On Continuum Models for Pedestrian Flows

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Macroscopic

Target:

Select a phenomenon:	When many people want to exit through a small door, the outflow may fall.
Basic assumptions:	The total number of pedestrians is conserved. $v = v(\rho)$. When-how-why does panic arise?
Simplify:	1D.
Write a model:	$\partial_t \rho + \partial_x \left(\rho\, v(\rho)\right) = 0 + \ldots$
Qualitative properties:	Does the model describe reduced outflows?
Optimal management	What are the optimal shape and position of an obstacle in front of the door?

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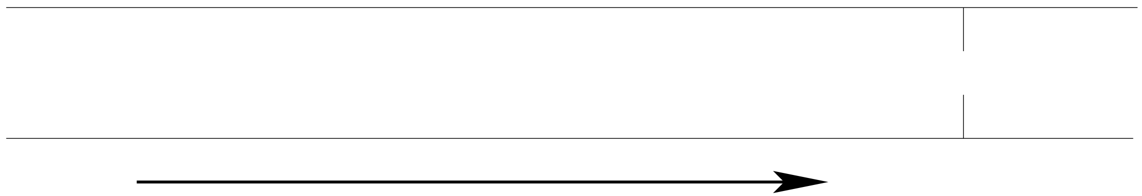
Simplify:	**1D.**
Write a model:	$\partial_t \rho + \partial_x (\rho\, v(\rho)) = 0$ + Non Classical Shocks
Qualitative properties:	Does the model describe reduced outflows?
Optimal management	What are the optimal shape and position of an obstacle in front of the door?

1D

1D

For example:

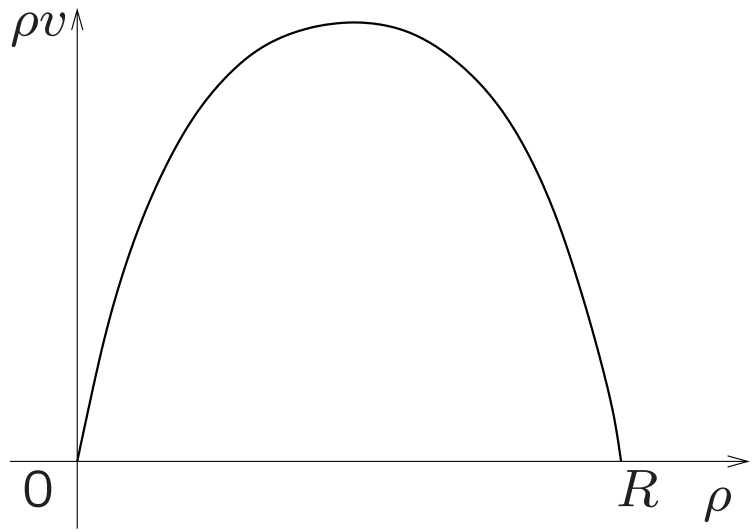


(2D work in progress)

The total number of pedestrians is conserved

$$v = v(\rho)$$

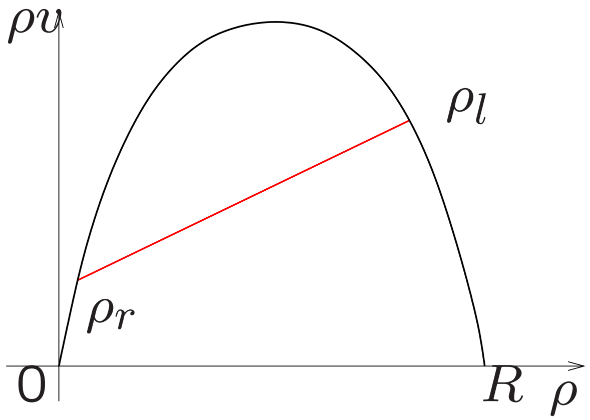
$$\partial_t \rho + \partial_x \left(\rho\, v(\rho)\right) = 0$$

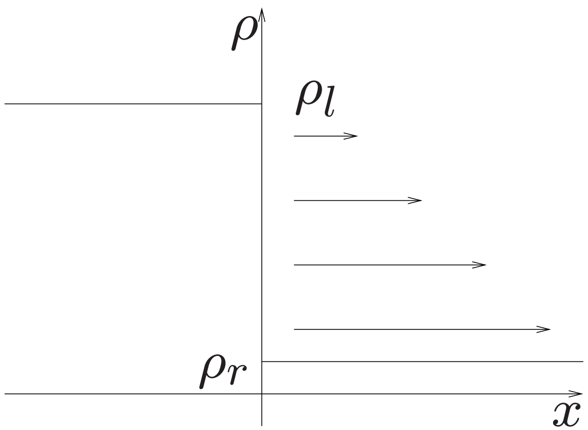


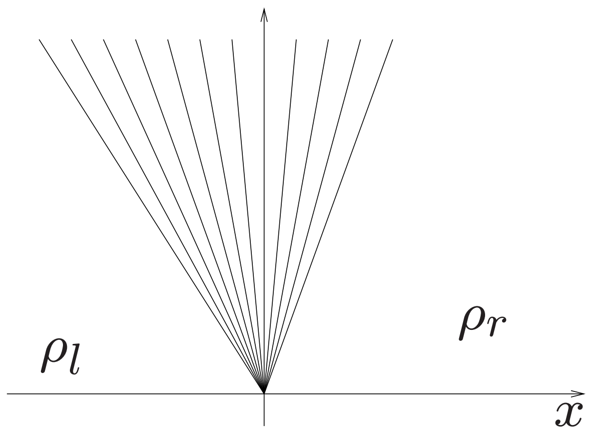
Standard situations

The Classical Riemann Problem $\begin{cases} \partial_t \rho + \partial_x (\rho\, v(\rho)) = 0 \\ \rho(0,x) = \begin{cases} \rho_l & x < 0 \\ \rho_r & x > 0 \end{cases} \end{cases}$

$$\rho_l > \rho_r ,$$







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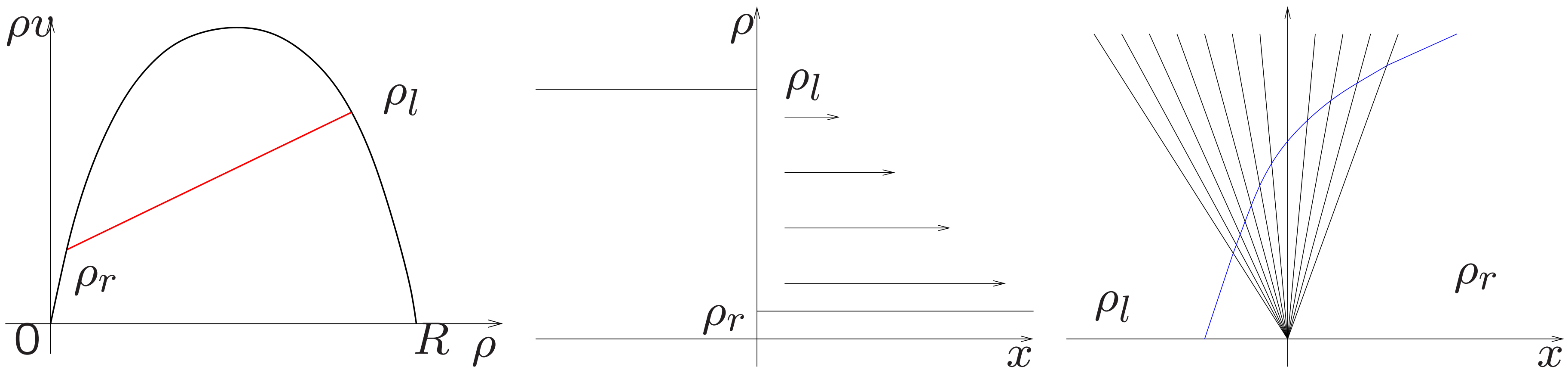

