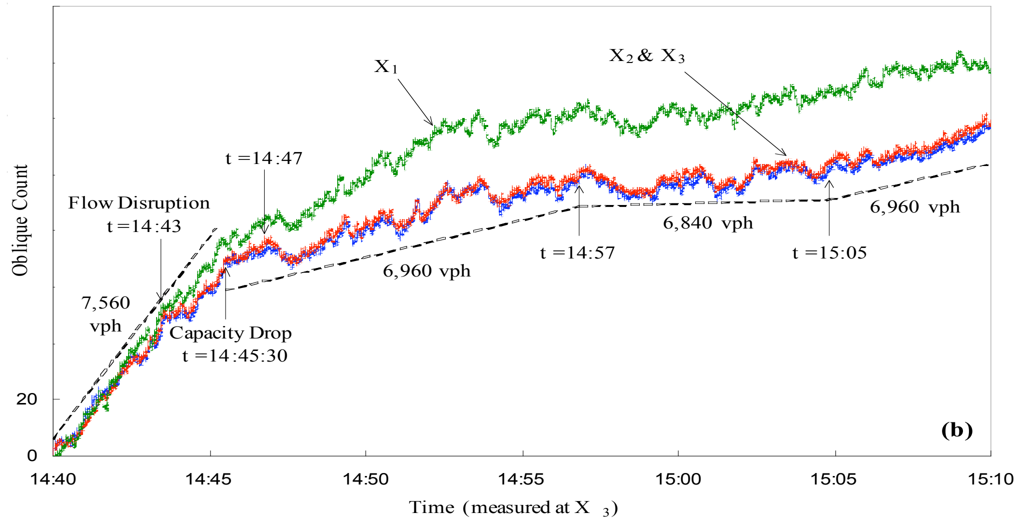
Discharge through bottleneck is greater with separation of traffic than without

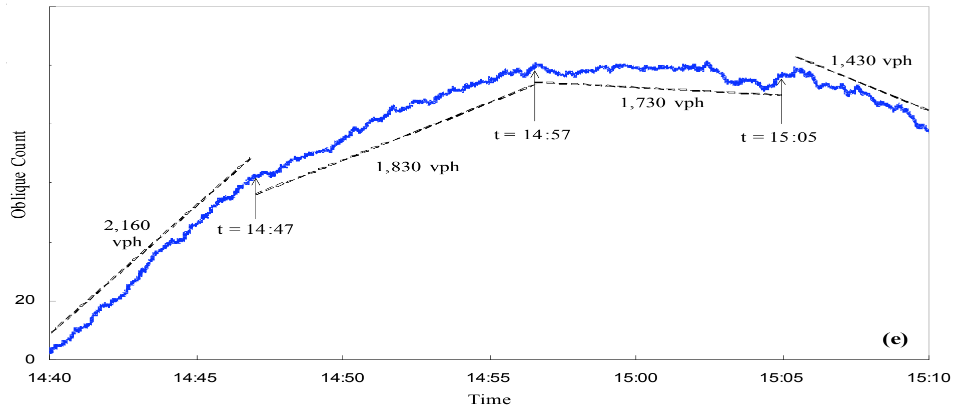
Bottleneck Discharge Flow

(Menendez & Daganzo, 2007)

The Smoothing Effect







(Cassidy & Daganzo & Jang and Chung, 2006)

Summary

- Variational Theory unifies different views of traffic
- Parallel streams can be composed with VT
- Results are parsimonious and realistic
- Larger systems, scaling?