Quantum complexity theory

A very brief introduction, with some topology

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Classical complexity

Some complexity classes (more or less formally)

Definition (Computing)

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  Fix a language $L \in \mathbf{P}$; there exists an "algorithm" and a polynomial p s.t. for any input $x \in \{0,1\}^n$, the algorithm returns *yes* if $x \in L$, and *no* otherwise, in at most $p(n)$ logical operations.

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- **NP**: *class of problems L where the yes-instances can be solved in polynomial time, when given a hint of polynomial size.*

  Ex: given a graph G, is there a Hamiltonian cycle (cycle visiting all nodes exactly once) ?
  $\longrightarrow$ the hint is the sequence of nodes forming the Hamiltonian cycle (in a *yes*-instance).
  Hard to find one, easy to verify one is valid.

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- **#P**: *class of counting problems.*

  Ex: given a graph, how many Hamiltonian cycles are there ?

Directly from the definitions: $\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{P}^{\#\mathbf{P}}$.

Quantum computing

Quantum states

Consider the $\mathbb{C}$-vector space $\mathbb{C}^2$ of basis: $|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Definition (Qubits)

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$\longrightarrow$ so far, quite similar to classical words $x \in \{0,1\}^n$.

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Definition (Quantum state)

A quantum state on n qubits is the superposition:

$$\alpha_0 |0\rangle + \cdots + \alpha_{2^n-1} |2^n - 1\rangle, \quad \sum_{j=0}^{2^n-1} |\alpha_j|^2 = 1,$$

or equivalently a norm 1 vector of $\mathbb{C}^{2^n}$.

Quantum gates & measurement

A quantum gate is a **unitary matrix** $U : \mathbb{C}^{2k} \to \mathbb{C}^{2k}, \quad \text{i.e., } U^{-1} = U^*.$

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$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ swaps bits,} \qquad R_\phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \text{ phase gate}$$

Hadamard $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $\quad H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, HH|0\rangle = |0\rangle$.

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Example on 2 qubits, in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$\rightarrow$ flips the second **target** qubit iff the first **control** qubit is 1.

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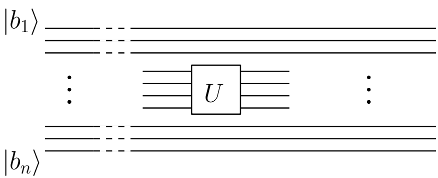
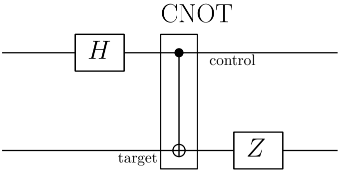
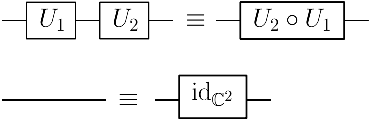
Measuring a quantum state $\sum_{j=0}^{2^n-1} \alpha_j |j\rangle$ returns the (non-superposed) state $|j\rangle$ with probability $|\alpha_i|^2$. (there exist more general projections)

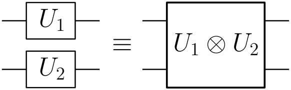
Quantum circuits

Horizontal collection of wires, one per qubit, on which gates are applied from left to right, followed by a final measurement.









can be mixed with "classical" computations.

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  Naturally, **P** $\subseteq$ **BQP**.

- There is a quantum counterpart to **NP** called **QMA**, with a rich theory of complexity (complete problems, etc). The hint is a quantum state.

The Solovay-Kitaev theorem

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Theorem (Solovay-Kitaev)

Let G be a **finite** *set of elements (and their inverses) of* $\mathbf{SU(d)}$*, and assume the group* $\langle G \rangle$ *they generate is* **dense** *in* $\mathbf{SU(d)}$*.*
Then, there exists a constant c such that, for any $\varepsilon > 0$ and element $M \in \mathbf{SU(d)}$, there exists $O(\log^c(1/\varepsilon))$ many elements $U_1, \dots, U_{O(\log^c(1/\varepsilon))}$ of G such that:

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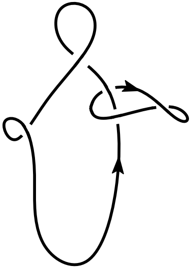
$\Longrightarrow$ there are finite sets of gates dense in $\mathbf{SU(2)}$. Several finite sets of gates are used depending on applications.