$$\dot{x} = v\,(\rho(t,x))$$

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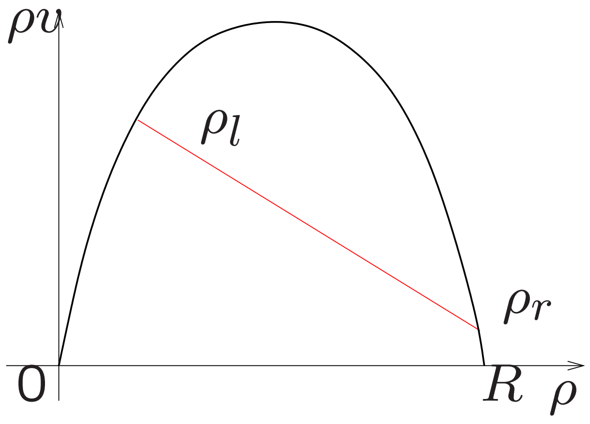

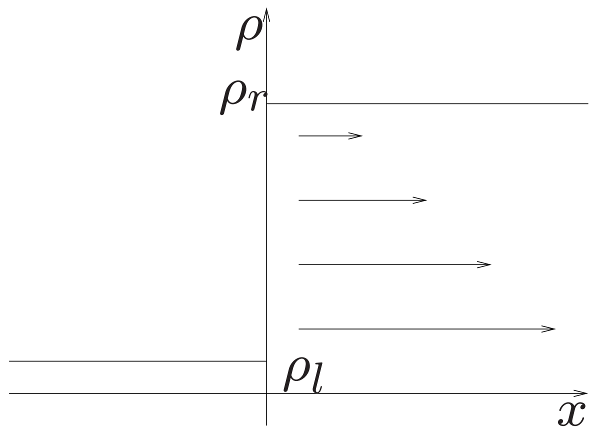

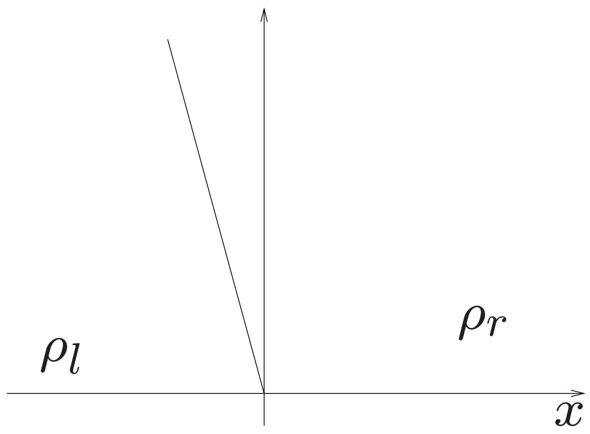

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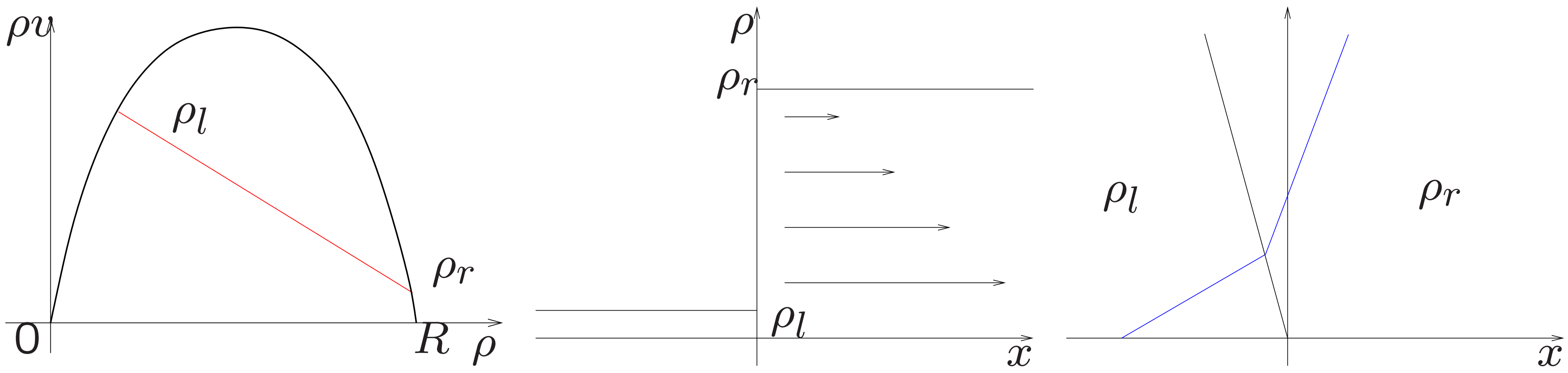
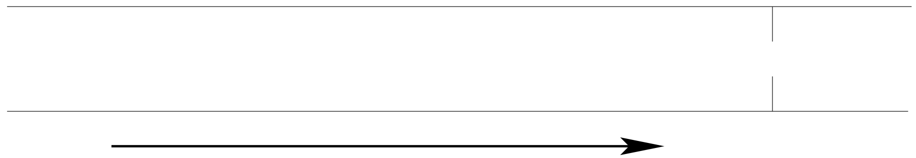


$$\dot{x} = v\left(\rho(t,x)\right)$$

Modeling Pedestrian Flow through $\partial_t \rho + \partial_x [\rho \, v(\rho)] = 0$



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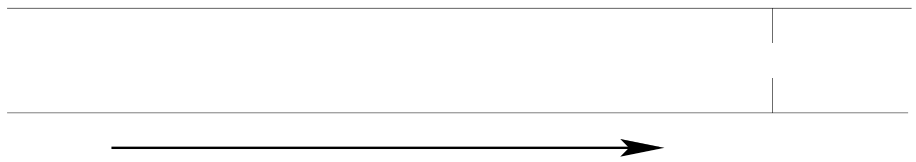
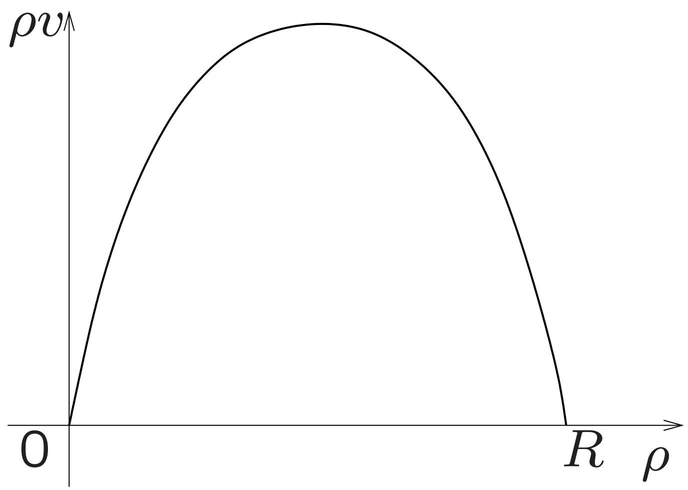
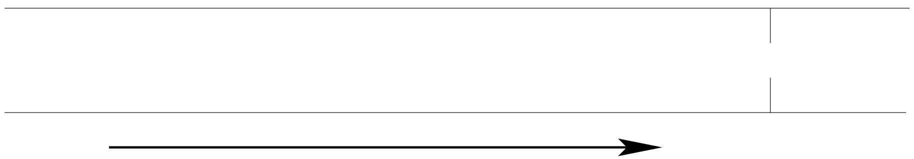


