Séminaire d'algèbre, topologie et géométrie
Jeudi 7 décembre à 14h
Salle I

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On the Hilbert function of general fat points in $\mathbb{P}^1 \times \mathbb{P}^1$

Polynomial interpolation problems have been largely studied in algebraic geometry and commutative algebra. The classical question is the following : how many independent conditions a general union of fat points in the projective space $\mathbb{P}^n$ give on the complete linear system of hypersurfaces of given degree ? The case of double points has a very long history which goes back to the classical school of algebraic geometry of XIX century, but a complete solution has been given by J. Alexander and A. Hirschowitz in 1995, after a series of enlightening papers where they introduced to powerful méthode d'Horace différentiel. For higher multiplicities, even the case of planar curves is in general open. A conjectural answer in this case is given the so-called SHGH Conjecture (due to B. Segre, B. Harbourne, A. Gimigliano and A. Hirschowitz).

In this talk, we consider a multi-graded version of the classical question. We take ideals defining schemes of fat points (with same multiplicity and generic support) in $\mathbb{P}^1 \times \mathbb{P}^1$ and we want to compute how many independent conditions they impose on the linear system of curves of given bi-degree. In 2005, M.V. Catalisano, A.V. Geramita and A. Gimigliano introduced a so-called multiprojective-affine-projective method that reduces this problem to the standard graded case of fat points in $\mathbb{P}^2$. In their work, they completely solve the case of double points in $\mathbb{P}^1 \times \mathbb{P}^1$. After a historical introduction and after explaining the Horace method and the multiprojective-affine-projective method, I will present a joint work with M.V. Catalisano and E. Carlini where we use these methods to give a complete answer in the case of triple points in $\mathbb{P}^1 \times \mathbb{P}^1$. Partial results for higher multiplicity will also be presented.