Quantum topology

Penrose functor: Diagram → (algebraic) invariant
[Sketch of construction]



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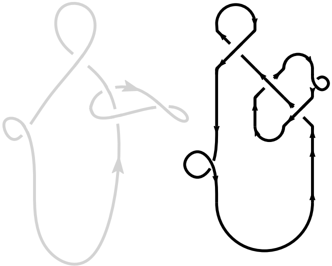

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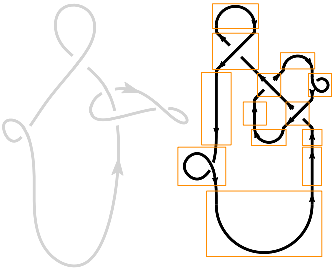

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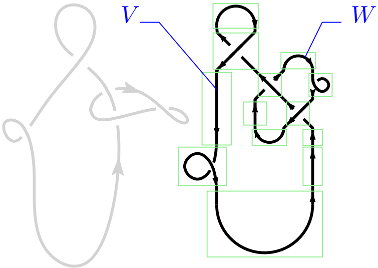

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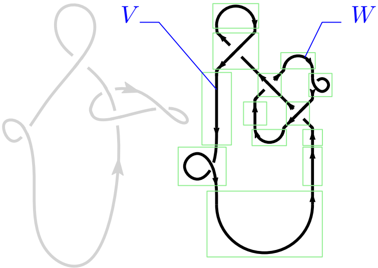

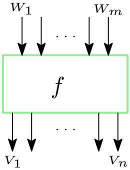

$$f\colon V_1 \otimes \ldots \otimes V_n \to W_1 \otimes \ldots \otimes W_m$$

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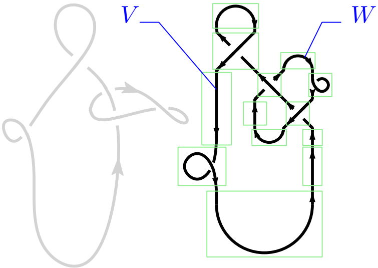

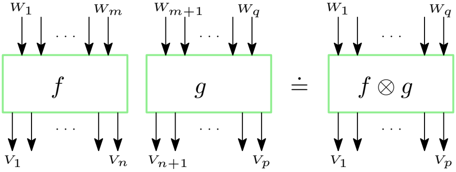

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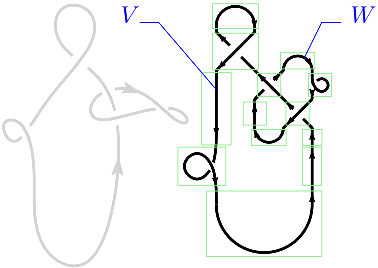

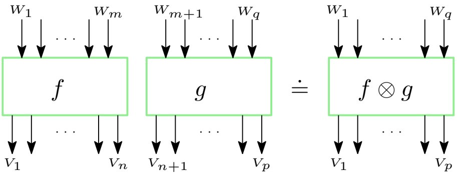

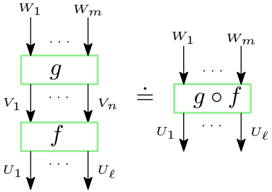

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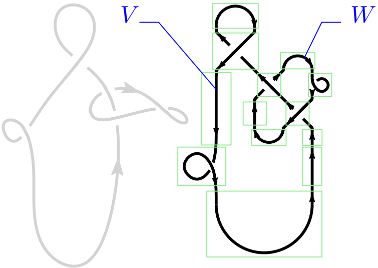

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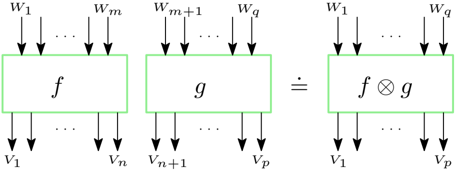

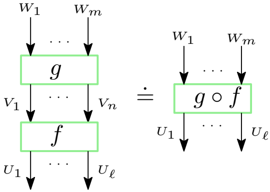

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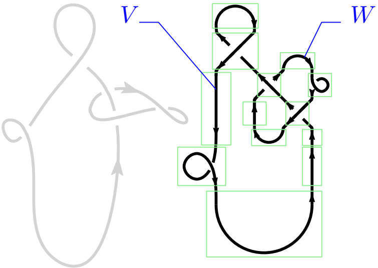

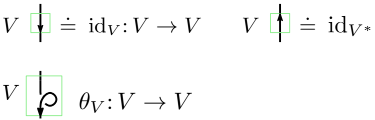

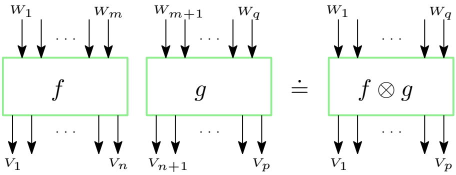

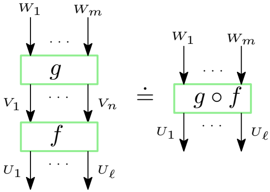

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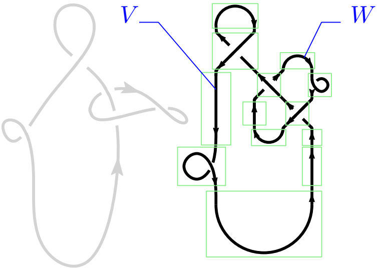