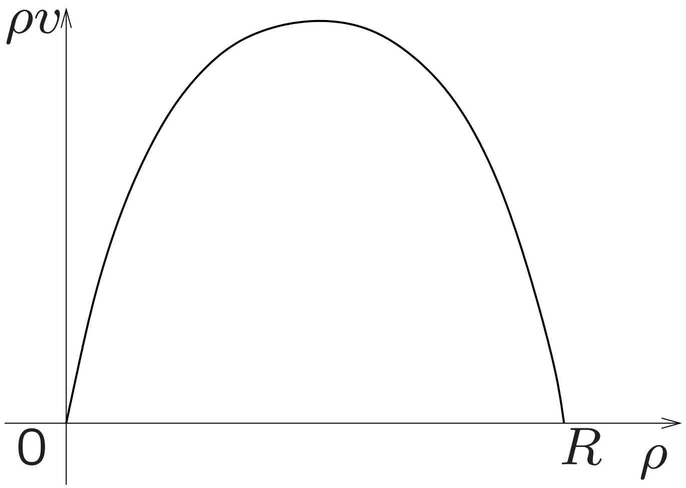


Maximum Principle

Standard Construction

Modeling Pedestrian Flow through $\partial_t \rho + \partial_x [\rho\, v(\rho)] = 0$





Maximum Principle

Standard Construction

No panic effects.

Even with unilateral constraints.

(Colombo & Goatin, JDE 2007)

Panic:

overcompression

panic states

transition to panic

Panic:

overcompression

panic states

transition to panic

$\Rightarrow$ drop in the door outflow

Panic:

overcompression

panic states

transition to panic

$$\left\{\begin{array}{l}\partial_t\rho + \partial_x\left(\rho\, v(\rho)\right) = 0\\ \rho(0,x) = \rho_o(x)\end{array}\right. \qquad \Rightarrow \qquad \boxed{\boxed{\text{Maximum Principle}}}$$

$$\begin{array}{llllllll}\text{if} & \rho_o(x) & \in & [\rho_{\min}, \rho_{\max}] & \forall x & \in & \mathbb{R} & \\ \text{then} & \rho(t,x) & \in & [\rho_{\min}, \rho_{\max}] & \forall x & \in & \mathbb{R} & \forall\, t \in [0, +\infty[\end{array}$$

Introduce “panic states”

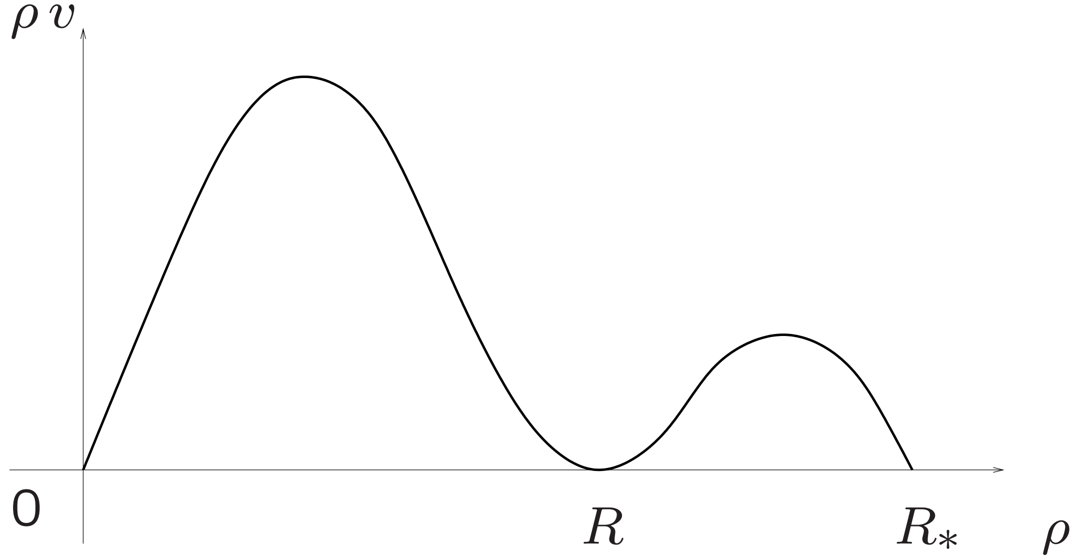
Extend the speed law

Change the evolution

$$\left.\begin{array}{l}\text{Introduce “panic states”}\\ \\ \text{Extend the speed law}\end{array}\right\} \Rightarrow \quad [0, R] \longrightarrow [0, R_*]$$

Change the evolution $\Rightarrow$ **Nonclassical Shocks**

Modeling Pedestrian Flow through $\partial_t \rho + \partial_x [\rho\, v(\rho)] = 0$



$[0, R]$ standard states $\quad$ $]R, R_*]$ "panic" states

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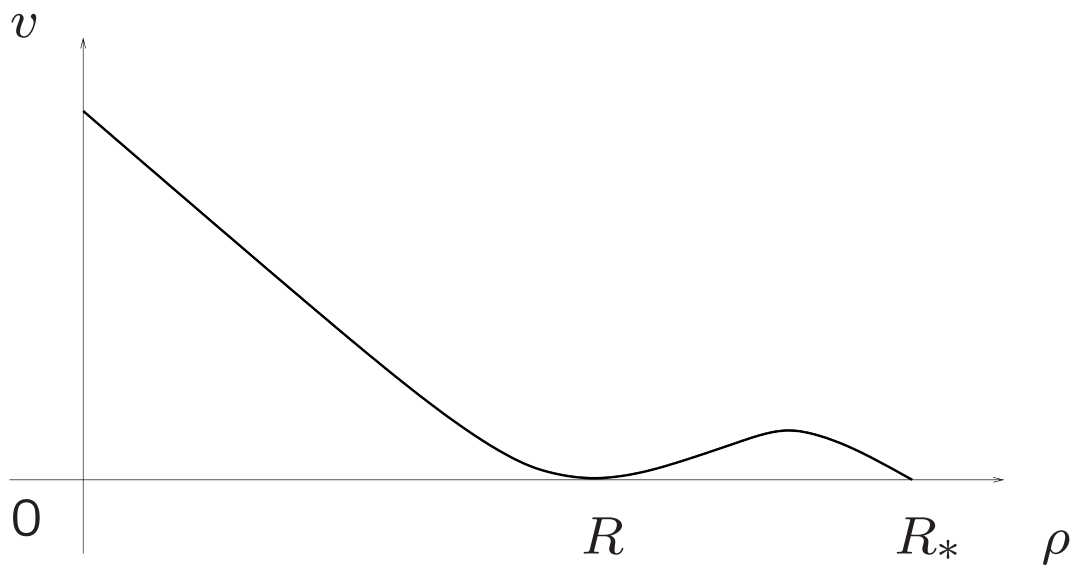

