

# Séminaire d'algèbre, géométrie et topologie

## Jeudi 17 mars à 14h

### Salle I

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Madrid

*Composing iterated integrals :  
the Hopf algebra of the feedback loop in nonlinear control theory*

In this talk we review recent developments on the combinatorial and algebraic structures underlying the interconnection theory of Fliess operators. Chen-Fliess functional expansions, as they are also known, play a central role in the theory of nonlinear control systems. Aiming at representing systems in terms of subsystems, Fliess operators are combined in several ways through interconnections. The four basic system interconnections found in most control systems are the parallel, product, cascade and feedback connections. The latter, also known as a feedback loop, is one of the building blocks of modern control theory. Roughly speaking, it can be described as feeding the output of one system into another system, the output of which is then fed back into the first one.

Through the work of Gray and Duffaut Espinosa, it has become clear that a composition of Chen-Fliess functional expansions characterizes a feedback loop and can be considered a natural generalization of classical composition of functions. As a result, a Faà di Bruno type Hopf algebra has emerged. Foissy made a crucial contribution by proposing an appropriate grading of this Hopf algebra. A combinatorial description of the feedback loop by realizing its (co)algebraic structures in terms of combinatorial operations on rooted circle trees has been described recently. A Zimmermann type forest formula for the antipode is also given.

This talk is based on joint work with W.S. Gray and L.A. Duffaut Espinosa.