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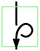
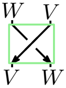
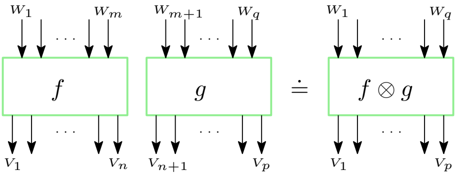
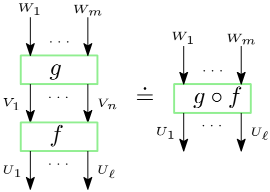
V  $\theta_V\colon V \to V$



$c_{V,W}\colon V \otimes W \to W \otimes V$

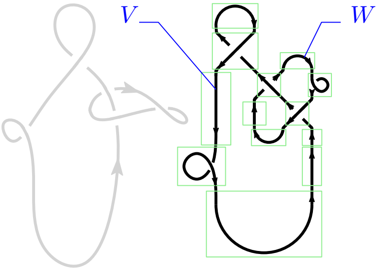


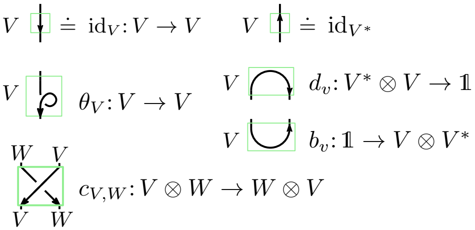


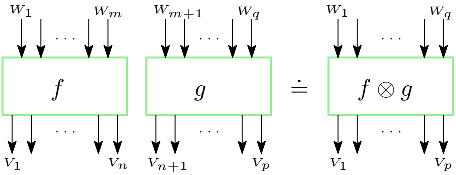
$f \otimes g\colon V_1 \otimes \ldots \otimes V_p \to W_1 \otimes \ldots W_q$

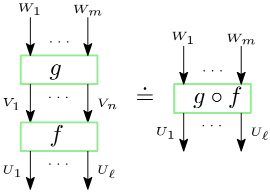
$g \circ f\colon U_1 \otimes \ldots \otimes U_\ell$

Penrose functor: Diagram $\to$ (algebraic) invariant
[Sketch of construction]









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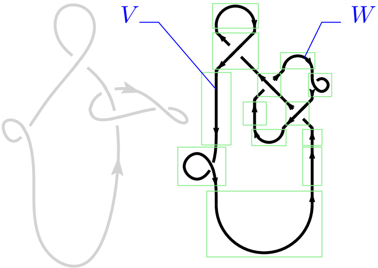
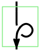


V  $\doteq \mathrm{id}_V\colon V \to V$ V  $\doteq \mathrm{id}_{V^*}$

V  $\theta_V\colon V \to V$

V  $d_v\colon V^* \otimes V \to \mathbb{1}$

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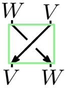
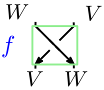
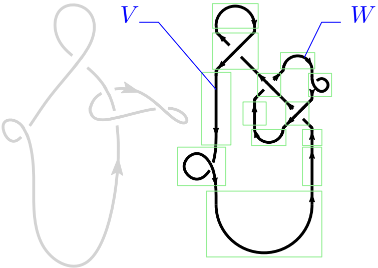


$c_{V,W}\colon V \otimes W \to W \otimes V$



Penrose functor: Diagram $\to$ (algebraic) invariant
[Sketch of construction]



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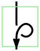
V  $\theta_V \colon V \to V$

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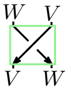
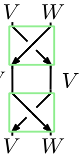
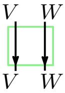


$c_{V,W} \colon V \otimes W \to W \otimes V$

$c_{W,V}$ 

f

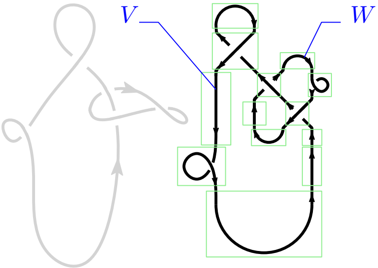
$\doteq$

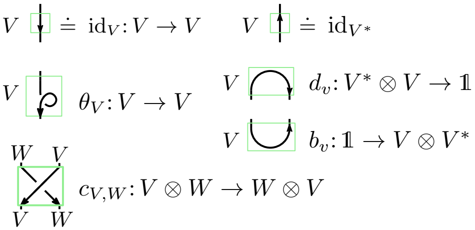


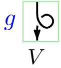
$\mathrm{id}_{V \otimes W}$

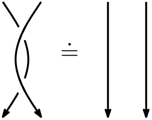
 $\doteq$ 

Penrose functor: Diagram $\to$ (algebraic) invariant
[Sketch of construction]