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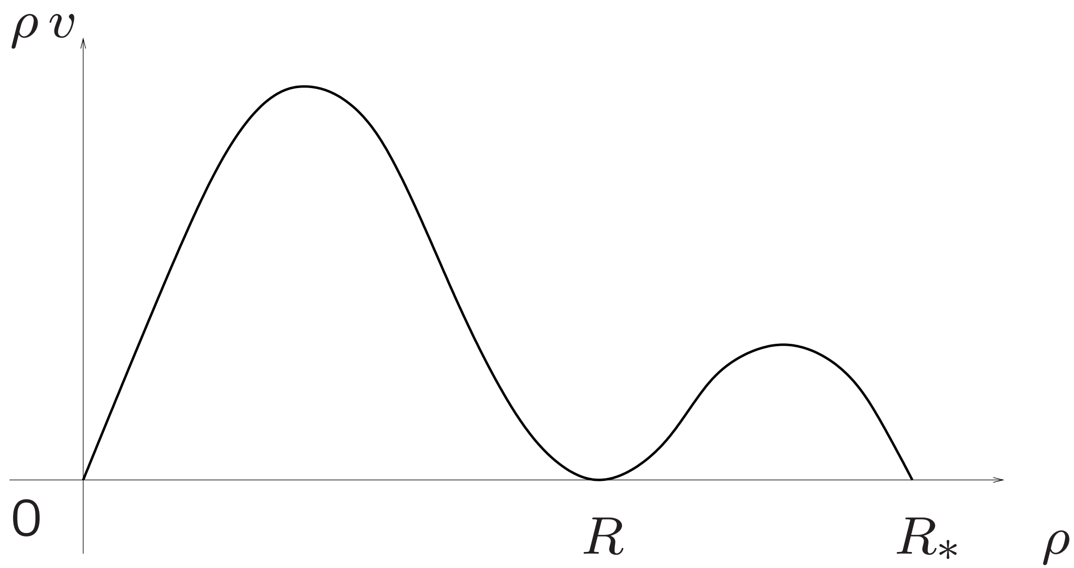

NON Classical Shocks $\Rightarrow$ No Maximum Principle $\Rightarrow$ Overcompression

(Colombo & Rosini, M^{2}AS, 2005)

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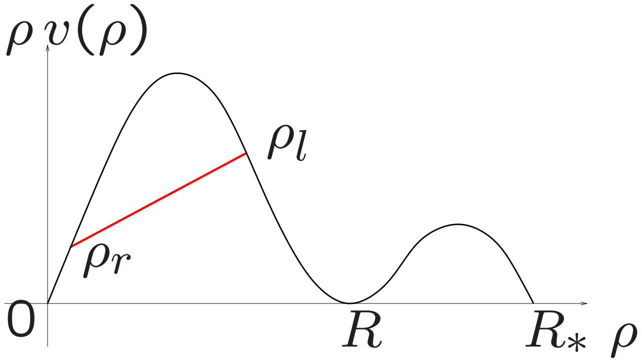

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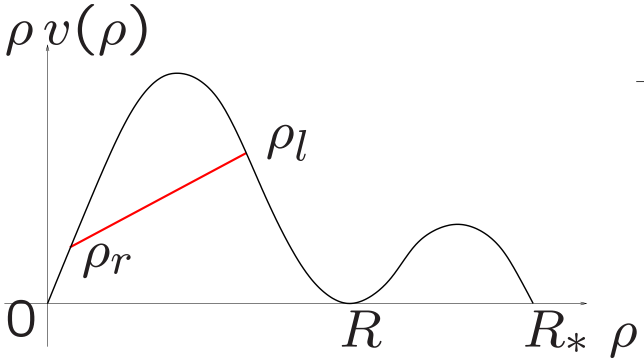

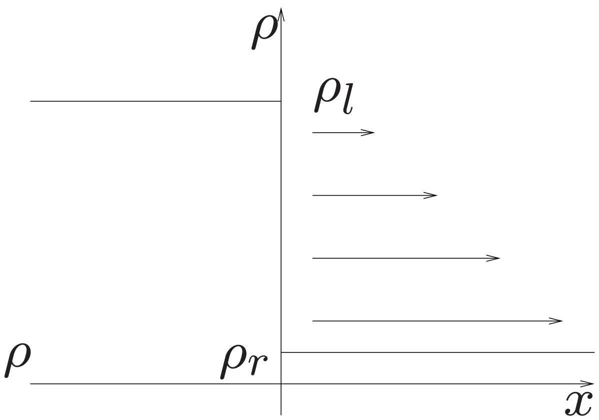

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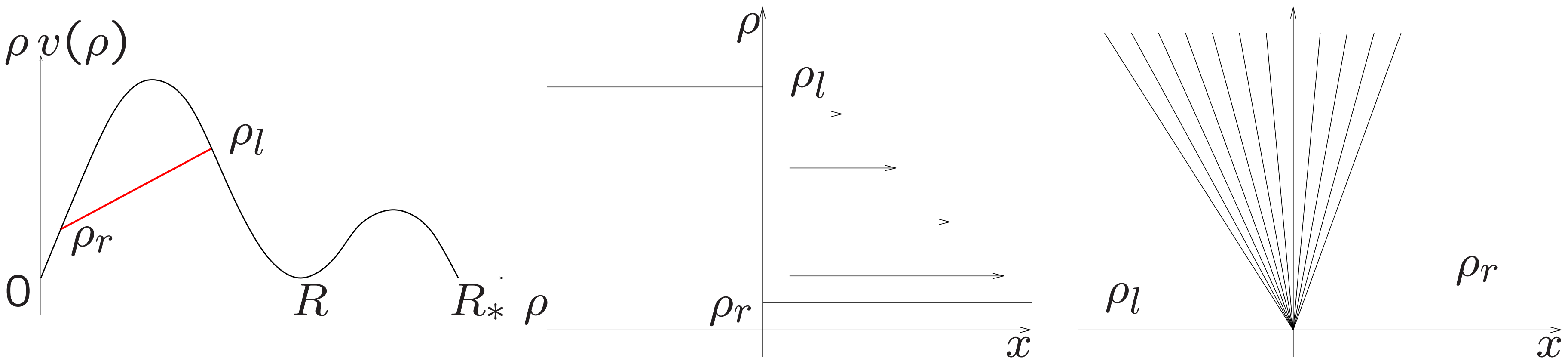

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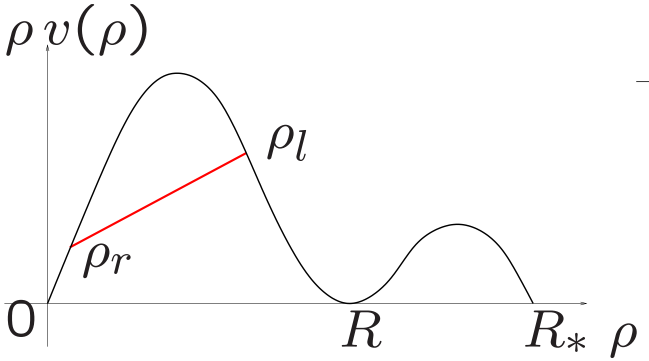

