Séminaire d'algèbre, topologie et géométrie
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Salle de conférences

Pablo González-Mazón

INRIA Sophia Antipolis

Trilinear birational maps in dimension three

A 3-dimensional trilinear rational map is a rational map

$$\begin{array}{rcl} \phi : \mathbb{P}^1_{\mathbb{C}} \times \mathbb{P}^1_{\mathbb{C}} \times \mathbb{P}^1_{\mathbb{C}} & \dashrightarrow & \mathbb{P}^3_{\mathbb{C}} \\ (x_0 : x_1) \times (y_0 : y_1) \times (z_0 : z_1) & \mapsto & (f_0 : f_1 : f_2 : f_3) \end{array}$$

where $f_i = f_i(x_0, x_1, y_0, y_1, z_0, z_1)$ is a trilinear polynomial. If it admits an inverse rational map $\phi^{-1} : \mathbb{P}^3_{\mathbb{C}} \dashrightarrow (\mathbb{P}^1_{\mathbb{C}})^3$, we say that ϕ is a trilinear birational map. Trilinear birational maps have a special interest in Computer-Aided Geometric Design, as they ensure global injectivity and the existence of an inverse has numerical advantages.

In the first part of this talk, we analyze the syzygies of the f_i's. We provide the list of all the possible minimal tri-graded free resolutions of the ideal generated by the f_i's. Moreover, we explain that the birationality of these maps can be exclusively decided from the first syzygies, and therefore using methods from linear algebra.

On the other hand, we discuss geometric aspects of trilinear birational maps. Namely, the set of classes up to automorphism of $\mathbb{P}^3_{\mathbb{C}}$ of trilinear birational maps can be endowed with the structure of an algebraic subset of some Grassmannian, and we describe its irreducible components. More interestingly, we give the complete classification of the isomorphism classes of the possible base loci of ϕ.

For the second part, we review the technical steps yielding the classification of the base loci. The strategy relies on the factorization of ϕ as the composition of a canonical rational map $\zeta : (\mathbb{P}^1_{\mathbb{C}})^3 \to \mathbb{P}^n_{\mathbb{C}}$, for some $n > 3$, and a linear projection $\pi : \mathbb{P}^n_{\mathbb{C}} \to \mathbb{P}^3_{\mathbb{C}}$. We explain how the group action given by $\mathrm{Aut}(\mathbb{P}^1_{\mathbb{C}})^3$ on $(\mathbb{P}^1_{\mathbb{C}})^3$ extends to the image of ζ. This new group action determines finitely many orbits, which correspond to the isomorphism classes of the base loci.