Penrose functor: Diagram → (algebraic) invariant
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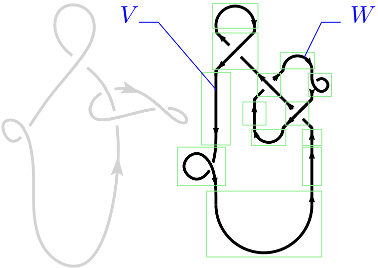

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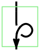
V  $\theta_V\colon V \to V$

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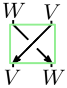

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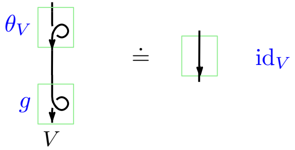
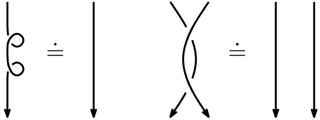
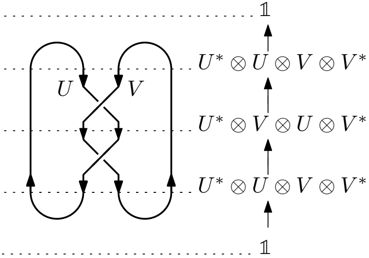




Penrose functor: Diagram $\to$ (algebraic) invariant



$d_U \otimes d_{V^*}$

$\mathrm{id}_{U^*} \otimes c_{U,V} \otimes \mathrm{id}_{V^*}$

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$b_{U^*} \otimes b_V$

Theorem (Reshetikhin, Turaev)

A ribbon category associates to every coloured *ribbon diagram a morphism* $\mathbb{1} \to \mathbb{1}$. *It is an isotopy invariant.*

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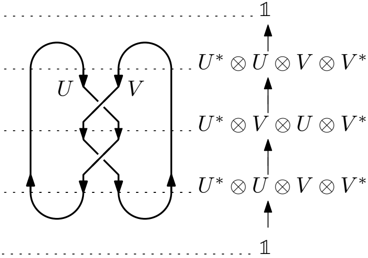

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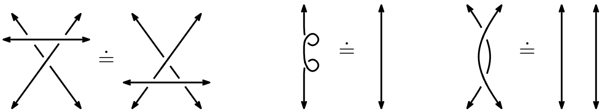
Proof: any isotopy of ribbon diagrams may be described by a sequence of *Reidemeister moves*. $\longrightarrow$ inv. by design.



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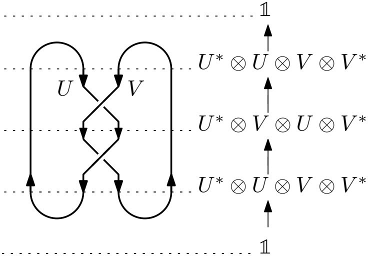

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These invariants have a rich computational complexity, around the classes **#P** and **BQP**. They create links between classical and quantum complexity, and offer topological tools for their study.

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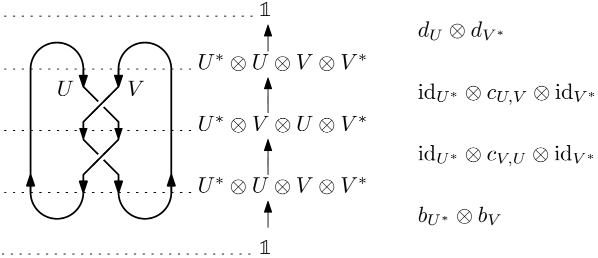

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$$\mathbf{BQP} \quad \subseteq \quad \mathbf{PostBQP} \quad = \mathbf{PP} \quad \text{and} \quad \mathbf{P}^{\mathbf{PP}} = \mathbf{P}^{\#\mathbf{P}}$$

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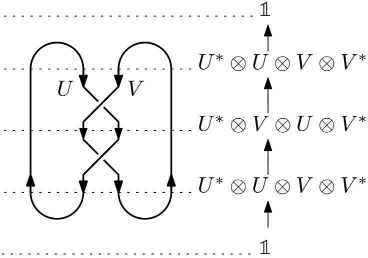

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Thank you!