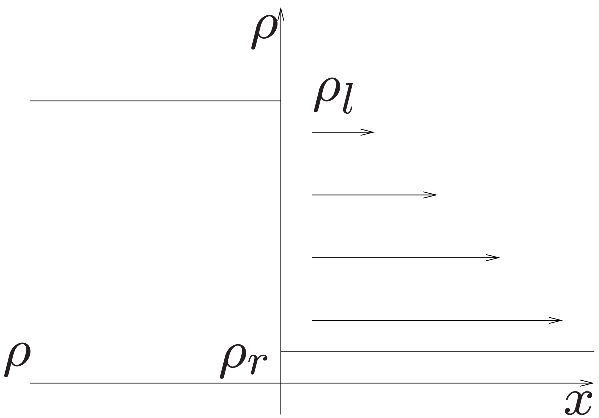

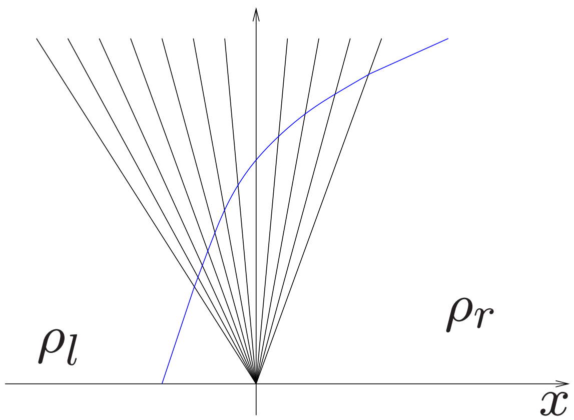

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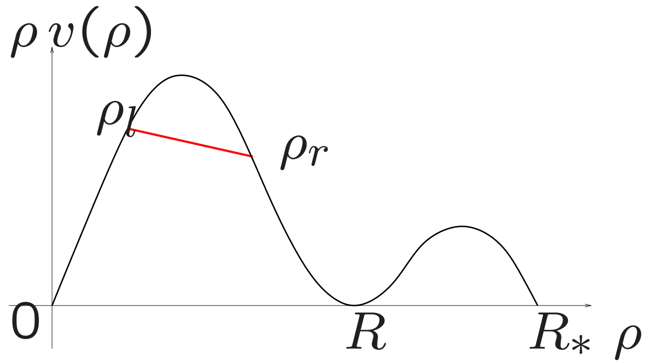

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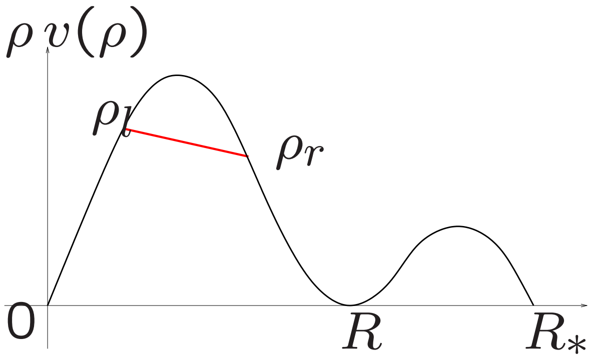

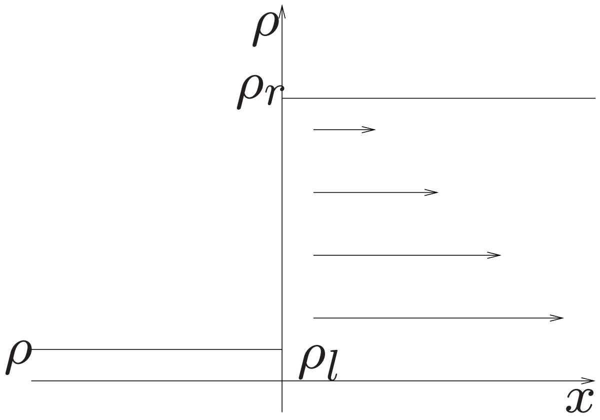

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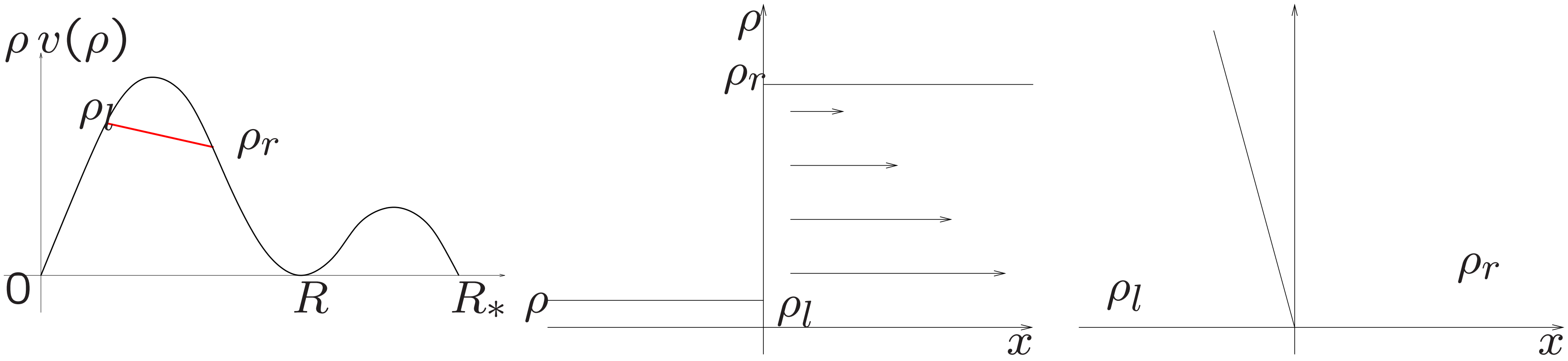

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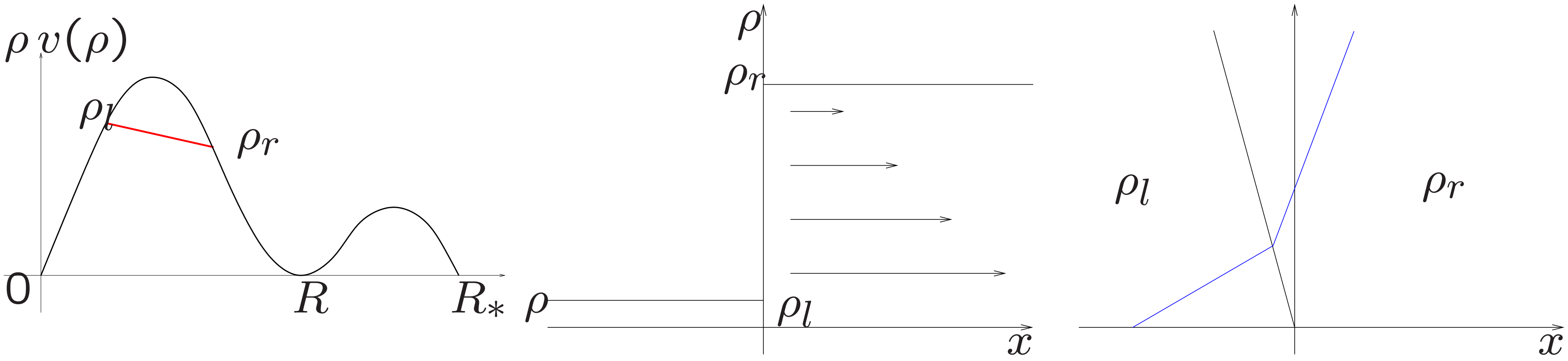

