Séminaire Commun Probabilités et Statistique - Géométrie, Analyse et Dynamique

Mardi 16 Mars à 15h30

ZOOM

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Bijective enumeration of planar bipartite maps with three tight boundaries

Maps, in the combinatorial sense, are graphs drawn on surfaces without edge crossings, and considered up to continuous deformation. Their enumerative theory was initiated by Tutte in the 1960's, and they then appeared in several branches of mathematics, theoretical physics and computer science. What especially motivates their study is that they model discrete random bidimensional geometries, and this point of view has been the subject of intense research in combinatorics and probability theory over the last 20 years.

After a general review of this field of research and of some of its current challenges, I will then focus on a recent work done in collaboration with Emmanuel Guitter and Grégory Miermont. We consider planar maps with three boundaries, colloquially known as pairs of pants. In the bipartite case, a simple expression for their generating function was found by Eynard using the framework of topological recursion. This expression was then proved bijectively by Collet and Fusy. We discovered an even simpler formula for "tight pairs of pants", namely such maps whose boundaries have minimal length in their homotopy class. Even though our formula is equivalent to the Eynard-Collet-Fusy formula, we provide a direct bijective proof. Our construction is reminiscent of a similar construction in hyperbolic geometry, which consists in building a pair of pants (with its hyperbolic metric) by pasting two ideal triangles. Our work is a first step in a long-term project which aims at getting a better geometric understanding of maps of arbitrary topologies, and at developing a combinatorial theory of topological recursion.