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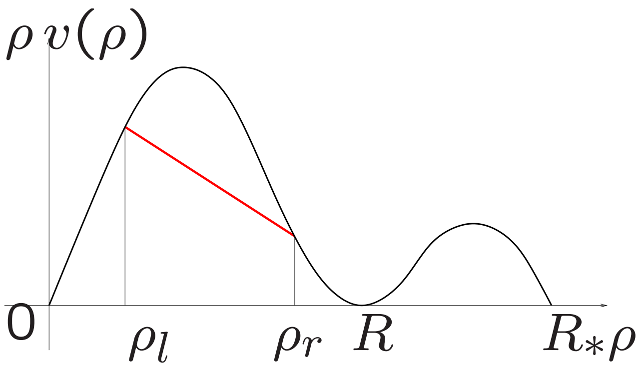

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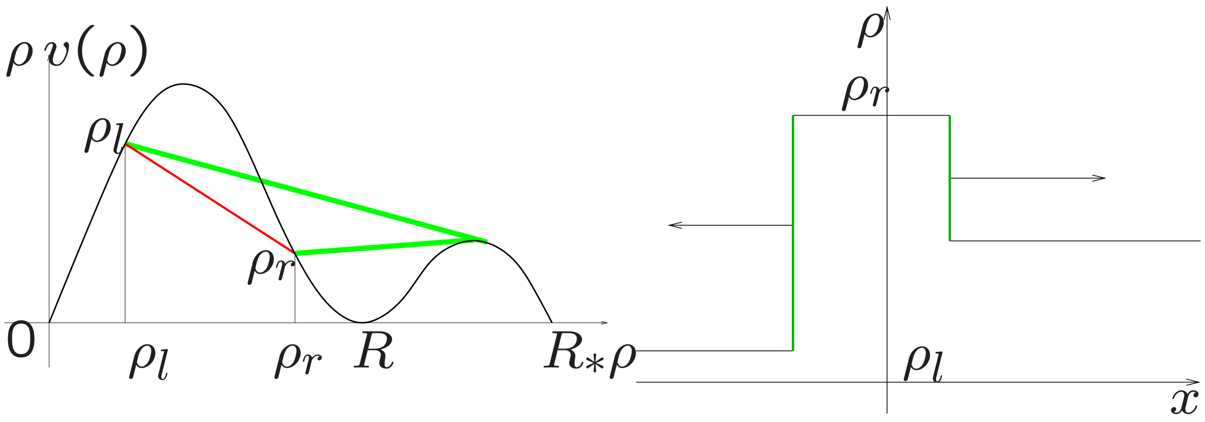

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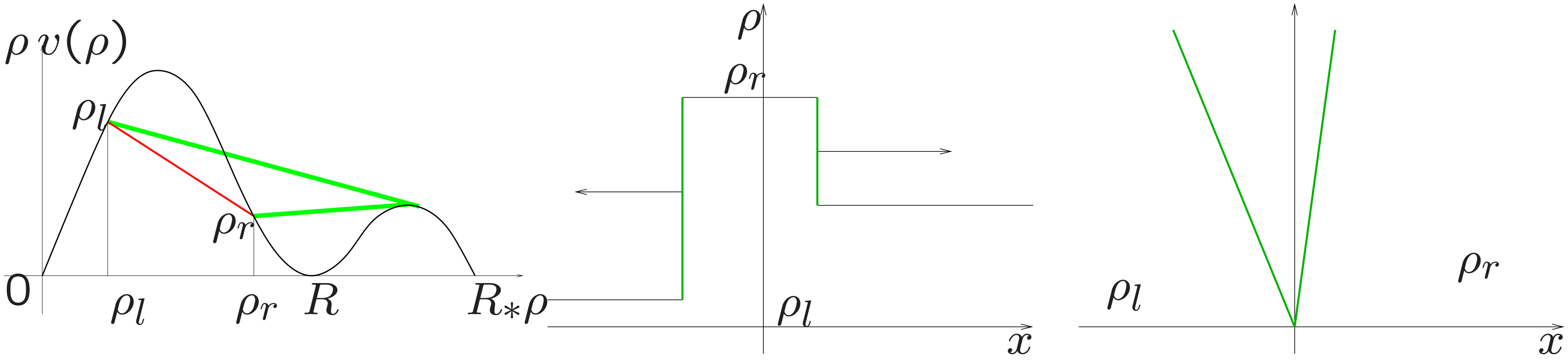

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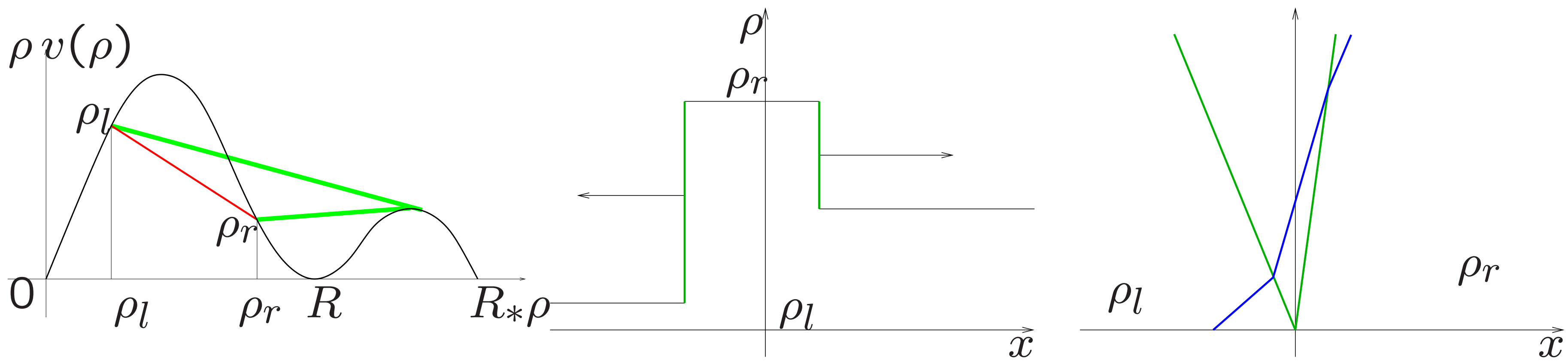

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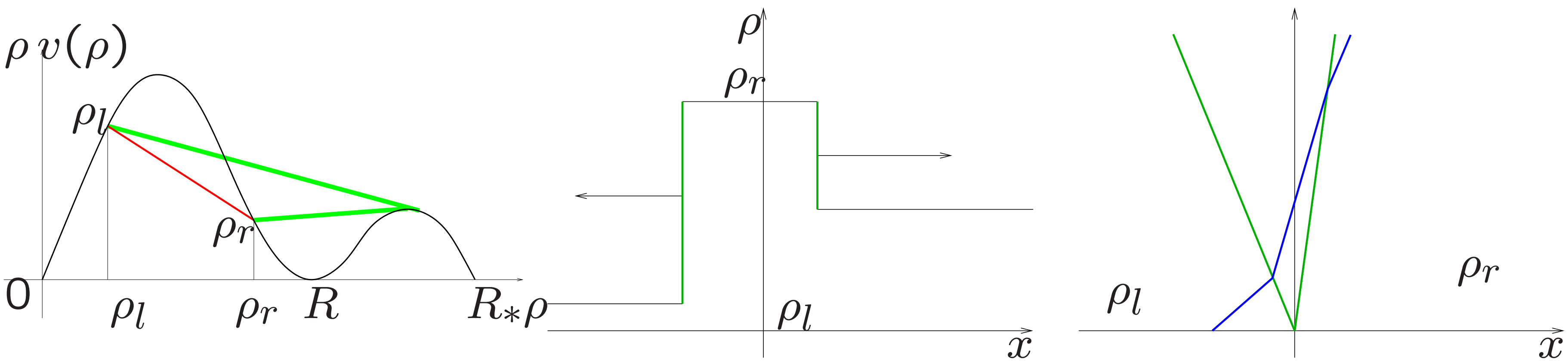
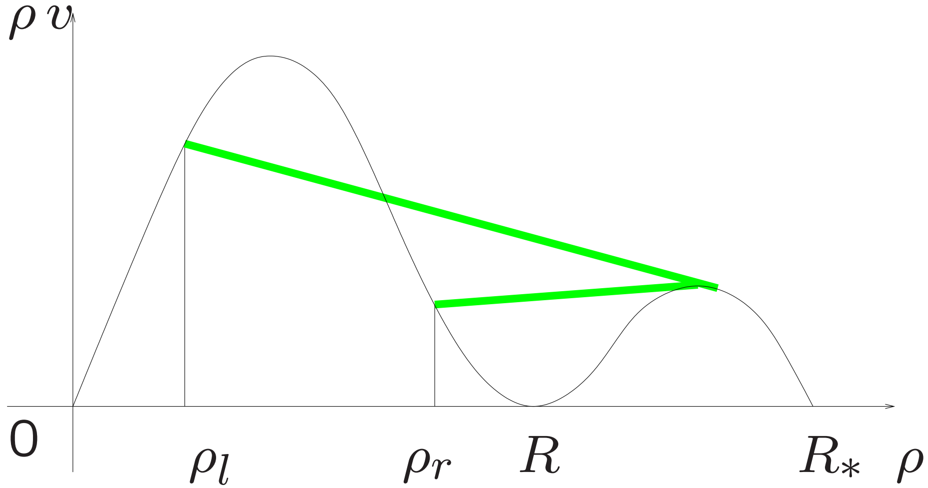


The Maximum Principle is violated!

Non Classical Solution to the Riemann Problem



$$\left.\begin{array}{r l} \rho_r - \rho_l > & \text{threshold} \\ \rho_l > & \text{threshold} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \textcolor{red}{\text{non}}\text{classical solution} \\ \text{no Maximum Principle} \\ \text{transition into panic states} \end{array}\right.$$

$$\text{Non Classical Solution to} \left\{ \begin{array}{l} \partial_t \rho + \partial_x \left(\rho\, v(\rho)\right) = 0 \\ \rho(0,x) = \left\{ \begin{array}{lll} \rho_l & x & < & 0 \\ \rho_r & x & > & 0 \end{array} \right. \end{array} \right.$$

$$\text{If} \quad \begin{array}{rcl} \rho_l & > & \text{threshold} \\ \rho_r - \rho_l & > & \text{threshold} \end{array} \quad \longrightarrow \quad \text{nonclassical shock}$$

(possibly also with classical waves).

$$\text{Otherwise} \quad \longrightarrow \quad \text{classical solution}$$

$$\text{Non Classical Solution to} \begin{cases} \partial_t \rho + \partial_x (\rho\, v(\rho)) = 0 \\ \rho(0,x) = \begin{cases} \rho_l & x < 0 \\ \rho_r & x > 0 \end{cases} \end{cases}$$

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(possibly also with classical waves).

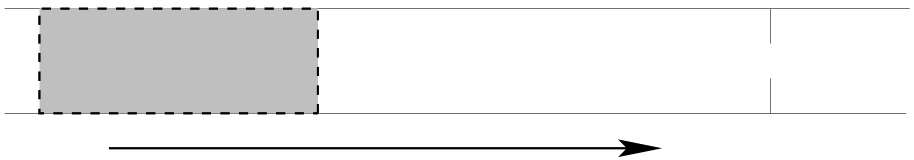
Otherwise $\rightarrow$ classical solution

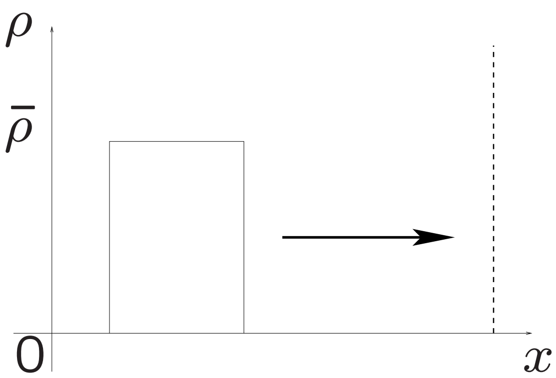
Theorem

The unique Riemann Solver so defined is
$\mathbf{L^1_{loc}}$ continuous whenever ρ_l and $\rho_r - \rho_l \neq$threshold.

(Colombo & Rosini, M^2AS, 2005)

Pedestrian Flow



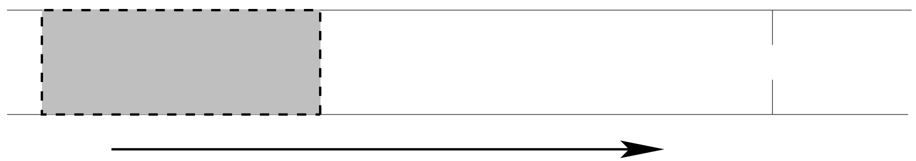


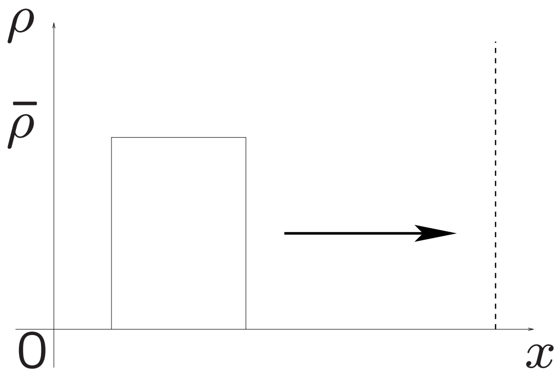
$$\begin{cases} \partial_t \rho + \partial_x \left(\rho\, v(\rho)\right) = 0 \\ \rho(0,x) = \text{as above} \\ \left(\rho\, v(\rho)\right)(t,L) \leq \bar{q} \end{cases}$$

Riemann Problems

Wave Front Tracking

Pedestrian Flow





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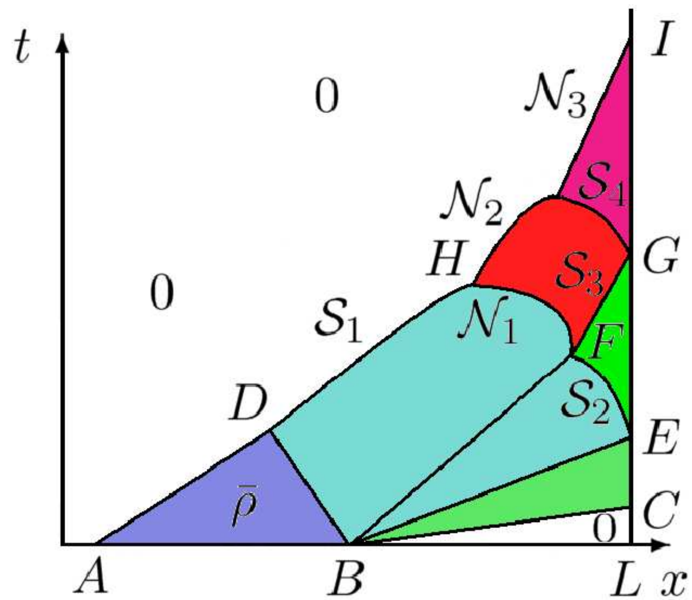
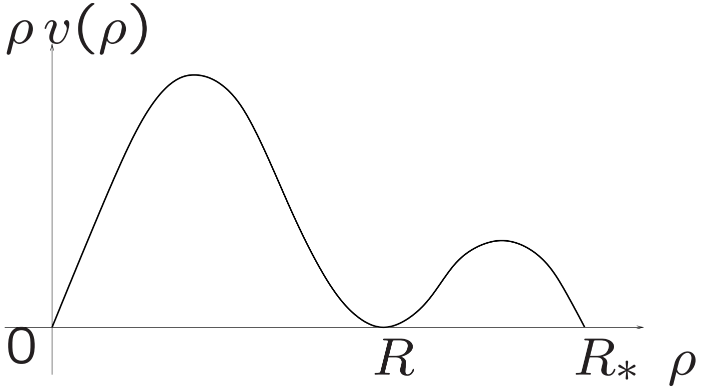
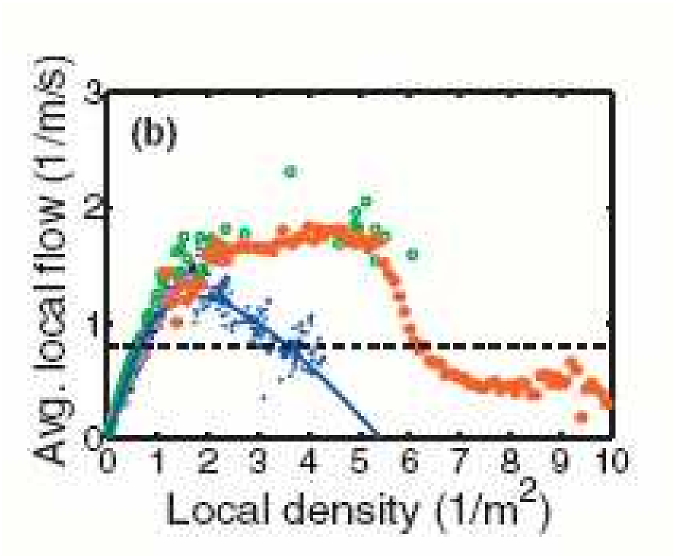


Criticisms:

1. 1D
2. bound on the total variation
3. lack of continuous dependence





Colombo & Rosini, M^2AS, 2005

Helbing, Johansson & Al-Abideen
Physical Review E, 2007

2D

$$\boxed{\boxed{\text{2D: } \partial_t \rho + \operatorname{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0}}$$

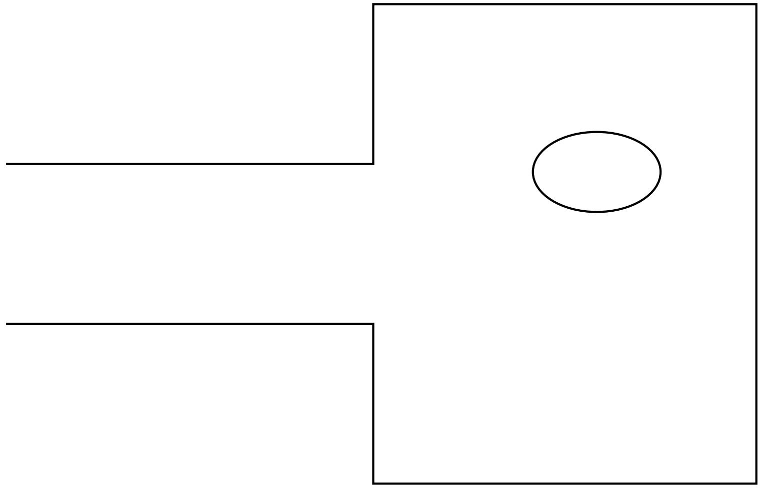
$$\mathbf{v} = ?$$

(Bressan & Colombo; Colombo & Facchi, works in progress.)

$$\boxed{\boxed{\text{2D: } \partial_t \rho + \mathrm{div}\,(\rho\,(1-\rho)\,\mathbf{v}) = 0}}$$

$$\begin{aligned} \mathbf{v} &= -\frac{\nabla\Phi(x,y)}{\|\nabla\Phi(x,y)\|} \\ \Phi(x,y) &= \text{distance from the destination} \\ &\Rightarrow \text{trails} \end{aligned}$$

$$\boxed{\text{2D: } \partial_t \rho + \operatorname{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0} \qquad \mathbf{v} = -\frac{\nabla\Phi(x,y)}{\|\nabla\Phi(x,y)\|}$$



$$\text{2D: } \partial_t \rho + \operatorname{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0 \qquad \mathbf{v} = -\frac{\nabla\Phi(x,y)}{\|\nabla\Phi(x,y)\|}$$

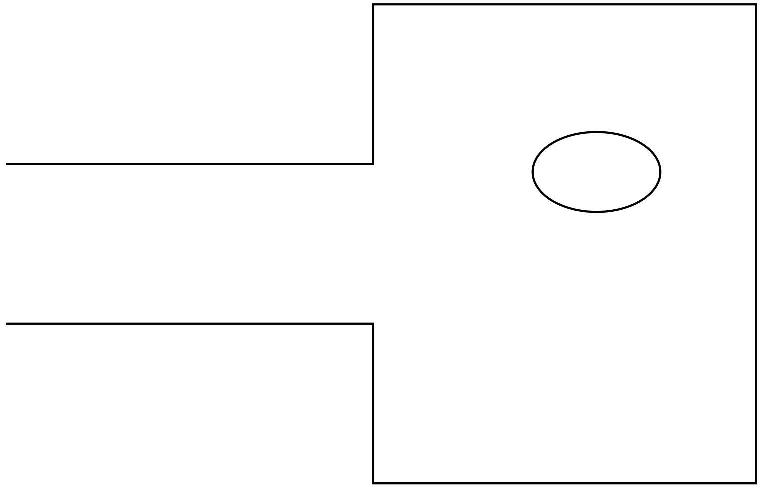
`http://dm.ing.unibs.it/rinaldo/panic/disco00.avi`

$$\boxed{\text{2D: } \partial_t \rho + \operatorname{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0}$$

$$\begin{aligned}
\mathbf{v} &= -\frac{\nabla\Phi(x,y)}{\|\nabla\Phi(x,y)\|} - \eta\frac{\nabla\Psi(\rho)}{\sqrt{1+\|\nabla\Psi(\rho)\|^2}} \\
\Phi(x,y) &= \text{distance from the destination} \\
\Psi(\rho) &= \text{discomfort}
\end{aligned}$$

$$\boxed{\text{2D: } \partial_t \rho + \operatorname{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0}$$

$$\mathbf{v} = -\frac{\nabla\Phi(x,y)}{\|\nabla\Phi(x,y)\|} - \eta\frac{\nabla\Psi(\rho)}{\sqrt{1+\|\nabla\Psi(\rho)\|^2}}$$



$$\boxed{\boxed{\text{2D: } \partial_t \rho + \mathrm{div}\left(\rho\,(1-\rho)\,\mathbf{v}\right) = 0}}$$

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`http://dm.ing.unibs.it/rinaldo/panic/disco03.avi`